

The ratio $\sigma(\gamma,p)/\sigma(\gamma,n)$ can now be calculated from Eq. (19) of the paper of Weisskopf and Ewing:²

$$\sigma(\gamma,p) \propto \int_0^{\epsilon_0} \epsilon S_p(\epsilon) \xi_p \ln\left(\frac{\epsilon_0 - \epsilon + a}{a}\right) d\epsilon,$$

$$\sigma(\gamma,n) \propto \int_0^{\epsilon_0} \epsilon S_n(\epsilon) \xi_n \ln\left(\frac{\epsilon_0 - \epsilon + a}{a}\right) d\epsilon.$$

The integrands give the relative numbers of protons and neutrons that are emitted with energy ϵ (in Mev), where the maximum energy ϵ_0 is the difference between the γ -ray energy and the binding energy of a nucleon. For the energy range under consideration, the sticking probabilities, ξ_p and ξ_n , are taken to be unity, and the cross section for neutron capture, $S_n(\epsilon)$, is taken to be the nuclear area. The proton capture cross section, $S_p(\epsilon)$, can be found from Fig. 1 of reference 2. The dependence of ϵ_0 and the residual nuclear level densities on the oddness or evenness of Z and A will be ignored here.

The relative energy distributions computed in this way for the emitted protons and neutrons are shown in Fig. 1 for $Z=30$ ($a=0.30$ Mev) and in Fig. 2 for $Z=50$ ($a=0.17$ Mev); it is assumed that $\epsilon_0=9$ Mev. The cross-section ratios can be found from the areas under the curves:

$$\frac{\sigma(\gamma,p)}{\sigma(\gamma,n)} = \begin{cases} 0.124, & Z=30, \\ 0.020, & Z=50. \end{cases}$$

These values are in good agreement with the experimental results,¹ but indicate a somewhat more marked dependence of cross-section ratio on Z than is observed. This suggests that the statistical model is inadequate for the smaller group of Z values covered by the experiments, especially since rather large fluctuations are observed there. Experiments on the energy distributions of protons and neutrons emitted by the same compound nucleus would be helpful in clarifying the nature of the processes that take place.

Note on the Stability of Systems Containing a Light Positive Particle

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It is of interest to consider the dynamic stability of atomic or molecular systems which contain a positron, e.g., e^+Cl^- . This paper examines what information about this stability can be obtained without elaborate calculations, (a) from experimental data descriptive of hydrides and of the hydrogen molecule or molecular ion, and (b) from existing calculations of the energy of light polyelectrons. The procedure (a) suffices to guarantee stability only when the positron is replaced by a positive meson which has a mass of the order of twenty-electron masses or more. The second procedure allows a certain lowering of these approximate minimum mass values, in a special case to a few electron masses. The various numerical results are obtained very easily. They indicate that negative atomic ions can have a positive positron affinity.

I. INTRODUCTION

RECENTLY Wheeler¹ envisaged the existence of short-lived atomic or molecular systems which contain a positron, e.g., e^+Cl^- . At present, even the question of the dynamic stability of such systems remains unanswered.

In a quantitative investigation of this stability the approximate methods of atomic or molecular theory do not seem well adapted. In the first place, the electric charge of the positron has the opposite sign of that of an atomic electron. Secondly, the adiabatic approximation, which is justified in the case of ordinary molecules (Born-Oppenheimer), cannot be expected

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¹ J. A. Wheeler, *Ann. N. Y. Acad. Sci.* **48**, 219 (1946).

to remain valid when the mass of one "nucleus" has been reduced to that of an electron. (If this approximation be valid for a "di-atomic molecule," irrespective of the mass of one "nucleus," and, provided we can consider the zero point energy inversely proportional to the square root of the reduced mass of the "nuclei," one easily finds that the mass of the hydrogen nucleus (e.g., in HCl) can be reduced to the order of a few electron masses before dissociation occurs.)

The simplest system of the kind under consideration is e^+H , which results when one proton is replaced by a positron in the molecular hydrogen ion, H_2^+ . Clearly, the same system can be obtained by adding a positron to a hydrogen atom. It is well known that a negative electron can combine with a hydrogen atom to form the stable negative atomic ion H^- , or e^-H . Classically, one would then expect that e^+H would be unstable. That this argument is not valid can be illustrated by the fact that the bi-electron, e^+e^- , can form a stable tri-electron when an electron of *either* sign is added. On the other hand, e^+H results from the stable tri-electron $e_2^+e^-$ when the mass of one of the positive particles is increased. Again, however, increasing the mass of one particle, keeping the charges unchanged, does not necessarily mean that a more stable system results. Thus, when in H^- one of the electrons is replaced by a negative meson (of mass 200m), the second electron will no longer be bound.² Hence, we conclude that qualitative arguments can hardly be applied to the problem in question.

It is the purpose of this note to examine what information about the dynamic stability of systems containing a light positive particle can readily be obtained from experimental data descriptive of ordinary molecules and from existing calculations of the energy of light polyelectrons. In an investigation of this stability the Schrodinger equation can be assumed to be valid, that is, we can neglect the finite probability of annihilation and we can apply non-relativistic quantum mechanics (cf. Wheeler, reference 1). Since the results one obtains in this way are not conclusive when the particle in question is a positron, we shall not attempt to discuss the

various questions which arise in connection with the formation of such systems.

II. APPLICATION OF MOLECULAR WAVE FUNCTIONS

We shall understand by "system M " an ordinary molecular system which contains at least one hydrogen nucleus (mass M), and by "system μ " the corresponding "molecule" in which a particle of mass μ plays the role of a light isotope of hydrogen. We shall proceed to determine approximate minimum values of μ which are consistent with dynamic stability for system μ in various cases.

In the following it will be understood that we are dealing exclusively with the ground states of the two systems under consideration.

The respective wave equations differ merely in the multiplicative constant of one of the kinetic energy terms. When the part of the Hamiltonians which is common for system M and system μ is denoted by H' , the two equations can be written

$$-\frac{\hbar^2}{2M}\nabla^2\psi_M + H'\psi_M = E_M\psi_M, \quad (1)$$

and

$$-\frac{\hbar^2}{2\mu}\nabla^2\psi_\mu + H'\psi_\mu = E_\mu\psi_\mu. \quad (2)$$

We will now let ψ_M approximate ψ_μ in the sense of the variational method. Assuming normalized ψ_M , we obtain

$$E_\mu < -\frac{\hbar^2}{2\mu} \int \psi_M^* \nabla^2 \psi_M d\tau + \int \psi_M^* H' \psi_M d\tau, \quad (3)$$

or

$$E_\mu - E_M < \left(\frac{M}{\mu} - 1\right) T \quad (4)$$

where

$$T = -\frac{\hbar^2}{2M} \int \psi_M^* \nabla^2 \psi_M d\tau. \quad (5)$$

According to the last equation, T is the mean kinetic energy of the proton in system M . This quantity can be obtained rather accurately from the zero-point energy of that molecule. The lowest vibrational level corresponds closely to that of a harmonic oscillator, that is, one-half of the zero-point energy represents the kinetic energy of the nuclear vibrations,

² E. Fermi and E. Teller, Phys. Rev. 72, 399 (1947).

TABLE I. Approximate lower limit values for the mass μ of a positive particle which can replace a hydrogen nucleus in a molecule (system M).

System M	H ₂ ⁺	H ₂	HLi ⁷	HBe ⁹	HC ¹²	HF ¹⁹	HN _a ²³	HCl ³⁵	HK ³⁹	HCa ⁴⁰
μ , (minimum) in units of m	24	24	22	40	38	32	22	33	21	34

The total energy E_M is known only for the lightest hydrides, but since it is only the *difference* $E_\mu - E_M$ that enters the left-hand side of (4), the dissociation energy D_M is all that we need know in order to be able to apply the inequality.

The last statement becomes evident when we write

$$E_M = -R_H h + E_{n-2} - D_M, \quad (6)$$

and

$$E_\mu = -\frac{\mu(M+m)}{M(\mu+m)} R_H h + E_{n-2} - D_\mu. \quad (7)$$

In (6) the term $-R_H h$ is the energy of the hydrogen atom, and E_{n-2} the energy of the partner atom, which consists of all but two of the n particles which form system M .

The reason why we need consider only the type of dissociation indicated in (7) is that the first term in this equation will have a value between -6.77 eV ($\mu = m$) and -13.53 eV ($\mu = M$), while no atom is known to have an electron affinity which exceeds a few electron volts.³

It should be understood, however, that in writing down Eqs. (6) and (7) we are assuming that system M is a neutral molecule. When system M is a molecular ion the equations will be somewhat different.

The exact wave function ψ_M is not known explicitly for any molecule. The preceding outline shows, however, that our ignorance as to the form of the function used in the variational calculation is remedied by available spectroscopic data. When we combine the inequality (4) with the proper experimental data, we readily find minimum values of μ consistent with stability ($D_\mu > 0$). Some of the results one finds in this way are given in Table I.**

Small minimum values are favored by low zero-point energy (hydrides of group I in the periodic

³ Cf. H. S. W. Massey, *Negative Ions* (Cambridge University Press, Teddington, England, 1938).

** The experimental data have been taken from the book by G. Herzberg, *Molecular Spectra and Molecular Structure* (Prentice-Hall, Inc., New York, 1939).

table) and by large dissociation energy. Since the latter energy is not known to a high degree of accuracy for all hydrides, the values of μ given in Table I may need slight corrections. Such corrections can be of but little significance in the present approximation, however.

According to the variational character of the treatment which has been outlined above, the various systems μ will be dynamically stable when μ has a value exceeding the minimum value, e.g., when the positive particle is a cosmic-ray meson. Furthermore, the possibility is not excluded that the correct lower limits are such as to include a positron.

However, the values given in Table I are still so high that the adiabatic approximation may be essentially valid.

III. MODIFICATION OF ψ_M IN ACCORD WITH THE VIRIAL THEOREM

We shall now proceed to modify the unknown ψ_M in such a way that we obtain a function which will be a better approximation for ψ_μ , at least as far as the energy is concerned. This is possible by what amounts to an application of the virial theorem.†

In the way familiar from variational calculations of binding energies, we introduce a parameter k by writing

$$\psi_M = \psi_M(\mathbf{r}_i) = \varphi(k\mathbf{r}_i), \quad i = 1, 2, \dots, n. \quad (8)$$

Furthermore, we shall use the following abbreviations:

$$N = \int |\psi_M|^2 d\tau; \quad (9)$$

$$T_{n-1} = -\sum_{n-1} \frac{\hbar^2}{2m_i} \int \psi_M^* \nabla_i^2 \psi_M d\tau,$$

and we denote by V the corresponding matrix element representing the total potential energy. An apostrophe will be used to indicate that

† Cf. V. Fock, *Zeits. f. Physik* **63**, 885 (1930).

matrix elements are given in terms of $\varphi(\mathbf{r}_i)$ rather than $\psi_M(\mathbf{r}_i)$, as in (9).

Corresponding to (3) we have

$$E_\mu N < \frac{M}{\mu} T + T_{n-1} + V, \quad (10)$$

whence

$$E_\mu N' < k^2 \left(\frac{M}{\mu} T' + T_{n-1}' \right) + k V'. \quad (11)$$

We minimize the right-hand side of (11) with respect to the parameter k . This minimization actually means that we modify ψ_M , given by (8), in such a way that we obtain an approximate ψ_μ which will be in accord with the virial theorem.

The result can be written as

$$E_\mu < -(V')^2 / 4N' \left(\frac{M}{\mu} T' + T_{n-1}' \right). \quad (12)$$

(Strictly, we should keep the possibility open that (12) be an equality, since the modification of ψ_M leads to the correct ψ_μ in the case where the system M is the hydrogen atom. This case, however, does not concern us at present.)

We can omit the apostrophes in (12). The omission is equivalent to a multiplication by k^2 of both numerator and denominator in the right-hand side expression.

Similarly, we can write

$$E_M = -V^2 / 4N(T + T_{n-1}). \quad (13)$$

Again we assume $N=1$. Furthermore, according to the virial theorem we can substitute $2E_M$ for V . From the preceding relations we then obtain the inequality

$$E_\mu < - \frac{E_M}{1 - \left(\frac{M}{\mu} - 1 \right) \frac{T}{E_M}}. \quad (14)$$

This inequality requires for its applicability the value of the total energy E_M and not merely the dissociation energy D_M , as was the case when we applied (4). E_M is not known with accuracy for heavier hydrides. However, even when system M is HLi, and more so for heavier hydrides, the two inequalities (14) and (4) are approximately equivalent.

When system M is either H_2^+ or H_2 one easily finds that the relation (14) guarantees stability for system μ provided the mass μ is greater than about 20-electron masses, as compared to roughly 24-electron masses which was the minimum value resulting from inequality (4) in these cases.

IV. APPLICATION OF APPROXIMATE POLYELECTRONIC WAVE FUNCTIONS

We shall once more consider the two special cases in which system M is either H_2 or H_2^+ .

When system M is H_2^+ then system μ consists of a hydrogen atom plus the positive particle of mass μ . This system can be obtained from the positive tri-electron $e_2^+e^-$ when in the latter we replace one positron by a proton and the other positron by the particle μ .

The wave equation for the tri-electron and for the corresponding system μ can be written

$$\left\{ \nabla_1^2 + \nabla_2^2 + \nabla_3^2 + 2 \left(\frac{1}{r_1} + \frac{1}{r_2} - \frac{1}{r_{12}} \right) + E_t \right\} \psi_t = 0, \quad (15)$$

and

$$\left\{ \frac{m}{M} \nabla_1^2 + \frac{m}{\mu} \nabla_2^2 + \nabla_3^2 + 2 \left(\frac{1}{r_1} + \frac{1}{r_2} - \frac{1}{r_{12}} \right) + E_\mu \right\} \psi_\mu = 0, \quad (16)$$

respectively. In both equations, 1 and 2 refer to the two positive particles and 3 refers to the negative electron. The unit of length is the atomic unit of length, a_H , and we have used $R_\infty h$ as energy unit.

The similarity of the two Eqs. (15) and (16) permits a simple application of Hylleraas' calculations of the energy of the tri-electron⁴ to the problem of the dynamic stability of our system μ .

Consider a function φ which is symmetric in r_1 and r_2 and which does not depend upon the third distance (r_{12}) between the particles. Since in this case

$$\int (\nabla_1 \varphi)^2 d\tau = \int (\nabla_2 \varphi)^2 d\tau = \frac{1}{2} \int (\nabla_3 \varphi)^2 d\tau, \quad (17)$$

⁴ E. A. Hylleraas, Phys. Rev. **71**, 491 (1947).

we readily obtain the following inequality

$$E_\mu < \frac{4E_i'}{\frac{m}{M} + \frac{m}{\mu} + 2}. \quad (18)$$

In (18) E_i' is the approximate energy of the tri-electron which results from a variational calculation by means of the function φ . In Hylleraas' calculations φ contains parameters which are given the values which minimize E_i . It follows from (18) that these values are at the same time the best values we can give the parameters when φ is taken to approximate the wave function of system μ .

Hylleraas obtained the approximate value $E_i' = -\frac{1}{2} \times 1.02762 R_\infty h$ when φ had the form

$$\varphi = e^{-\frac{1}{2}s} (1 + c_2 t^2 + c_4 t^4 + c_6 t^6); \quad (19)$$

$$s = r_1 + r_2, \quad t = r_2 - r_1.$$

When this value of E_i' is inserted into (18), one now finds that system μ will be stable provided $\mu > 18m$. Thus, we have obtained a slightly lower value for μ , above which stability is guaranteed, as compared to the values 24 m and 20 m of the preceding sections.

Next, let us consider the case in which system M is the neutral hydrogen molecule, H_2 . System μ can then also be obtained from the quadri-electron, $e_2^+ e_2^-$, by replacement of the two positrons by one proton and by the particle μ , respectively.

The existing calculations of the energy of the quadri-electron have been made by using variational functions which do not contain the two distances between identical particles and which are symmetric with respect to exchange of electrons, exchange of positrons, and exchange of the two electrons against the two positrons.

Assuming this kind of function, and by a procedure similar to the one above, we arrive at an inequality which differs from (18) merely in that E_i' has been replaced by the approximate energy E_q' of the quadri-electron.

If we take the value $-1.0085 R_\infty h$ for E_q' ,

which results from a variational function having the character of a linear combination of somewhat generalized "atomic" functions,⁵ we find that the system μ under consideration will be stable for μ greater than about 5 m .

A linear combination of "atomic" and "ionic" functions⁶ yielded a slightly lower value for E_q' . The corresponding minimum value of the mass μ , consistent with dynamic stability, is found to be roughly 4.5 m .

V. CONCLUSION

The successive lower limit values which we have obtained in the particular case when system M is the hydrogen molecule, namely, 24-, 20-, and 4.5-electronic masses, show that the choice of variational function is essential for the result. However, not even the function which gave the lowest value, 4.5 m , can be considered to be a very good approximation for the system μ when μ is small compared to the mass of a proton. The symmetry properties mentioned above are not proper in this case. Furthermore, the function does not contain the total number of variables.

For these reasons it seems likely that this system μ will be stable even for $\mu = m$. Furthermore, the values given in Table I then indicate that at least in some cases we can replace the hydrogen nucleus in a hydride molecule by a positron and obtain a dynamically stable system, of the kind envisaged by Wheeler.

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⁵ E. A. Hylleraas and A. Ore, Phys. Rev. **71**, 493 (1947).

⁶ A. Ore, Phys. Rev. **71**, 913 (1947).