One of us (A.S.) hopes to return to the electromagnetic case.

[1] A. Storruste and H. Wergeland, Kgl. N. V. Selsk. Forh. (1948) (in print).
[2] Stratton, Morse, Chu, and Hutner, *Elliptic Cylinder and Spheroidal Wave Functions* (1941).
[3] E. A. Hylleraas, Zeits. f. Physik **71**, 739 (1931).

Correlation between the States of Polarization of the Two Quanta of Annihilation Radiation*

E. Bleuler and H. L. Bradt**
Purdue University, Lafayette, Indiana
April 13, 1948

It has been pointed out by J. A. Wheeler[1] that according to pair theory the planes of polarization of the two quanta originating in the annihilation of a positron should be perpendicular to each other. This correlation is the equivalent of angular momentum conservation in the process of annihilation of an electron pair with relative velocity zero in the singlet state. The azimuthal variation of intensity of the simultaneous Compton scattering of the two quanta, resulting from this correlation between their respective states of polarization, has been calculated by Pryce and Ward[2] and by Snyder, Pasternack, and Hornbostel.[3] An experimental verification has been attempted with the aid of the arrangement shown in Fig. 1.

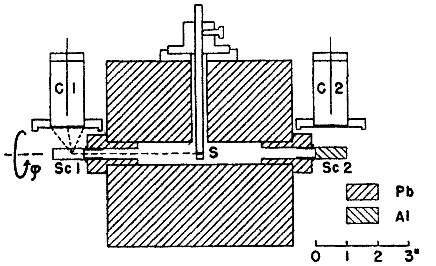

FIG. 1. Coincidence measurement of Compton scattering.

The annihilation radiation of the source S (Cu^{64}, prepared by deuteron irradiation of copper in the Purdue cyclotron) is collimated by a ⅜-in. channel in the lead block. The quanta are scattered by cylindrical aluminum scatterers *Sc* and detected with bell-shaped Geiger counters with lead cathodes. Coincidences were measured for azimuth differences (φ) of 0°, 90°, 180°, and 270° between the counter axes. In order to eliminate all asymmetries both counters were rotated in turn. As a result of absorption in the scatterer the mean scattering angle is slightly less than 90°, near the theoretical maximum of anisotropy calculated for a scattering angle of 82°. Taking into account the finite solid angle subtended by the counters, a ratio $C_\perp/C_\parallel \approx 1.7$ is expected for the coincidence rates at $\varphi = 90°$ ($C_\perp$) and $\varphi = 180°$ ($C_\parallel$). Four different runs were made with different sources consistently showing $C_\perp > C_\parallel$. Data for a characteristic run of 16 hours are given in Table I.

TABLE I.

Average single counts without scatterers	3000/min.
Average single counts with scatterers	5370/min.
Chance coincidences ($T = 1.2 \cdot 10^{-7}$ sec.)	0.117/min.
Genuine coincidences $C_\perp$	0.152/min.
Genuine coincidences $C_\parallel$	0.073/min.
Asymmetry ratio $C_\perp/C_\parallel$	2.1 ±0.64

The observed average asymmetry ratio for all runs is

$$C_\perp/C_\parallel = 1.9_4 \pm 0.3_7.$$

The indicated error is the statistical mean standard deviation. The theoretical prediction is therefore confirmed by this experiment.

* Work done under Navy Contract N6ori-222, Task Order I.
** Now at the University of Rochester, Rochester, New York.
[1] J. A. Wheeler, Ann. N. Y. Acad. Sci. **48**, 219 (1946).
[2] M. H. L. Pryce and J. C. Ward, Nature **160**, 435 (1947).
[3] H. S. Snyder, S. Pasternack, and J. Hornbostel, Phys. Rev. **63**, 440 (1943).

Piezoelectric or Electrostrictive Effect in Barium Titanate Ceramics

W. P. Mason
Bell Telephone Laboratories, Murray Hill, New Jersey
April 13, 1948

In a recent Letter to the Editor,[1] Matthias and Von Hippel have discussed the resonances obtained in a piece of multicrystalline barium titanate ceramic and have called the effect a "quadratic" piezoelectric effect. While the term used is to some extent a matter of definition, it appears worth while to point out that the effect in the titanate ceramic does not conform to the original definition of a "quadratic" piezoelectric effect, but is, on the other hand, the analog of a magnetostrictive effect in a ferromagnetic material.

According to Mueller,[2] a "quadratic" piezoelectric effect is one following the same equations as an electrostrictive effect but depending on a strain caused by a spontaneous polarization or an applied field acting on the piezoelectric constant. The discovery[3] that a shear vibration can be set up when an a.c. field is applied at right angles to a d.c. polarization and the quantitative check between the value of this constant and the radial and thickness constants show that the effect cannot be a "quadratic" piezoelectric effect. This follows from the fact that the only type of symmetry that the ceramic can have in the presence of an applied field is that known as transverse isotropy. For this case the $c = Z$ axis lies along the direction of the field, and the properties in any direction perpendicular to the field are independent of direction. The effect of this symmetry is to reduce the constants to the terms $d_{31} = d_{32}$, d_{33}, $d_{15} = d_{24}$, and there is no necessary relationship between d_{15} and d_{31} and d_{33}.

On the other hand, if the effect is regarded as a second-order electrostrictive effect, it was shown in a previous paper[4] that the stress strain and electric relations are given by the tensor equations (when other second-order effects are neglected)

$$\begin{aligned} S_{ij} &= T_{kl}s_{ijkl}{}^{D} + \delta_n[g_{ijn} + Q_{ijn0}\delta_0], \\ E_m &= -T_{kl}[g_{mkl} + 2Q_{klmn}\delta_n] + \delta_n[4\pi\beta_{mn}{}^{T}], \end{aligned} \tag{1}$$

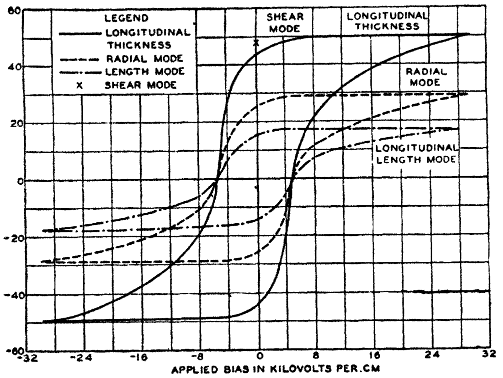

FIG. 1. Measured values of electromechanical coupling as a function of the voltage bias for 4 modes of motion in barium titanate ceramics.

where S_{ij} and T_{kl} are, respectively, the strains and stresses, $\delta_n = D_n/4\pi$ where D_n are the electric displacements, s_{ijkl}^D are the constant displacement elastic compliances, g_{ijn} the piezoelectric constants, Q_{ijn0} the electrostrictive constants, and β_{mn}^T the impermeability constants (i.e., inverse of dielectric constants).

On the assumption that the material is isotropic and non-piezoelectric, these equations reduce to the forms expressed in terms of the usual two index symbols:

$$S_1 = s_{11}^D T_1 + s_{12}^D [T_2 + T_3] + Q_{11}\delta_1^2 + Q_{12}[\delta_2^2 + \delta_3^2],$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$S_{12} = \frac{S_6}{2} = [s_{11}^D - s_{12}^D] T_6 + [Q_{11} - Q_{12}]\delta_1\delta_2,$$

$$\begin{aligned} E_1 = \delta_1[4\pi\beta_{11}^T] - 2[&Q_{11}[\delta_1 T_1 + \delta_2 T_6 + \delta_3 T_5] \\ &+ Q_{12}[\delta_1[T_2 + T_3] - [T_6\delta_2 + T_5\delta_3]]], \end{aligned} \qquad (2)$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$\begin{aligned} E_3 = \delta_3[4\pi\beta_{11}^T] - 2[&Q_{11}[\delta_3 T_3 + \delta_1 T_5 + \delta_2 T_4] \\ &+ Q_{12}[\delta_3[T_1 + T_2] - [\delta_1 T_5 + \delta_2 T_4]]]. \end{aligned}$$

Hence as shown by the equation for the shear strain S_6, a shear mode is excited when two electric displacements occur at right angles.

Using these equations, the electromechanical coupling factors have been calculated for the longitudinal length mode, the radial mode, the thickness longitudinal mode, and the thickness shear mode. These are given, respectively, by the formulae

$$k_l = \frac{2Q_{12}\delta_{3_0}}{(s_{11}^E(4\pi\beta_{11}^T))^{\frac{1}{2}}}; \quad k_r = \left(\frac{2}{1-\sigma}\right)^{\frac{1}{2}} k_l,$$

$$k_t = \frac{2\delta_{3_0}\left[Q_{11} - \dfrac{2s_{12}^D}{s_{11}D + s_{12}D} Q_{12}\right]}{\left(\dfrac{4\pi\beta_{11}^T}{c_{11}^E}\right)^{\frac{1}{2}}}; \quad k_s = \frac{2(Q_{11} - Q_{12})\delta_{3_0}}{\left(\dfrac{4\pi\beta_{11}^T}{\mu^E}\right)^{\frac{1}{2}}}.$$

Measurements for the coupling of these four modes as a function of the applied voltage are shown by Fig. 1. The fact that the coupling follows a hysteresis loop shows conclusively that the electrostrictive effect is a function of the electric displacement rather than the applied field. The longitudinal and thickness couplings determine the electrostrictive constants to be

$$Q_{12} = -2.15 \times 10^{-12} \left(\frac{\text{cm}^2}{\text{stat. coulombs}}\right)^2;$$

$$Q_{11} = +6.9 \times 10^{-12} \left(\frac{\text{cm}^2}{\text{stat. coulombs}}\right)^2.$$

Using these constants, the shear mode for a remanent displacement due to the application of 30,000 volts/cm has a calculated coupling agreeing well with the experimental value, which shows conclusively that the effect follows the second-order electrostrictive equations and not the "quadratic" piezoelectric relations. Since a ferromagnetic magnetostrictive material follows a similar set of equations, it appears more logical to call the effect electrostrictive.

---

[1] B. Matthais and A. Von Hipple, "Structure, electrical and optical properties of barium titanate," Phys. Rev. **73**, 268 (1948).

[2] H. Mueller, "Properties of rochelle salt IV," Phys. Rev. **58**, 805 (1940).

[3] W. L. Cherry and Robert Adler, "Piezoelectric effect in barium titanate," Phys. Rev. **72**, 981 (1947).

[4] W. P. Mason, "First and second order equations for piezoelectric crystals expressed in tensor form," Bell Sys. Tech. J. **26**, 80 (1947).

Slow Neutron Spectrometer Studies of Oxygen, Nitrogen, and Argon*

E. MELKONIAN, L. J. RAINWATER, W. W. HAVENS, JR.,
AND J. R. DUNNING
Columbia University, New York, New York

April 18, 1948

THE Columbia University slow neutron velocity spectrometer[1] is being used to investigate materials in the gaseous phase. Preliminary measurements on several elements are presented in this letter (Figs. 1–3). Following the usual convention, all cross sections are in units of 10^{-24} cm²/atom.

The samples were contained in aluminum alloy cylinders 1 meter long and having 10-cm inside diameters. Pressures up to 75 atmospheres can be used.

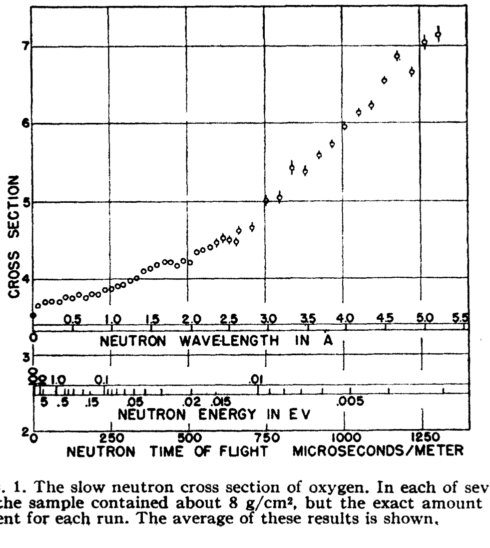

FIG. 1. The slow neutron cross section of oxygen. In each of several runs the sample contained about 8 g/cm², but the exact amount was different for each run. The average of these results is shown.