

mass in the universe. The equation of motion of the center of mass (position vector \mathbf{R}_i) of the vehicle and proof body in an inertial reference frame is

$$d^2\mathbf{R}_i/dt^2 = \mathbf{F}/(M+m) + \mathbf{g}, \tag{1}$$

where M is the mass of the vehicle and m that of the proof body. The equation of motion of the proof body in a non-rotating reference frame is

$$(\mu/m)(d^2\mathbf{r}/dt^2) + (\mathbf{L}/m) = -\mathbf{F}/(M+m), \tag{2}$$

where \mathbf{r} is the radius vector of the proof body relative to an origin fixed in the vehicle and μ is the reduced mass of the proof body relative to the vehicle. The quantity \mathbf{b}_{obs} , defined by

$$\mathbf{b}_{\text{obs}} = (\mu/m)(d^2\mathbf{r}/dt^2) + (\mathbf{L}/m), \tag{3}$$

is dynamically measurable within the vehicle (assuming m and μ known), and an instrument designed to measure it is usually termed an accelerometer. The quantity \mathbf{b} , defined by

$$\mathbf{b} = \mathbf{F}/(M+m), \tag{4}$$

is the acceleration of the vehicle (and proof body) due to external non-gravitational forces. Equation (2) states:

$$\mathbf{b}_{\text{obs}} = -\mathbf{b}, \tag{5}$$

or, in words: A vehicle-borne accelerometer measures the negative of the acceleration due to external non-gravitational forces. From the fact that Eq. (2) does not contain \mathbf{g} , it is clear that an accelerometer can never measure \mathbf{g} directly. Equation (6) below shows that an accelerometer (or gravimeter) can determine \mathbf{g} indirectly by measuring the negative of the equilibrant non-gravitational forces per unit mass when the vehicle is unaccelerated (but cannot determine the existence of the equilibrium).

Returning to Eq. (1), one obtains

$$(d^2\mathbf{R}_i/dt^2) - \mathbf{g} = \mathbf{b}(t) \tag{6}$$

as the equation of motion of the vehicle. In this equation, only the term \mathbf{b} (a function of time if a clock is carried) is an observable from the standpoint of the observer within the vehicle. To solve Eq. (6) for \mathbf{R}_i , the observer must know the mathematical form of the point function \mathbf{g} , and likewise that of the centrifugal and Coriolis accelerations corresponding to the earth's rotation, in their dependence on position (and velocity) coordinates in space. In a geocentric reference frame fixed in the earth (an approximately inertial frame), Eq. (6) becomes

$$(d^2\mathbf{R}/dt^2) + g_0(R_0^2/R^3)\mathbf{R} = \mathbf{b}(t), \tag{7}$$

where \mathbf{R} is the position vector of the vehicle and only the most significant term in the gravitational acceleration (that due to the earth considered as an equivalent sphere of radius R_0 and gravitational acceleration on its surface of g_0) has been retained. To determine \mathbf{R} , the vehicle-borne observer must solve the differential equation (7) subject to the appropriate initial conditions on vehicle position and velocity.

The fundamental equation (7) is non-linear, and, except in special cases, can be solved only approximately or numerically. If \mathbf{b} is zero, for example, it becomes the (soluble) differential equation of a rocket in an elliptic

trajectory. For a vehicle initially launched vertically from the earth's surface, the linearized solutions in a reference frame with the y axis vertical, the x axis horizontal, and origin on the earth's surface are

$$x = (v_{x,0}/\omega_x) \sin\omega_x t + (1/\omega_x) \int_0^t b_x(\tau) \sin\omega_x(t-\tau) d\tau, \tag{8a}$$

$$y = (v_{y,0}/\omega_y) \sinh\omega_y t - (2g_0/\omega_y^2) \sinh^2(\omega_y t/2) + (1/\omega_y) \int_0^t b_y(\tau) \sinh\omega_y(t-\tau) d\tau, \tag{8b}$$

where $v_{x,0}$ and $v_{y,0}$ are initial velocities, b_x and b_y are components of \mathbf{b} , $\omega_x = (g_0/R_0)^{1/2}$ and $\omega_y = (2g_0/R_0)^{1/2}$. The zero-order solutions corresponding to a flat earth are obtained from Eqs. (8a) and (8b) by letting $R_0 \rightarrow \infty$, which yields the familiar

$$x = v_{x,0} t + \int_0^t \int_0^\tau b_x(\lambda) d\lambda d\tau, \tag{9a}$$

$$y = v_{y,0} t - \frac{1}{2} g_0 t^2 + \int_0^t \int_0^\tau b_y(\lambda) d\lambda d\tau. \tag{9b}$$

A correction factor of type corresponding to Eqs. (9a) and (9b) was used in connection with the accelerometer which fixed the fuel cut-off point of the German V-2 rocket.^{3,4}

If Einstein's equivalence principle is formulated analytically⁵ as

$$\mathbf{g}_{\text{obs}} = \mathbf{g} - (d^2\mathbf{R}_i/dt^2), \tag{10}$$

where \mathbf{g}_{obs} is the apparent gravitation and $(d^2\mathbf{R}_i/dt^2)$ is the acceleration of the observer's reference frame, this formulation is the same as Eq. (7) with the identifications

$$\mathbf{g}_{\text{obs}} = \mathbf{b}_{\text{obs}} = -\mathbf{b}, \tag{11}$$

and

$$\mathbf{g} = -g_0(R_0^2/R^3)\mathbf{R}. \tag{12}$$

The equivalence principle is frequently interpreted² to imply that no dynamic experiment made by an observer within a windowless box can discriminate between a gravitational field due to attracting matter and the apparent field due to an acceleration of the box. This categorical interpretation is obviously too restrictive, since the procedure outlined above of solving Eq. (7) for \mathbf{R} , which yields \mathbf{g} , is always possible unless the proviso be made that the hypothetical observer is confined to the windowless box *throughout all time*.

¹ A. Einstein, *Ann. d. Physik* **35**, 898 (1911).
² F. K. Richtmyer and E. H. Kennard, *Introduction to Modern Physics* (McGraw-Hill Book Company, Inc., New York, 1942), third edition, p. 151.
³ W. G. A. Perring, *J. Roy. Aero. Soc.* **50**, 483 (1946).
⁴ T. M. Moore, *Elec. Eng.* **65**, 303 (1946).
⁵ M. Born, *Die Relativitätstheorie Einsteins* (Verlag Julius Springer, Berlin, 1920), p. 208.

Erratum: Does the Electron Have an Intrinsic Magnetic Moment?

[Phys. Rev. **72**, 984 (1947)]

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A LETTER to the Editor with above title has been published.¹ It has been stated by Schwinger² that the writer's conclusions regarding the magnitude of the ex-

pected effect for s terms disagree with Schwinger's, which were based on a development of quantum electrodynamics. A re-examination of the writer's calculations showed that an error was made in a mathematical transformation. The corrected result is in agreement with Schwinger's as well as with the now established connection of the anomalously large hyperfine structure of the ground state of hydrogen and deuterium³ with the anomalous magnetic moment of the electron spin.⁴

¹ G. Breit, *Phys. Rev.* **72**, 984 (1947).

² Julian Schwinger, *Phys. Rev.* **73**, 415 (1948).

³ J. E. Nafe, E. B. Nelson, and I. I. Rabi, *Phys. Rev.* **71**, 914 (1947); D. E. Nagel, R. S. Julian, and J. R. Zacharias, *Phys. Rev.* **72**, 971 (1947).

⁴ P. Kusch and H. M. Foley, *Phys. Rev.* **72**, 1256 (1947); H. M. Foley and P. Kusch, *Phys. Rev.* **73**, 412 (1948).

Erratum: Neutron-Proton Scattering at 100 Mev

[*Phys. Rev.* **73**, 641 (1948)]

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WE are sorry to report two misprints which occurred in our Letter to the Editor under the above title.

For the Yukawa potential the singlet range should read 1.18×10^{-13} cm instead of 1.8×10^{-13} cm and is, as stated, the same as the triplet range. The cross section at 100 Mev in the Born approximation is 0.101×10^{-24} cm² and not 0.111×10^{-24} cm².

Note on the "Natural Radioactivity of Rhenium"

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IN examining the isobaric pair Re^{187} – Os^{187} for radioactivity, Naldrett and Libby¹ found an activity in Re which they attributed to β^- -radiations from Re^{187} . The range of the particles was 3.5 mg Al/cm² with a corresponding upper energy limit of 43 kev. The samples were purified sufficiently to separate other elements with the exception of the chemically similar Tc (element 43). Since it appeared possible to the authors that the radiations observed might originate in a natural isotope of Tc present in the Re, the Re was subjected to chemical operations which were shown in tracer studies to free it of Tc, and then measured. The activity was essentially unchanged. The details of the chemical separation of Re from Tc and the results of the activity measurements follow.

The Tc tracer used was prepared by irradiation of ammonium molybdate in the "thimble" of the Argonne heavy water pile. The 5.9-hr. Tc^{99} was isolated by using a modification of the radiochemical procedure of Glendenin,²

The Tc activity is distilled from fuming H_2SO_4 in an air stream. Re carrier is added to the distillate and Re_2S_7 precipitated. After solution of the precipitate in conc. HNO_3 and two evaporations with 3 ml conc. HCl, the solution is diluted to 20 ml with H_2O . The solution is neutralized with NH_4OH and four precipitations of

TABLE I. Activity measurements on Re and purified Re_2S_7 .

Sample	Area (cm ²)	Sample thickness (mg/cm ²)	Activity (c/m)	Half-life Re^{187} (yrs.)
Re No. 1	260	4.2	38.5 ± 1.5	6.2×10^{12}
Re No. 2	170	5.0	39.3 ± 1.6	4.0×10^{12}
Re_2S_7 (No. 1a)	240	2.9	21.5 ± 1.4	6.3×10^{12}
Re_2S_7 (No. 1b)	250	2.9	20.4 ± 1.5	6.9×10^{12}

$\text{Fe}(\text{OH})_3$ are performed. The level of Tc activity in the tracer solution is determined in an aliquot, more Re carrier is added, and tetraphenylarsonium perrhenate precipitated. The radiochemical purity of the samples was shown to be very high (>99 percent Tc activity) by decay and absorption measurements.

Two methods of separating Re from Tc are given by Perrier and Segré.³ These consist of distilling Re but not Tc from H_2SO_4 solution in a stream of moist HCl, and precipitating the sulfide of Re alone from 10N HCl. In studying the separation of Re from Tc tracer by sulfide precipitation, another method was found in which Tc is volatilized from a solution by alternate evaporations with HNO_3 and HCl. After five HNO_3 –HCl cycles less than 2 percent of the Tc remained while the loss of Re was negligible.

The separation was tested by adding an aliquot of the Tc tracer to two HNO_3 solutions, each containing ~ 0.6 g Re. The solutions were evaporated almost to dryness. Two successive evaporations were then performed with 15 ml conc. HCl. Alternate evaporations of one addition of 10 ml conc. HNO_3 and two of 15 ml conc. HCl were performed until five such cycles had been completed. Finally, the solutions were diluted to 125 ml, aliquots removed, neutralized, and tetraphenylarsonium perrhenate precipitated. A comparison of the activity in the precipitates to that of the activity added, after correction for the fraction of Re counted, showed that more than 98 percent of the Tc had been removed.

Two one-gram samples of metallic Re (British Drug Houses, Ltd.) were dissolved in conc. HNO_3 and subjected to the separation procedure described above. Although macro-Tc might be expected to behave differently from tracer Tc in some chemical operations, it is probable that Tc, if present, would have been separated from Re. The two Re samples were converted to Re_2S_7 , counted, and the activity compared to that of untreated Re metal.

Activity measurements were made in the "screen-wall" counter used by Naldrett and Libby. Anticoincidence shielding added to the counter by Mr. E. C. Anderson reduced the background from 160 c/m to 50 c/m and greatly shortened the counting procedure. Samples of metallic Re (Re No. 1, British Drug Houses, Ltd.; Re No. 2,