TABLE I.

Absorber thickness	Time (hours) with C	back	Electrons/hour with C	back	Difference per hour	Corrected for time lags
11.2 cm Pb	159.1	152.5	2.41±0.12	0.96±0.08	1.45±0.14	1.42±0.14
31.6 cm Pb	104.1	186.0	3.24±0.17	1.37±0.09	1.87±0.20	1.81±0.20
31.6 cm Pb+ 25.4 cm Fe	135.3	93.7	2.38±0.13	0.74±0.09	1.64±0.16	1.59±0.16

495±30, and 890±30 Mev.** For the purpose of comparison, the point at 890 Mev has been fitted to the Wilson curve, given here on an energy scale; both experiments were performed at approximately the same geomagnetic latitude.

While the statistical errors in this experiment are about the same, or slightly larger than, the errors in Wilson's points, the resolution is sharper, and indicates the usefulness of this technique in the examination of the differential spectrum.

These results were obtained during the course of an investigation into the energy dependence of the positive meson excess, which will be reported on later, and we are indebted to Professor I. S. Lowen for suggesting the initial problem.

* This work was supported by the Office of Naval Research under Contract No. N6ori-201 Task Order II.

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** This does not take into account the possible spread in energy resulting from the distribution in path lengths through the upper absorber. If the deviations attributable to this effect are taken to be independent of those due to the stopper, the *maximum* energy spread at each point becomes: 210±40, 495±80, and 890±140 Mev. Inasmuch as qualitative considerations indicate that the true spread is much closer to those shown on the diagram than to the maximum, it was decided to show the former rather than the maximum spread. It should be noted, however, that even with the latter, the conclusion given above concerning the improved resolution remains valid.

On Two Complementary Diffraction Problems

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April 20, 1948

OCCASIONED by a discussion on Babinet's theorem, we have worked out exact solutions for the diffraction of sound waves by a circular disk, and in the corresponding hole, *i*, an infinite plane screen.[1]

Two such complementary obstacles are quite different in topological respect, one being singly connected and the other doubly connected. Therefore the mathematical expressions for the diffracted waves will also be entirely different in the two cases.

So Babinet's theorem does not at all emerge simply from the exact solution. It is a limiting principle, like the Huygens-Kirchhoff method on which it is based, valid at short wave-lengths.

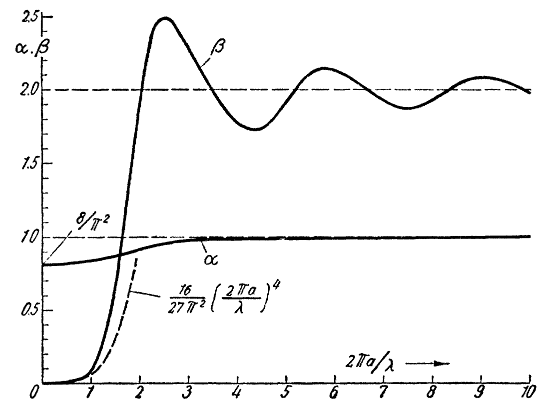

FIG. 1.

The most direct approach to an exact solution is the introduction of spheroidal coordinates. If these—μ, ν, ϕ—are defined by

$$x+iy=a((1+\mu^2)(1-\nu^2))^{\frac{1}{2}}\cdot e^{i\varphi},$$
$$z=a\mu\nu,$$

where

$$a=\text{radius of the hole},$$

the disk and the screen, in the xy-plane, will coincide with the coordinate surfaces $\mu=0$ and $\nu=0$, respectively.

Developing the whole sound field ϕ in the corresponding wave functions

$$\phi_\Lambda=M_\Lambda(\mu)N_\Lambda(\nu),$$

where $\mathbf{N}$ and $\mathbf{\Lambda}$ denote eigenfunctions and eigenvalues of the angular differential equation:

$$(d/d\nu)[(1-\nu^2)(dN/d\nu)]+(k^2a^2\nu^2-\Lambda)N=0,$$

and utilizing integral representations of the form (plane waves):

$$M_\Lambda(\mu)=\int \exp(ika\mu\nu)N_\Lambda(\nu)d\nu,$$

it is easy to write down compact expressions for the wave field satisfying the boundary conditions.

In the numerical elaboration of the results use has been made of the tables of Stratton *et al.*[2] and especially of earlier work of Hylleraas.[3]

We shall here give only the result for the total energy scattered by the disk or transmitted through the hole (Fig. 1).

$$\left.\begin{matrix}\alpha\\ \beta\end{matrix}\right\}=\text{total energy}\left\{\begin{matrix}\text{transmitted}\\ \text{scattered}\end{matrix}\right\}\Big/\begin{matrix}\pi a^2\cdot\text{intensity of}\\ \text{primary wave;}\end{matrix}$$
$$\lambda=\text{wave-length;}$$
$$a=\text{radius of hole and disk.}$$

The transmission coefficient, α, starts at a value of 81 percent for long wave-lengths, and the scattering coefficient β increases of course from zero $\sim 1/\lambda^4$ according to Rayleigh's law.

Babinet's theorem applies to the region $\lambda \ll a$, where both curves deviate negligibly from their asymptotes $\alpha=1$, $\beta=2$.

One of us (A.S.) hopes to return to the electromagnetic case.

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Correlation between the States of Polarization of the Two Quanta of Annihilation Radiation*

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April 13, 1948

It has been pointed out by J. A. Wheeler[1] that according to pair theory the planes of polarization of the two quanta originating in the annihilation of a positron should be perpendicular to each other. This correlation is the equivalent of angular momentum conservation in the process of annihilation of an electron pair with relative velocity zero in the singlet state. The azimuthal variation of intensity of the simultaneous Compton scattering of the two quanta, resulting from this correlation between their respective states of polarization, has been calculated by Pryce and Ward[2] and by Snyder, Pasternack, and Hornbostel.[3] An experimental verification has been attempted with the aid of the arrangement shown in Fig. 1.

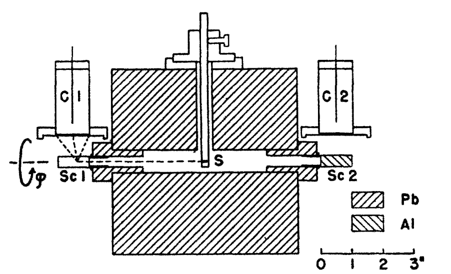

Fig. 1. Coincidence measurement of Compton scattering.

The annihilation radiation of the source S (Cu^{64}, prepared by deuteron irradiation of copper in the Purdue cyclotron) is collimated by a $\frac{3}{8}$-in. channel in the lead block. The quanta are scattered by cylindrical aluminum scatterers Sc and detected with bell-shaped Geiger counters with lead cathodes. Coincidences were measured for azimuth differences (φ) of 0°, 90°, 180°, and 270° between the counter axes. In order to eliminate all asymmetries both counters were rotated in turn. As a result of absorption in the scatterer the mean scattering angle is slightly less than 90°, near the theoretical maximum of anisotropy calculated for a scattering angle of 82°. Taking into account the finite solid angle subtended by the counters, a ratio $C_\perp/C_{\parallel}\approx 1.7$ is expected for the coincidence rates at $\varphi=90°$ ($C_\perp$) and $\varphi=180°$ ($C_\parallel$). Four different runs were made with different sources consistently showing $C_\perp > C_\parallel$. Data for a characteristic run of 16 hours are given in Table I.

Table I.

Average single counts without scatterers	3000/min.
Average single counts with scatterers	5370/min.
Chance coincidences ($T=1.2\cdot 10^{-7}$ sec.)	0.117/min.
Genuine coincidences $C_\perp$	0.152/min.
Genuine coincidences $C_\parallel$	0.073/min.
Asymmetry ratio $C_\perp/C_\parallel$	2.1±0.64

The observed average asymmetry ratio for all runs is

$$C_\perp/C_\parallel = 1.9_4 \pm 0.3_7.$$

The indicated error is the statistical mean standard deviation. The theoretical prediction is therefore confirmed by this experiment.

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** Now at the University of Rochester, Rochester, New York.
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Piezoelectric or Electrostrictive Effect in Barium Titanate Ceramics

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April 13, 1948

In a recent Letter to the Editor,[1] Matthias and Von Hippel have discussed the resonances obtained in a piece of multicrystalline barium titanate ceramic and have called the effect a "quadratic" piezoelectric effect. While the term used is to some extent a matter of definition, it appears worth while to point out that the effect in the titanate ceramic does not conform to the original definition of a "quadratic" piezoelectric effect, but is, on the other hand, the analog of a magnetostrictive effect in a ferromagnetic material.

According to Mueller,[2] a "quadratic" piezoelectric effect is one following the same equations as an electrostrictive effect but depending on a strain caused by a spontaneous polarization or an applied field acting on the piezoelectric constant. The discovery[3] that a shear vibration can be set up when an a.c. field is applied at right angles to a d.c. polarization and the quantitative check between the value of this constant and the radial and thickness constants show that the effect cannot be a "quadratic" piezoelectric effect. This follows from the fact that the only type of symmetry that the ceramic can have in the presence of an applied field is that known as transverse isotropy. For this case the $c=Z$ axis lies along the direction of the field, and the properties in any direction perpendicular to the field are independent of direction. The effect of this symmetry is to reduce the constants to the terms $d_{31}=d_{32}$, d_{33}, $d_{15}=d_{24}$, and there is no necessary relationship between d_{15} and d_{31} and d_{33}.

On the other hand, if the effect is regarded as a second-order electrostrictive effect, it was shown in a previous paper[4] that the stress strain and electric relations are given by the tensor equations (when other second-order effects are neglected)

$$\begin{aligned} S_{ij} &= T_{kl}s_{ijkl}{}^D + \delta_n[g_{ijn} + Q_{ijno}\delta_o], \\ E_m &= -T_{kl}[g_{mkl} + 2Q_{klmn}\delta_n] + \delta_n[4\pi\beta_{mn}{}^T], \end{aligned} \qquad (1)$$