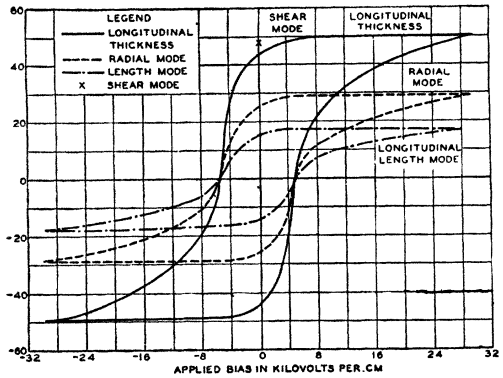

FIG. 1. Measured values of electromechanical coupling as a function of the voltage bias for 4 modes of motion in barium titanate ceramics.

where S_{ij} and T_{kl} are, respectively, the strains and stresses, $\delta_n = D_n/4\pi$ where D_n are the electric displacements, s_{ijkl}^D are the constant displacement elastic compliances, g_{ijn} the piezoelectric constants, Q_{ijn0} the electrostrictive constants, and β_{mn}^T the impermeability constants (i.e., inverse of dielectric constants).

On the assumption that the material is isotropic and non-piezoelectric, these equations reduce to the forms expressed in terms of the usual two index symbols:

$$S_1 = s_{11}^D T_1 + s_{12}^D[T_2+T_3] + Q_{11}\delta_1^2 + Q_{12}[\delta_2^2+\delta_3^2],$$
$$\cdots\cdots\cdots\cdots\cdots\cdots$$
$$S_{12} = \frac{S_6}{2} = [s_{11}^D - s_{12}^D]T_6 + [Q_{11}-Q_{12}]\delta_1\delta_2,$$
$$E_1 = \delta_1[4\pi\beta_{11}^T] - 2[Q_{11}[\delta_1 T_1 + \delta_2 T_6 + \delta_3 T_5] + Q_{12}[\delta_1[T_2+T_3] - [T_6\delta_2 + T_5\delta_3]]], \quad (2)$$
$$\cdots\cdots\cdots\cdots\cdots\cdots$$
$$E_3 = \delta_3[4\pi\beta_{11}^T] - 2[Q_{11}[\delta_3 T_3 + \delta_1 T_5 + \delta_2 T_4] + Q_{12}[\delta_3[T_1+T_2] - [\delta_1 T_5 + \delta_2 T_4]]].$$

Hence as shown by the equation for the shear strain S_6, a shear mode is excited when two electric displacements occur at right angles.

Using these equations, the electromechanical coupling factors have been calculated for the longitudinal length mode, the radial mode, the thickness longitudinal mode, and the thickness shear mode. These are given, respectively, by the formulae

$$k_l = \frac{2Q_{12}\delta_{30}}{(s_{11}^E(4\pi\beta_{11}^T))^{\frac{1}{2}}}; \quad k_r = \left(\frac{2}{1-\sigma}\right)^{\frac{1}{2}} k_l,$$
$$k_t = \frac{2\delta_{30}\left[Q_{11} - \frac{2s_{12}^D}{s_{11}^D + s_{12}^D}Q_{12}\right]}{\left(\frac{4\pi\beta_{11}^T}{c_{11}^E}\right)^{\frac{1}{2}}}; \quad k_s = \frac{2(Q_{11}-Q_{12})\delta_{30}}{\left(\frac{4\pi\beta_{11}^T}{\mu^E}\right)^{\frac{1}{2}}}.$$

Measurements for the coupling of these four modes as a function of the applied voltage are shown by Fig. 1. The fact that the coupling follows a hysteresis loop shows conclusively that the electrostrictive effect is a function of the electric displacement rather than the applied field. The longitudinal and thickness couplings determine the electrostrictive constants to be

$$Q_{12} = -2.15\times10^{-12}\left(\frac{\text{cm}^2}{\text{stat. coulombs}}\right)^2;$$
$$Q_{11} = +6.9\times10^{-12}\left(\frac{\text{cm}^2}{\text{stat. coulombs}}\right)^2.$$

Using these constants, the shear mode for a remanent displacement due to the application of 30,000 volts/cm has a calculated coupling agreeing well with the experimental value, which shows conclusively that the effect follows the second-order electrostrictive equations and not the "quadratic" piezoelectric relations. Since a ferromagnetic magnetostrictive material follows a similar set of equations, it appears more logical to call the effect electrostrictive.

[1] B. Matthais and A. Von Hipple, "Structure, electrical and optical properties of barium titanate," Phys. Rev. **73**, 268 (1948).
[2] H. Mueller, "Properties of rochelle salt IV," Phys. Rev. **58**, 805 (1940).
[3] W. L. Cherry and Robert Adler, "Piezoelectric effect in barium titanate," Phys. Rev. **72**, 981 (1947).
[4] W. P. Mason, "First and second order equations for piezoelectric crystals expressed in tensor form," Bell Sys. Tech. J. **26**, 80 (1947).

Slow Neutron Spectrometer Studies of Oxygen, Nitrogen, and Argon*

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THE Columbia University slow neutron velocity spectrometer[1] is being used to investigate materials in the gaseous phase. Preliminary measurements on several elements are presented in this letter (Figs. 1–3). Following the usual convention, all cross sections are in units of 10^{-24} cm²/atom.

The samples were contained in aluminum alloy cylinders 1 meter long and having 10-cm inside diameters. Pressures up to 75 atmospheres can be used.

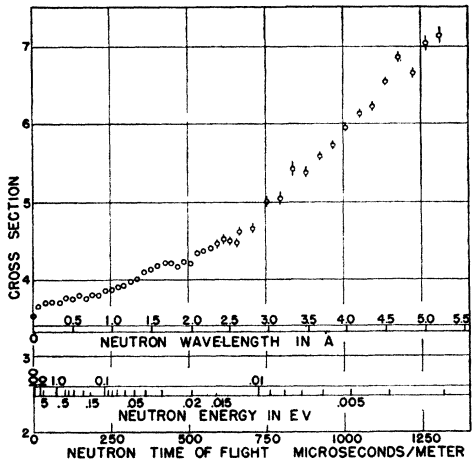

FIG. 1. The slow neutron cross section of oxygen. In each of several runs the sample contained about 8 g/cm², but the exact amount was different for each run. The average of these results is shown.

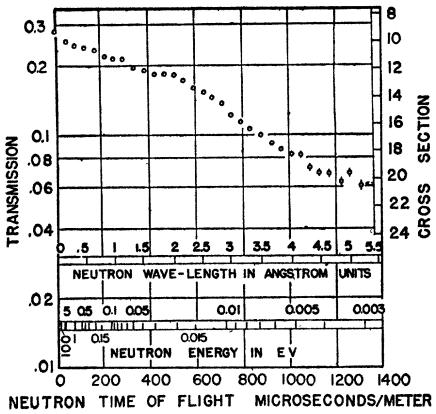

FIG. 2. The slow neutron transmission of 3.168 g/cm² of nitrogen.

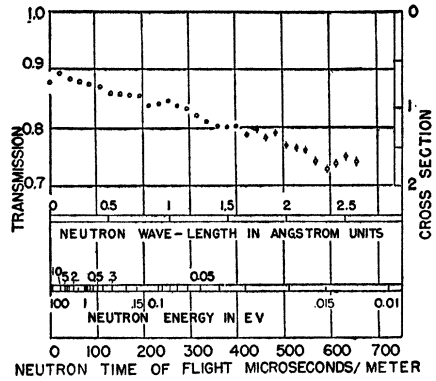

FIG. 3. The slow neutron transmission of 11.71 g/cm² of argon.

In the case of O_2, a beryllium filter (which discriminates against neutrons having energy greater than 0.01 ev) was used at timings greater than 665 microseconds per meter to reduce the small background of faster thermal neutrons. The investigation of nitrogen in this region will be repeated using this filter.

The statistical accuracy is indicated by vertical bars. Where no bars are shown, the statistical accuracy is of the same order as or less than the size of the circles.

The rapid changes in transmission near zero time of flight are due to the resolution width of the instrument which integrates the transmission of the sample over all neutron energies up to 14 Mev.

In addition to nuclear interactions with the neutrons, the following phenomena contribute to the cross section when a gaseous sample is used:

(a) Thermal motion of the gas molecules which causes an increase in cross section with decreasing neutron energy.
(b) Coherent scattering from the atoms in the molecule.
(c) Inelastic collisions (i.e., changes in vibrational and rotational levels of the molecule).

In addition to these effects, part of the increase of the oxygen cross section with decreasing neutron energy is believed to be due to the paramagnetism of oxygen, i.e., to an interaction between the magnetic moments of the oxygen molecules and the neutrons. Professor O. Halpern has separated by theoretical analysis the parts of the cross section corresponding to magnetic and non-magnetic interactions.[2] Oxygen has negligible capture in the thermal region.

The nitrogen cross section for neutron energies greater than 0.025 ev can be represented by

$$\sigma_{\text{thermal}} = [9.60 + 0.48E^{-\frac{1}{2}}] \times 10^{-24} \text{ cm}^2 \text{ per atom},$$

where E is the neutron energy in ev. At least a portion of this slope is attributed to capture.

The argon cross section can be represented by

$$\sigma_{\text{thermal}} = [0.640 + 0.123E^{-\frac{1}{2}}] \times 10^{-24} \text{ cm}^2 \text{ per atom}.$$

Since argon is monatomic so that molecular effects (b) and (c) cannot occur, and since (a) is not appreciable in the region studied, this slope is attributed almost entirely to capture.

Measurements at higher energies with better resolution give the following cross sections per atom for the energy range 15 to 200 ev: O_2, 3.68; N_2, 9.74; A, 0.67.

Fermi and Marshall[3] have reported measurements on the cross section of O_2 and N_2 using BeO filtered pile neutrons having an average wave-length of 5.1A. Their values (O_2, 8.1 per atom and N_2, 23.7 per atom) are about 14 percent higher than those reported here for the same wave-length. The use of a Be filter in the case of N_2 will increase the measured cross section and should decrease considerably or even eliminate the discrepancy between the two results. However, the discrepancy in the case of O_2, presumably outside of experimental error for both measurements, remains unexplained.

* This letter is based on work performed under Contract AT-30-1-Gen-72, and the information covered therein will appear in the Manhattan Project Technical Series as part of the contribution of Columbia University.

[1] Rainwater, Havens, Wu, and Dunning, Phys. Rev. **71**, 65 (1947).
[2] O. Halpern, Phys. Rev. **72**, 746 (1947).
[3] E. Fermi and L. Marshall, Phys. Rev. **71**, 666 (1947).

A Rapid Method for the Determination of the Maximum Energies of β-Emitters with Simple Spectra

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DURING the course of work on the back-scattering of electrons it became apparent that the back-scattering is a sensitive function of the maximum energy of the β-emitter, especially in the lower energy range.

Samples were mounted on Formvar films (50 mg/cm²) and had such specific activity that no solid deposit was visible after the solution was evaporated on the film. Measurements were made using an "end-on" Geiger counter with mica window 2.8 mg/cm². The sample was placed in position about 1 cm from the counter window and counted first without any back-scattering, save for air, and