

counter shield. A counting rate of several thousand counts per minute was to be expected from the value of the half-life given by Lougher and Rowlands. The actual counting rate after subtraction of background was  $2.9 \pm 0.2$  counts per minute for the xenon counter and  $2.6 \pm 0.2$  counts per minute for the argon-filled counter. These very small counting rates are equal within the limits of statistical accuracy and are due to beta-rays of approximately 1.7-Mev energy, from a trace of radioactive contamination. Hence, no activity caused by  $K$  capture of  $\text{Os}^{187}$  is indicated. This negative result leads to an estimated minimum value of  $4 \times 10^{12}$  years for the half-life of  $\text{Os}^{187}$  so far as possible  $K$  capture is concerned. Taking into account the counter efficiencies for rhenium  $L$  x-rays and the absorption in the counter walls, a minimum value for the half-life in the case of  $L$  capture is estimated at  $0.6 \times 10^{12}$  years.

The above result, showing the absence of x-rays due to an orbital electron capture of  $\text{Os}^{187}$ , is in agreement with the results of Nalderett and Libby, who, using counters filled with argon and  $\text{OsO}_4$ , respectively, showed that no Auger electrons are emitted by  $\text{Os}^{187}$ . They showed, moreover, that the other member of the pair,  $\text{Re}^{187}$ , is  $\beta^-$ -unstable.

This experiment was completed in January 1948.

\* Work performed under Navy Contract N6ori-222, Task Order I.

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<sup>1</sup> Nalderett and Libby, *Phys. Rev.* **73**, 487 (1948).

<sup>2</sup> Scherrer and Zingg, *Helv. Phys. Acta* **12**, 283 (1939); Zingg, *Helv. Phys. Acta* **13**, 219 (1940).

<sup>3</sup> Lougher and Rowlands, *Nature* **153**, 374 (1944).

reasonable ratio  $R$ , which is found experimentally to be  $\sim 3$  in the 100-Mev region.<sup>3</sup> Inclusion of the tensor force will modify these results. A preliminary estimate, based on the Born approximation, predicts that  $R$  will be reduced by

TABLE I.

	$\sigma$ Total cross section $\times 10^{24}$ cm <sup>2</sup>	$R$ Ratio of intensity at 180° to that at 90° in the c.g. system
Yukawa-exact	0.140	6.81
Yukawa-born approximation	0.150	12.5
Square well (range $2.8 \times 10^{-13}$ cm) <sup>2</sup>	0.111	159

about a factor of 2, while the total cross section is increased slightly.

The constants for the tensor force case with a Yukawa potential have been calculated and tentative values are:  $\gamma B\mu = 85 \pm 2$  Mev,  $(1-2g)B\mu = 46.5$  Mev,  $B\mu = 0 \pm 3$  Mev, in the Rarita-Schwinger notation. The exact calculation of the high energy cross section for the tensor force case is being carried out.

\* National Research Council Predoctoral Fellow.

<sup>1</sup> R. G. Sachs and M. Goeppert-Mayer, *Phys. Rev.* **53**, 991 (1938); L. E. Hoisington, S. S. Share, and G. Breit, *Phys. Rev.* **56**, 884 (1939).

<sup>2</sup> M. Camac and H. Bethe, *Phys. Rev.* **73**, 191 (1948).

<sup>3</sup> J. Hadley, C. Leith, H. York, E. Kelly, and C. Wiegand, *Bull. Am. Phys. Soc.* **23**, 15 (1948).

## High Energy Neutron-Proton Scattering

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WE have investigated the scattering of high energy neutrons by protons with the interaction potential given by the symmetrical meson theory with the tensor force omitted:

$$V = \frac{1}{3} [1 - \frac{1}{2}g + \frac{1}{2}g\sigma_1 \cdot \sigma_2] \tau_1 \cdot \tau_2 \sigma_1 \cdot \sigma_2 (\mu B) (e^{-\mu r} / \mu r)$$

$\sigma_1, \sigma_2, \tau_1, \tau_2$  are the usual spin and isotopic spin operators. The constants  $B, g,$  and  $\mu$  were chosen to give the correct binding energy of the deuteron and low energy neutron-proton scattering.<sup>1</sup> The values taken were  $B\mu = 67.8$  Mev,  $1/\mu = 1.18 \times 10^{-13}$  cm,  $g = 0.157$ . [In order to fit the low energy proton-proton cross section,  $g$  should be taken as 0.162.]

The method of phase shifts was used throughout since it was found that the Born approximation gives unreliable results especially with respect to the angular distribution, as has been stressed by Camac and Bethe.<sup>2</sup> At 2.2 Mev and 20 Mev the results are substantially the same as those obtained from the corresponding square well potential. The results at 80 Mev are cited in Table I, with the square well and Born approximation figures given for comparison.

One sees that the Yukawa potential gives an appreciably higher cross section than the square well, but a much more

## Einstein's Equivalence Principle and the Problem of Blind Navigation

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THE importance of Einstein's equivalence principle<sup>1</sup> in the problem of blind navigation of aerial or space vehicles has been appreciated<sup>2</sup> for a long time. A formulation of the limitations this principle imposes in the practical solution of the problem has, however, never been published. The following discussion assumes a vehicle having no radiation connection with the earth and confining an observer who is posed with the problem of determining the vehicle's position with respect to the earth purely by dynamic measurements. A dynamic measurement is defined as a force measurement on a proof body, or a measurement of acceleration, velocity, or displacement on such a body. The gravitational field in the neighborhood of the vehicle is assumed locally uniform, and Newtonian mechanics is assumed.

The forces acting on the vehicle can be analyzed into three sets: forces, whose sum is  $\mathbf{F}$ , due to external, non-gravitational forces on the vehicle; forces, whose sum is  $\mathbf{L}$ , due to the reaction on the vehicle of the internal forces exerted on a proof body; and forces, whose sum (per unit mass) is  $\mathbf{g}$ , due to the gravitational attraction of all other

mass in the universe. The equation of motion of the center of mass (position vector  $\mathbf{R}_i$ ) of the vehicle and proof body in an inertial reference frame is

$$d^2\mathbf{R}_i/dt^2 = \mathbf{F}/(M+m) + \mathbf{g}, \tag{1}$$

where  $M$  is the mass of the vehicle and  $m$  that of the proof body. The equation of motion of the proof body in a non-rotating reference frame is

$$(\mu/m)(d^2\mathbf{r}/dt^2) + (\mathbf{L}/m) = -\mathbf{F}/(M+m), \tag{2}$$

where  $\mathbf{r}$  is the radius vector of the proof body relative to an origin fixed in the vehicle and  $\mu$  is the reduced mass of the proof body relative to the vehicle. The quantity  $\mathbf{b}_{\text{obs}}$ , defined by

$$\mathbf{b}_{\text{obs}} = (\mu/m)(d^2\mathbf{r}/dt^2) + (\mathbf{L}/m), \tag{3}$$

is dynamically measurable within the vehicle (assuming  $m$  and  $\mu$  known), and an instrument designed to measure it is usually termed an accelerometer. The quantity  $\mathbf{b}$ , defined by

$$\mathbf{b} = \mathbf{F}/(M+m), \tag{4}$$

is the acceleration of the vehicle (and proof body) due to external non-gravitational forces. Equation (2) states:

$$\mathbf{b}_{\text{obs}} = -\mathbf{b}, \tag{5}$$

or, in words: A vehicle-borne accelerometer measures the negative of the acceleration due to external non-gravitational forces. From the fact that Eq. (2) does not contain  $\mathbf{g}$ , it is clear that an accelerometer can never measure  $\mathbf{g}$  directly. Equation (6) below shows that an accelerometer (or gravimeter) can determine  $\mathbf{g}$  indirectly by measuring the negative of the equilibrant non-gravitational forces per unit mass when the vehicle is unaccelerated (but cannot determine the existence of the equilibrium).

Returning to Eq. (1), one obtains

$$(d^2\mathbf{R}_i/dt^2) - \mathbf{g} = \mathbf{b}(t) \tag{6}$$

as the equation of motion of the vehicle. In this equation, only the term  $\mathbf{b}$  (a function of time if a clock is carried) is an observable from the standpoint of the observer within the vehicle. To solve Eq. (6) for  $\mathbf{R}_i$ , the observer must know the mathematical form of the point function  $\mathbf{g}$ , and likewise that of the centrifugal and Coriolis accelerations corresponding to the earth's rotation, in their dependence on position (and velocity) coordinates in space. In a geocentric reference frame fixed in the earth (an approximately inertial frame), Eq. (6) becomes

$$(d^2\mathbf{R}/dt^2) + g_0(R_0^2/R^3)\mathbf{R} = \mathbf{b}(t), \tag{7}$$

where  $\mathbf{R}$  is the position vector of the vehicle and only the most significant term in the gravitational acceleration (that due to the earth considered as an equivalent sphere of radius  $R_0$  and gravitational acceleration on its surface of  $g_0$ ) has been retained. To determine  $\mathbf{R}$ , the vehicle-borne observer must solve the differential equation (7) subject to the appropriate initial conditions on vehicle position and velocity.

The fundamental equation (7) is non-linear, and, except in special cases, can be solved only approximately or numerically. If  $\mathbf{b}$  is zero, for example, it becomes the (soluble) differential equation of a rocket in an elliptic

trajectory. For a vehicle initially launched vertically from the earth's surface, the linearized solutions in a reference frame with the  $y$  axis vertical, the  $x$  axis horizontal, and origin on the earth's surface are

$$x = (v_{x,0}/\omega_x) \sin\omega_x t + (1/\omega_x) \int_0^t b_x(\tau) \sin\omega_x(t-\tau) d\tau, \tag{8a}$$

$$y = (v_{y,0}/\omega_y) \sinh\omega_y t - (2g_0/\omega_y^2) \sinh^2(\omega_y t/2) + (1/\omega_y) \int_0^t b_y(\tau) \sinh\omega_y(t-\tau) d\tau, \tag{8b}$$

where  $v_{x,0}$  and  $v_{y,0}$  are initial velocities,  $b_x$  and  $b_y$  are components of  $\mathbf{b}$ ,  $\omega_x = (g_0/R_0)^{1/2}$  and  $\omega_y = (2g_0/R_0)^{1/2}$ . The zero-order solutions corresponding to a flat earth are obtained from Eqs. (8a) and (8b) by letting  $R_0 \rightarrow \infty$ , which yields the familiar

$$x = v_{x,0} t + \int_0^t \int_0^\tau b_x(\lambda) d\lambda d\tau, \tag{9a}$$

$$y = v_{y,0} t - \frac{1}{2} g_0 t^2 + \int_0^t \int_0^\tau b_y(\lambda) d\lambda d\tau. \tag{9b}$$

A correction factor of type corresponding to Eqs. (9a) and (9b) was used in connection with the accelerometer which fixed the fuel cut-off point of the German V-2 rocket.<sup>3,4</sup>

If Einstein's equivalence principle is formulated analytically<sup>5</sup> as

$$\mathbf{g}_{\text{obs}} = \mathbf{g} - (d^2\mathbf{R}_i/dt^2), \tag{10}$$

where  $\mathbf{g}_{\text{obs}}$  is the apparent gravitation and  $(d^2\mathbf{R}_i/dt^2)$  is the acceleration of the observer's reference frame, this formulation is the same as Eq. (7) with the identifications

$$\mathbf{g}_{\text{obs}} = \mathbf{b}_{\text{obs}} = -\mathbf{b}, \tag{11}$$

and

$$\mathbf{g} = -g_0(R_0^2/R^3)\mathbf{R}. \tag{12}$$

The equivalence principle is frequently interpreted<sup>2</sup> to imply that no dynamic experiment made by an observer within a windowless box can discriminate between a gravitational field due to attracting matter and the apparent field due to an acceleration of the box. This categorical interpretation is obviously too restrictive, since the procedure outlined above of solving Eq. (7) for  $\mathbf{R}$ , which yields  $\mathbf{g}$ , is always possible unless the proviso be made that the hypothetical observer is confined to the windowless box *throughout all time*.

<sup>1</sup> A. Einstein, *Ann. d. Physik* **35**, 898 (1911).

<sup>2</sup> F. K. Richtmyer and E. H. Kennard, *Introduction to Modern Physics* (McGraw-Hill Book Company, Inc., New York, 1942), third edition, p. 151.

<sup>3</sup> W. G. A. Perring, *J. Roy. Aero. Soc.* **50**, 483 (1946).

<sup>4</sup> T. M. Moore, *Elec. Eng.* **65**, 303 (1946).

<sup>5</sup> M. Born, *Die Relativitätstheorie Einsteins* (Verlag Julius Springer, Berlin, 1920), p. 208.

### Erratum: Does the Electron Have an Intrinsic Magnetic Moment?

[Phys. Rev. **72**, 984 (1947)]

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A LETTER to the Editor with above title has been published.<sup>1</sup> It has been stated by Schwinger<sup>2</sup> that the writer's conclusions regarding the magnitude of the ex-