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Neutron Polarization*

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The production of polarized neutrons in magnetized iron has been studied, using the intense neutron beams available at the Argonne heavy water pile. The theoretical work of Halpern *et al.*, used as a guide in the experiments, has been checked in many respects, with the exception that the polarization cross section p has a measured value of 3.15 barns compared to the theoretical 1 barn. The application of neutron polarization to the measurement of the approach to saturation in ferromagnets is described and preliminary results are reported.

I. INTRODUCTION

THE possibility of polarizing the neutrons in a beam was first suggested by Bloch[1] who showed that the scattering cross section for neutrons by magnetized iron should be different for the two possible orientations of the neutron spin relative to the magnetic field. The difference in cross sections implies that a neutron beam, after passing through magnetized iron, should have a preponderance of neutrons with spins orientated in the same direction. In this way almost complete polarization could be produced in a very thick block of magnetized iron. The difference in cross section was shown by Bloch to arise from the term in the intensity caused by the interference of the neutron wave scattered by the magnetic field of the iron atom with the wave scattered by the nucleus. It was suggested that neutron polarization experiments would give valuable information on the interaction of neutrons with electrons and on the nature of ferromagnetism.

The earliest experiments to demonstrate the production of neutron polarization by passage through magnetized iron were performed by Hoffman, Livingston and Bethe,[2] by Frisch, von Halban and Koch,[3] and by Powers *et al.*[4] These experiments showed very small effects which indicated the polarization of neutrons but the results were such that quantitative interpretation was very difficult. Some later experiments of Powers[5] showed qualitative agreements with the theory but again the effects were so small that any detailed comparison was impossible. In 1939 Alvarez and Bloch[6] used the polarization of neutrons to determine the value of the neutron magnetic moment. In this work they were able to obtain effects somewhat larger than in the earlier experiments. Later, Bloch, Hammermesh, and Staub[7] made the first investigation of the

* Declassified (in sections) 4/24/47, 3/7/47, 12/22/47.

[1] F. Bloch, Phys. Rev. **50**, 259 (1936); **51**, 994 (1937).

[2] Hoffman, Livingston, and Bethe, Phys. Rev. **51**, 214 (1937).

[3] Frisch, von Halban, and Koch, Nature **139**, 756 (1937); Nature **140**, 360 (1937); Phys. Rev. **53**, 719 (1938).

[4] Powers, Carroll, and Dunning, Phys. Rev. **51**, 371, 1112 (1937); Dunning, Powers, and Beyer, Phys. Rev. **51**, 382 (1937).

[5] P. N. Powers, Phys. Rev. **54**, 827 (1938).

[6] L. Alvarez and F. Bloch, Phys. Rev. **57**, 111 (1939).

[7] Bloch, Hammermesh, and Staub, Phys. Rev. **64**, 47 (1943).

process of polarization in which results were obtained which were large enough to permit careful comparison with theory. Although the polarization effects obtained were somewhat larger than the theory predicted, the qualitative theoretical results could be verified quite well.

On the theoretical side, the initial suggestion of Bloch was followed by more detailed analysis of Schwinger[8] and especially of Halpern and co-workers.[9] The complete theoretical calculation of the polarization is quite complicated, as it requires detailed knowledge of the distribution of the magnetic scattering caused by the orbital electrons (form factor), as well as the calculation of the somewhat complicated crystalline effects in the iron. A short review of the main results will be given here but the reader is referred to the excellent papers of Halpern *et al.* for the complete theory.

The scattering in iron is caused not only by the usual nuclear scattering, which is isotropic, but also by the orbital electrons because of the interaction of the electron spins with the neutron magnetic moment. The magnetic scattering shows a marked angular dependence and a reversal of sign for the two possible orientations of the neutron spin relative to the magnetic moment of the atom (parallel or antiparallel). The coherent scattering in a microcrystalline substance (which is the major part of the scattering as will be seen later) is obtained by adding the amplitude of the scattered waves, both nuclear and magnetic, and squaring the resultant amplitude. In the summation, of course, contributions are obtained only for microcrystals orientated

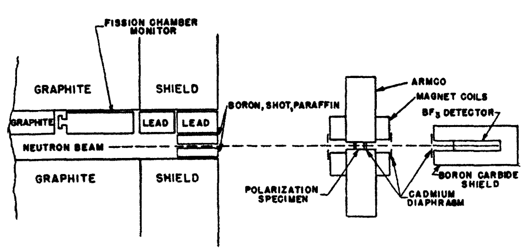

FIG. 1. Apparatus for measurement of single transmission effect, E.

[8] J. Schwinger, Phys. Rev. **51**, 544 (1937).

[9] O. Halpern and M. Johnson, Phys. Rev. **55**, 898 (1939); O. Halpern and T. Holstein, Phys. Rev. **59**, 960 (1941); Proc. Nat. Acad. Sci. **28**, 112 (1942); O. Halpern, M. Hammermesh, and M. Johnson, Phys. Rev. **59**, 981 (1941); M. Hammermesh, Phys. Rev. **61**, 17 (1942).

at such angles that the Bragg conditions of scattering obtain.

If the iron is unmagnetized, that is, if the magnetizations of the various domains are at random, then there is no preferential scattering depending on orientation of neutron spin in the incident beam. If, however, the iron is completely saturated, the scattering cross section can be written in the following way for the two spin orientations:

$$\sigma=\sigma_0\pm p. \quad (1)$$

The total cross section σ, the cross section σ_0 for unmagnetized iron, and the polarization cross section, p, all depend on neutron velocity in a complex way because of the crystal effects. Because all the cross sections of Eq. (1) are made up of the coherent scattering only, they will disappear completely for neutrons of wave-length great enough so that no Bragg reflections are possible. This critical wave-length is 4.04A for iron.

The polarization effects are thus calculated only for the coherent part of the iron scattering. It is true that some of the iron scattering is incoherent; this incoherent scattering is caused by (a) presence of isotopes, (b) spin dependent scattering, (c) inelastic scattering, and (d) crystal irregularities. If the incoherent scattering in iron is the result of (a) and (b) only, then no additional polarization will be caused by the incoherent scattering. However, if the incoherent scattering is caused by (c) and (d), then polarization effects will be present in the incoherent scattering as well. Because of the possible polarization effects in the incoherent scattering it would be necessary to understand the nature of the incoherent scattering to present a complete picture of neutron polarization. However, the fact that only a small part of the scattering is incoherent means that the limitation of the theory to coherent scattering is not a serious inadequacy.[10]

[10] O. Halpern, Phys. Rev. **72**, 261 (1947), has recently suggested the incoherent polarization as a method of study of the inelastic scattering. His suggestion was based on the finding of Shull and Wollan, Phys. Rev. **72**, 168A (1947), that inelastic scattering in crystals was surprisingly large. Although the inelastic scattering is certainly not large in iron (as shown in Section II) it is still possible that the experiment suggested by Halpern is feasible and it is planned to do work along that line.

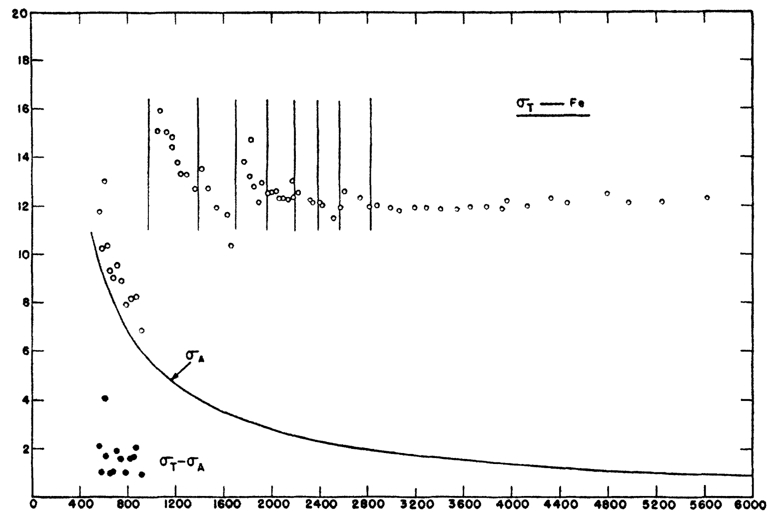

FIG. 2. Total cross section of iron as a function of neutron velocity (in meters/sec.). The points marked $\sigma_T-\sigma_A$ show the amount of incoherent scattering.

Halpern *et al.* have calculated the polarization cross section p as a function of neutron velocity, obtaining a value of about 1*b* averaged over the thermal velocity spectrum, and have shown the relationship between p and the amount of polarization produced in a block of magnetized iron. The most direct way to detect the polarization is by means of the relative increase in transmission of an iron block upon magnetization. This relative increase is called the single transmission effect E_s, and it depends on the polarization cross section p according to the equation

$$E_s=\cosh Npd-1\approx\tfrac{1}{2}(Npd)^2, \qquad (2)$$

where N is the number of iron nuclei/cm^3, and d the thickness of iron in cm. The quantity E_s of Eq. (2) refers to the transmission effect that would be obtained if the iron were completely saturated. If the iron is not quite saturated, as is always the case experimentally, then the small regions whose magnetizations are not in line with the applied field will have a serious effect in depolarizing the neutron beam. The result of depolarization is that the actual effect observed is related to E_s in the following way:

$$E=E_sf(x);\quad x=\lambda/\epsilon d. \qquad (3)$$

$f(x)$ is a saturation function (shown in Figs. 3 and 5) which depends on the degree of magnetization (ϵ is the fractional departure from saturation), the grain size of the iron (λ is a length related to the grain size), and the thickness of the iron block, d. Actually, Eq. (3) is a close approximation to the exact formula** given by Halpern and Holstein (their Eq. (9.4a)).

The general objectives of measurements with neutron polarization are (1) a measurement of the polarization cross section p to see how it compares with the theoretical expectation, (2) a study of the polarization phenomenon as a function of material, grain size, magnetization, etc., and (3) the use of polarized neutrons for the determination of magnetic properties as well as nuclear properties which depend on spin orientation. The earlier experiments had been hampered greatly by the lack of intense neutron beams, and it was felt that with the availability of much stronger beams at the Argonne chain reacting pile a detailed investigation of the production of polarized neutrons would be of value. Some measurements involving polarized neutrons had already been made at Argonne in the summer of 1946 by Arnold and Roberts[11] in connection with the redetermination of the neutron magnetic moment by the method of Alvarez and Bloch.

II. SCATTERING IN UNMAGNETIZED IRON

A fundamental problem in the study of neutron polarization is the measurement of the polarization cross section, p. If complete saturation of

** Equations (9.4a) and (9.4b) of Halpern and Holstein contain a slight misprint. The factor e^{-px} should be written $e^{-px/2}$ in each case.

[11] W. Arnold and A. Roberts, Phys. Rev. **71**, 878 (1947).

the iron block could be obtained, then p could be measured very easily by the use of Eq. (2). However, because complete saturation is never obtained it is necessary to extrapolate from the actual E to E_s in order to get the value of p. Also the value of p which results is actually an average over the distribution of neutron velocities used because intensities are usually low enough so that monoenergetic neutron beams cannot be used. In making the extrapolation to E_s it is necessary to know the shape of the neutron distribution and the change in this shape as the neutron beam passes through the iron blocks. For this reason and because it is important to know how much of the scattering in iron is coherent, it was felt important to study in some detail the scattering in unmagnetized iron before attempting a complete analysis of the production of polarized neutrons in magnetized iron.

The neutron beam used in the present experiments is one obtained from the thermal column of the Argonne deuterium moderated chain reacting pile (Fig. 1). The neutrons in the beam are those moderated by collisions with graphite of sufficient thickness so that they are in approximate thermal equilibrium. If the neutron distribution is Maxwellian, then the number of neutrons of velocity v will be proportional to $v^2 \exp(-v^2/v_0^2)$, where v_0 is the most probable velocity, and the flux (nv) will be proportional to $v^3 \exp(-v^2/v_0^2)$. The detector is a BF_3 proportional counter and such a detector, if very thin, has a sensitivity proportional to $1/v$. Thus the neutron intensity detected by the counter in such a case will again be proportional to the Maxwell distribution, $v^2 \exp(-v^2/v_0^2)$.

The neutron distribution in the beam was measured first using a mechanical velocity selector[12] ("chopper") and a B^{10} enriched BF_3 counter. The neutron distribution was shown by the chopper measurements to be approximately Maxwellian with a most probable velocity, v_0, about 2250 meters per sec. (The standard value of v_0 for room temperature neutrons is 2200 m/sec.) However, because of the finite resolution of the chopper it is difficult to infer the actual neutron distribution from the measured distribution. Because of the difficulty of determining the neutron distribution accurately with the chopper and because a distortion in the distribution is caused by the fact that the detector used was not thin enough to be an ideal $1/v$ detector, additional experiments were performed to determine the actual distribution and the effective distribution used in the experiments.

The most probable velocity of the neutron distribution as measured by a thin detector was determined by measuring the transmission of the beam through a piece of calibrated Pyrex using a thin $1/v$ detector. The transmission of this Pyrex plate as a function of neutron velocity had already been determined with the "chopper." Assuming a Maxwell distribution, it is possible to calculate v_0 from such a transmission experiment using the correction for hardening of the beam in the Pyrex as calculated by Bethe.[13] The transmission measured in this way gave the most probable velocity for the incident neutron beam as 2180 meters per sec., in excellent agreement with the theoretical value for room temperature of 2200 m/s.

The actual detector used for the polarization work (the enriched counter mentioned above) is less sensitive for low neutron velocities because it is not thin and hence, in effect, shifts the spectrum to somewhat higher energies. The effective neutron distribution as detected by the counter was calculated from the pressure of BF_3 gas in the counter and the cross section of boron, and it was found that the resulting distribution was very closely a Maxwellian again with a most probable velocity of 2250 instead of 2180 m/s. The value of 2250 thus checks the value obtained with the chopper. The transmission of the Pyrex plate was then calculated for this modified Maxwell distribution (as seen by the counter), the transmission through Pyrex again measured using the actual counter and it was found to be the same within the small experimental error as the calculated value. The results thus showed quite conclusively that the neutron distribution as used in the experiments could be taken to be very nearly a Maxwell distribution with a most probable velocity of 2250 m/s while

[12] The design of the rotating shutter velocity selector is given by T. Brill and H. Lichtenberger, Phys. Rev. **72**, 585 (1947).

[13] H. A. Bethe, Rev. Mod. Phys. **9**, 136 (1937).

the true neutron distribution in the beam had a most probable velocity of 2180 m/s.

In order to determine the change in the neutron spectrum with transmission through iron and also to determine how much of the scattering was coherent, a set of experiments was carried out on the iron cross section as a function of velocity. These experiments were performed using the chopper in the standard manner for the determination of total cross section as a function of velocity by transmission. The total cross section for a sample of Armco iron thus measured is shown in Fig. 2. It is seen that the total cross section is about 12 barns for high neutron velocities and with lower neutron velocities the cross section becomes irregular, as is to be expected from crystal effects. Below 1000 m/s there is a sudden drop in total cross section, then a rapid rise as the velocity becomes even lower. The short vertical lines represent the velocities at which discontinuities in the iron cross section are to be expected when the neutron wave-length exceeds twice the lattice spacing for a particular set of Miller planes. The last discontinuity is expected at 980 m/s (4.04A) as calculated from the lattice spacing for iron.

For neutron velocities less than the last cut-off value of 980 m/s, the cross section is caused only by incoherent scattering and capture. The neutron capture cross section was measured for the same iron sample by observing its effect on the reactivity of the graphite pile (danger coefficient method of measuring absorption cross sections) with the result that the absorption cross section is 2.5 b at 2200 m/s. Knowing the value of the absorption at 2200 meters per second and assuming a $1/v$ dependence, the solid line labeled σ_A can then be drawn. When the absorption is subtracted the points $\sigma_T-\sigma_A$ are obtained. These points show that the incoherent scattering in the region 400 to 1000 meters per sec. is about 1.5 barns and is roughly constant with velocity. If it is assumed that the incoherent scattering is 1.5 b at thermal energy also, it is possible to calculate the amount of coherent scattering at thermal energies. The coherent scattering at 2200 m/s is thus 12 b minus 2.5 b for absorption and 1.5 b for incoherent scattering giving 8 b for the coherent scattering cross section. The value of 1.5 b for the incoherent scattering is much less than that found by Whitaker[14] (3.5 b) which was used by Halpern in his theoretical calculation of neutron polarization. However, the present analysis is in good agreement with the value of 8.1 b for the coherent scattering cross section found by Fermi[15] in his work on the total reflection of neutrons from an iron mirror. The small value of the incoherent scattering means that it is unnecessary to assume that the scattering amplitudes for the different iron isotopes are of opposite sign as was assumed by Halpern. The fact that the incoherent scattering is small is also relevant to the discussion of reference 10 of the present paper.

The scattering cross section curve of Fig. 2 was used to calculate the change in shape of the neutron spectrum with penetration through iron. In this way the spectrum was calculated for various values of the iron thickness up to 6 cm. Knowing the shape of the spectrum at each d value, it was then possible to calculate the average value of the polarization cross section p which was appropriate for each particular iron thickness used in the polarization experiments. The effect of the change in shape, on "hardening" of the spectrum is to lower the average value of p as d increases.

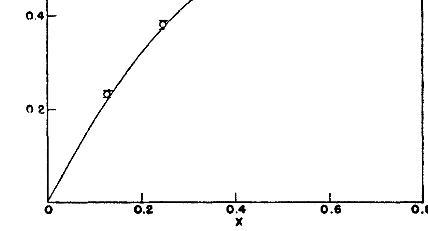

FIG. 3. The single transmission effect as measured in Armco iron. The function $f(x)$, which is the ratio of the observed to the saturated effect for a cross section of 3 barns, is shown for blocks of different thicknesses.

[14] M. D. Whitaker and H. G. Beyer, Phys. Rev. 55, 1101 (1939).
[15] E. Fermi and L. Marshall, Phys. Rev. 71, 666 (1947).

TABLE I. $H = 7000$ oersteds, Armco.

d, cm	E(percent)
1.0	2.0 ± 0.1
1.3	3.1 ± 0.2
2.0	5.9 ± 0.2
3.0	10.1 ± 0.2
6.0	25.4 ± 0.3

III. SINGLE TRANSMISSION EFFECT IN ARMCO IRON

In order to determine E_s it is necessary to measure E and then extrapolate to the saturated value by means of $f(x)$. If E is measured for different values of x, then the results can be fitted to the $f(x)$ curve of Fig. 3, assuming various values of p. In this way the extrapolated value E_s and hence p, can be determined. As x is equal to $\lambda/\epsilon d$, the value of x can be changed by changing λ, ϵ, or d. Bloch, Hammermesh, and Staub kept λ and d constant (using only a single block) and varied ϵ by changing the magnetic field H. In fitting their results to the $f(x)$ curve they had thus two adjustable constants, λ and p. They found that their results could be made to fit the $f(x)$ curve with a p value of 2.0 barns.† The difficulty in this procedure is that it is extremely hard to measure ϵ accurately, especially for high values of H. In the present work it was decided to vary d and keep ϵ constant by using the same value of the magnetizing field H for the different d values. In fitting the $f(x)$ curve there would then be the two disposable constants p and the ratio λ/ϵ.

The apparatus used for the measurements is shown schematically in Fig. 1. The neutron beam emerges from a 4-in.×4-in. hole in the thermal column of the pile. The electromagnet has exciting coils which operate at 2000 volts; the current is supplied by electronically regulated power supplies fed by 110 a.c. The total power dissipation is 3 kilowatts and at this power sufficient cooling is obtained by several fans placed near the magnet. The iron blocks are 1.2 cm high, 2 cm wide, and d cm in length while diaphragms of cadmium delineate a beam 1 cm square at the iron blocks. The neutron detector is an 8-in. long BF_3 proportional counter filled to a 20-cm pressure with enriched boron and placed so that the neutron beam passes down the axis of the counter. Even though the intensity in the beam is quite steady it was found necessary to monitor the beam to secure the desired accuracy. The monitor used was a fission ionization chamber placed just inside the graphite of the thermal column and counting rates were always recorded in terms of counts per monitor count. Careful checks were made to insure that the drop in voltage caused by the magnet current did not affect the counting rate of the BF_3 counter or the monitor counter. In order to determine whether the magnet caused any change in the counting rate aside from the polarization effect, a zero check was made by using a brass block instead of an iron block with all other conditions unchanged. With such an arrangement it was found that there was no change in the transmission with application of the magnetic field.

Measurements of the single transmission effect were made for the various block thicknesses d, keeping the magnetic field H equal to 7000 oersteds for each block by adjusting the magnet current; the field was measured at the surface of the block by a small search coil and flux meter. The values of the single transmission effect obtained are shown in Table I, with standard errors computed from the number of counts.

In order to compare the experimental values with the $f(x)$ curve it is necessary to consider the changing shape of the neutron distribution with d. In order to make this correction, the average value of p was calculated for the appropriate neutron distribution determined by the method described in Section II. In calculating the average p it was assumed that p varied with

TABLE II. Single transmission effect for different materials.

$d = 3$ cm	$H = 11{,}000$ oersteds			
Swedish iron	9.8 percent			
Armco iron	13.0			
Hot-rolled	16.6			
Cold-rolled:				
H direction	E		(Neutron direction)	
A	22.7 percent	(B)	22.2 percent	(C)
B	19.6	(A)	18.8	(C)
C	16.3	(A)	17.1	(B)
45° to A and B	19.1	(C)		

† The p value was raised to about 2.2 b in some later work: F. Bloch, R. I. Condit, and H. H. Staub, Phys. Rev. 70, 972 (1946).

neutron velocity as shown by the theoretical curve calculated by Halpern. The effect of the hardening is that the average p decreases with d and that the saturated effect for a certain d is less than would be expected if the Maxwell distribution remained unchanged through the iron block. The expected saturated effect corrected for this "hardening" of the neutron distribution is called $E_{S,H}$ and it was calculated for different values of p for the various d's used. The ratio of the experimental E to $E_{S,H}$ is the experimental $f(x)$ and the values were fitted to the $f(x)$ curve, assuming different p values and adjusting the constant λ/ϵ for each particular p. The best fit was obtained for a p of 3.0 ± 0.2 b and a λ/ϵ of 0.74 cm as shown in Fig. 3. The value of p is surprisingly high when compared with the expected value of 1.1 b from Halpern's calculation and with Bloch's experimental value of 2.0 b.

In fitting the experimental points to the $f(x)$ curve it is assumed that x values are inversely proportional to d. This is actually correct only if all velocities in the distribution have the same x value. If the x is different for various velocities, then it would be necessary to calculate an average x for the neutron distribution, and as this average x would be different for different d's (because of hardening of the beam) it would no longer be correct to assume that x would be simply proportional to $1/d$. Because λ depends on velocity if the grain size is below a certain critical size (about 0.5×10^{-3} cm, see Section VII) but is independent of velocity above the critical size, the correctness of Fig. 3 depends on the grain size in Armco. Bloch assumed that the grain size in his experiments was less than the critical size but the assumption is made here that the grain size is above the critical size. A direct determination of the grain size for the Armco blocks was made for us by Mr. H. Paine, of the Argonne Laboratory, and by Dr. J. Berger, of the University of Pittsburgh, with a resulting average grain size of 7×10^{-3} cm which is well above the critical size. If it is assumed that the grain size is small and Fig. 3 changed accordingly, the resulting p value would be about 0.2 b higher.

It is noteworthy that the values of $f(x)$ for all the blocks are distinctly less than unity and the possibility suggests itself that the high p value might be due to a misinterpretation, that is, that the $f(x)$ values are really much closer to unity and hence p much smaller than 3.0 barns. The work to be described in the next section was designed in order to eliminate this possibility by obtaining higher values of $f(x)$ and lessening the amount of extrapolation involved in obtaining E_s and p.

TABLE III. $H=12{,}000$ oersteds, cold-rolled steel.

d, cm	E, percent
0.7	1.5 ± 0.2
1.0	2.9 ± 0.2
1.7	7.7 ± 0.3
3.0	20.1 ± 0.4
6.0	59.9 ± 0.6

IV. MEASUREMENT OF p FOR COLD-ROLLED STEEL

Because the values of $f(x)$ obtained for Armco were substantially less than unity, it was decided to attempt to get higher values of x by obtaining an increase in the ratio λ/ϵ. It is very difficult to estimate in advance what materials will give high values for this ratio because of the complicated dependence of both λ and ϵ on material. As far as grain size is concerned, it is to be expected that going to either very large or very small grains would give an increase in x (see Section VII). In fact the grain size for Armco happened to be such that λ was quite small. However, using materials of larger grain size could result in smaller values of x because the ϵ obtained for a given field might be much larger.

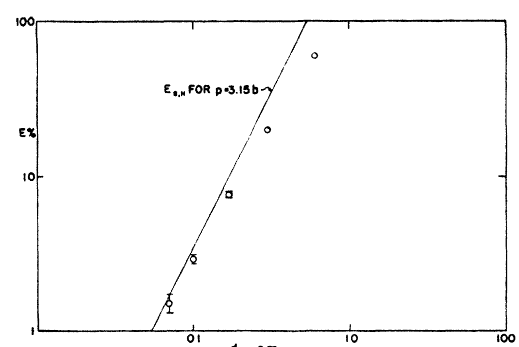

FIG. 4. The single transmission effect, E, for cold-rolled steel as a function of block thickness. The solid line shows the theoretical value for saturated iron for a p of 3.15 barns.

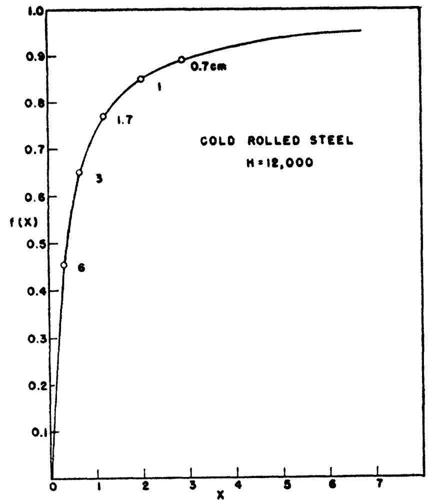

FIG. 5. The saturation function $f(x)$, and the experimental values for cold-rolled steel which lead to a p value of 3.15 barns.

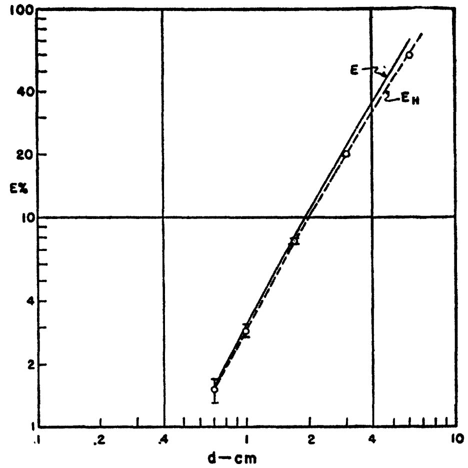

FIG. 6. Experimental values of E compared to the exact calculations for a p of 3.15 barns. E_H is the calculated curve corrected for hardening of the neutron beam.

In order to make a rapid survey of materials, various samples of iron and steel were selected and the single transmission effect was measured for each material using a d of 3 cm and an H of 11,000 oersteds (a substantial increase over the field of Section III). The values of E obtained for the different materials are listed in Table II. It is obvious from Table II that Armco is by no means the best material for production of polarized neutrons and that, of the samples tried, cold-rolled steel is the best. Several blocks of cold-rolled steel were cut to investigate the directional properties of the polarization and the results of these tests are shown in the second part of Table II. The blocks were cut so that the magnetic field was in the direction of rolling (A), perpendicular to the direction of rolling but in the plane of the rolling (B), and in the direction perpendicular to the plane of rolling (C). For each of these directions of magnetization, the neutrons were passed in two directions also. The data in Table II show that the largest effects were obtained with H in the direction of rolling with no evident variation depending on the neutron direction. The direction B gave intermediate results and C the lowest values; neither of these cases shows any variation with neutron direction. All these results are most simply interpreted as being caused by variation in ϵ, this quantity being the smallest when the magnetization is in the direction of rolling. Apparently any difference in effective grain size depending on the direction of neutron propagation is not very important.

In order to determine the value of p for cold-rolled steel, a series of blocks were cut of different d values such that H would always be in the direction of rolling. The single transmission effects measured for these blocks with a magnetic field of 12,000 oersteds are listed in Table III and are plotted against d in Fig. 4. In Fig. 4 the values for small d's show a slope of nearly 2 which means that the effect is proportional to d^2 at small thicknesses. The fact that proportionality to d^2 is obtained means that $f(x)$ must be very close to 1, for it is only when $f(x)$ does not change with d (that is with x) that a proportionality to d^2 would be observed. The solid line in Fig. 4 is the expected saturated effect corrected for hardening (as described in Section III), assuming a p value of 3.15 b. The points for small d fall very close to the solid line, showing that high values of $f(x)$ were obtained for these thicknesses. The decreasing values of $f(x)$ with increasing d are shown by the dropping off of the points below the solid line. The same results are shown in Fig. 5 where the data are plotted in terms of $f(x)$, again for a p of 3.15 b. Here, as for Armco, x is taken as inversely proportional to d; in other words, it is assumed that the grains are above the critical size. However, metallographic examination (Section VI) as well as direct experiments with polarized neutrons (Section VII) show that the grain size is large for cold-rolled steel. The experimental $f(x)$ values fit the theoretical curve very well and the resulting λ/ϵ is 2.03 cm, much higher than for Armco. The error in p, estimated by varying p for the $f(x)$ curve, is 0.1 b.

As was stated in Section I, Eq. (3) which was used for Fig. 5 is not quite exact, so the exact values of E were calculated from Halpern and Holstein's formulas. The solid line of Fig. 6 shows the exact values of E for $p=3.15$ and $\lambda/\epsilon=2.03$ while the dotted line includes the correction for hardening. The experimental points agree extremely closely with the latter curve. In conclusion, the work with cold-rolled steel substantiates the high p value obtained from Armco and makes it much more certain because $f(x)$ values were obtained which were much closer to unity. There is very little doubt that the magnetic interaction between neutrons and iron atoms and the resulting polarization are much greater than would be expected from theory.

V. POLARIZATION AS A FUNCTION OF NEUTRON VELOCITY

As discussed in Section I, the polarization cross section p is a complicated function of neutron velocity, changing discontinuously as different Miller planes become effective and decreasing suddenly for neutron wave-lengths for which no Bragg reflections are possible. The work already described had shown that the actual value of p, averaged over the Maxwell distribution, was much larger than expected theoretically, and it was felt important to investigate the values of p for monoenergetic neutrons as well.

In order to study p as a function of velocity, the experimental set-up was modified by addition to the chopper velocity selector. All the velocity selector measurements were made with a single piece of iron, a sample of cold-rolled steel of the same type used for the measurements of Section IV, with a d of 1.3 cm. The method of using the chopper to obtain narrow velocity bands of neutrons will not be described here because it is discussed in the paper of Brill and Lichtenberger. The instrument was first used with low resolution in order to make a rapid survey of a large velocity range. This survey showed that the single transmission effect increased with decreasing neutron velocity as expected, then decreased suddenly to extremely low or zero values at velocities below about 1000 m/sec. These results indicated that the theoretical curve for the variation of p with velocity was qualitatively correct and could be used as a guide in more detailed investigations.

As the region just above the cut-off is of particular importance from the theoretical standpoint (only one set of planes is involved and the form factor is constant), the region from 980 to 1380 m/sec. (the 110 planes) was studied with high resolution. The values of E measured are shown in Fig. 7 where the solid line is the E that would be expected for a d of 1.3 cm based on the theoretical curve of Halpern, assuming $f(x)=1$. It is obvious that the experimental curve behaves qualitatively according to theory, but is very much greater in magnitude. In order to

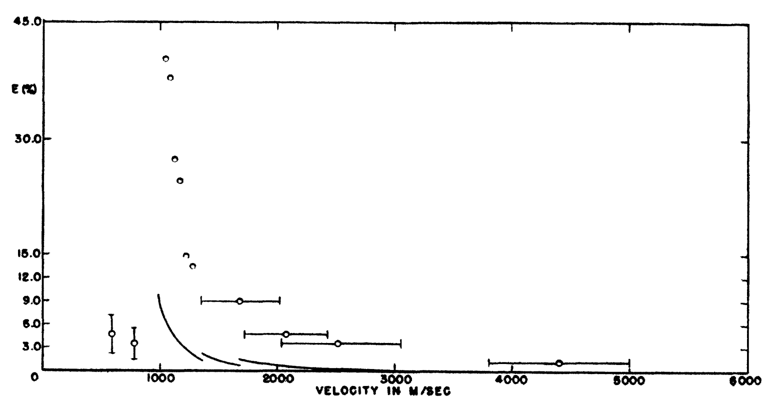

FIG. 7. The single transmission effect for a 1.32-cm block of cold-rolled steel as a function of neutron velocity. The solid curve is Halpern's theoretical curve.

see if the results for monoenergetic neutrons were consistent with those for the thermal distribution, the curve of Fig. 7 was integrated over the Maxwell distribution (of $v_0=2250$) with a resulting E of 5 percent. From Fig. 6 it is seen that 5 percent of a d of 1.3 cm agrees with the earlier thermal distribution measurements.

It is obviously of great interest to investigate whether any change in the theoretical treatment can account for the large quantitative difference between theory and experiment for the values of Fig. 7. Any changes in form factor of the iron atom would have to be quite large in order to explain the discrepancy. While it is not intended at present to attempt the determination of the magnetic form factor experimentally, it is worth pointing out that an increase in the theoretical factor would be relatively larger at high velocities. There is some evidence from Fig. 7 that the ratio of the experimental to the theoretical values is larger at high velocities in line with the behavior to be expected from a larger form factor. However, Sachs[16] has shown that a decrease in the size of the iron atom by a factor of 4 would be necessary to give p values as large as those actually observed. Halpern[17] has stated recently that some assumptions made in the theory are no longer true for such high p values as the experimental ones and correction to the theory for these assumptions might change the theoretical predictions by a sizeable amount. Wick[18] has pointed out that the assumption of randomly orientated crystal grains of the Halpern theory might not be correct for rolled steel and that a favorable distribution of grain orientation might cause a marked increase in magnetic scattering.††

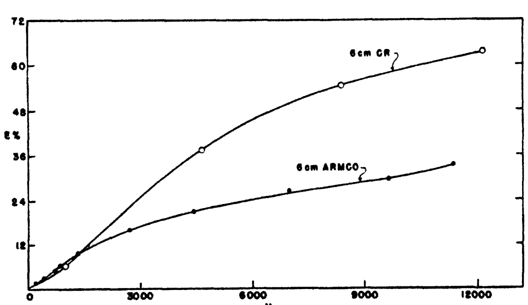

FIG. 8. The single transmission effect for different values of the magnetizing field, H.

[16] R. G. Sachs, private communication.
[17] O. Halpern, private communication.
[18] G. C. Wick, private communication.

As the theoretical curve is based only on coherent scattering it is zero below 980 m/sec. The experimental points on the other hand show a small single transmission effect at velocities below the cut-off. The existence of polarization for velocities below the cut-off means that there must exist scattering caused by crystal irregularities or inelastic scattering. This follows from the fact discussed in Section I that of the possible causes for scattering below the cut-off ("residual" scattering) only the above two possibilities would result in production of polarization. The question of the existence of polarization below the cut-off velocity is of interest[10] because of the fact that it reveals something of the nature of the incoherent scattering in iron, but it is difficult to investigate because the intensity of such slow neutrons is very low. Additional measurements were made at low velocity using BeO and graphite filters to produce slow neutron beams instead of the chopper. The filters can produce low velocity neutrons of moderate intensity even though the neutron distributions are not truly monoenergetic. Thus the BeO filter produces a neutron group of velocity about 700 m/sec., and the graphite filter a group of about 500 m/sec. The filtered neutrons showed a single transmission effect of about the same magnitude as the velocity selected neutrons but it could not be measured accurately. Recently Fryer[19] has published results of polarization measurements with cyclotron velocity selected neutrons down to 1300 m/s in velocity. He finds about a threefold increase over the thermal E for this range but his p values are much less than the present ones.

VI. THE APPROACH TO MAGNETIC SATURATION

When the polarization cross section p is known, then it is possible to obtain $f(x)$, and x, values by measuring the single transmission effect. In this way variations of the quantities entering into x, that is λ and ϵ, can be studied. The quantity ϵ shows the fraction of the magnet-

†† J. Steinberger and G. C. Wick have recently made a series of corrections to the theoretical p value with the result that agreement with the present experiments is obtained (Bull. Am. Phys. Soc. **23**, 9 (1948).
[19] E. M. Fryer, Phys. Rev. **70**, 235 (1946).

ization in the iron which is not lined up with the magnetizing field H. Thus, measurements of x can be used to determine the departure of the magnetization from the saturation value. In ordinary magnetic measurements the departure from saturation is never measured directly but only the increase in magnetization as H is increased. In the ordinary measurements, therefore, it is always necessary to assume some law for ϵ as a function of H in order to extrapolate to infinite H in obtaining the saturated value of the magnetization. Because the quantity ϵ as measured by the neutron polarization shows directly how much of the magnetization is still not aligned, the method should give valuable information on the law of approach to magnetic saturation with increasing H.

As x is very sensitive to small changes in $f(x)$ when the latter is near unity (that is for small d values) it is much better to work with small $f(x)$ values (large d's) in order to measure x from observed changes in $f(x)$. Hence, for the measurements made to study the law of approach to magnetic saturation, 6-cm blocks of iron were used and values of E were measured as a function of magnetizing field H. Figure 8 shows the data for Armco and cold-rolled steel in the most obvious manner, E as a function of H. The values of $f(x)$ as a function of H were calculated directly from the ratio of E to E_s and x obtained from the analytical form of $f(x)$. The reciprocals of the x values thus determined are strictly proportional to ϵ, the constant of proportionality being just the ratio d/λ, and Fig. 9 shows $1/x$ for Armco iron plotted against $1/H$. According to ferromagnetic theory[20] it is expected that ϵ should vary as $1/H^2$. Magnetic measurements[20] have shown the $1/H^2$ dependence for moderate values of H (2000 to 6000 oersteds), but a departure from this law in the direction of a smaller exponent of H for higher magnetizing fields. Figure 9 shows a very good $1/H$ dependence for fields ranging from 3000 to 12,000 oersteds and it is only for fields as low as 1000 that the variation becomes more nearly like $1/H^2$. An additional fact of interest is that the straight line does not extrapolate to zero as H becomes infinite. This fact is probably to be interpreted

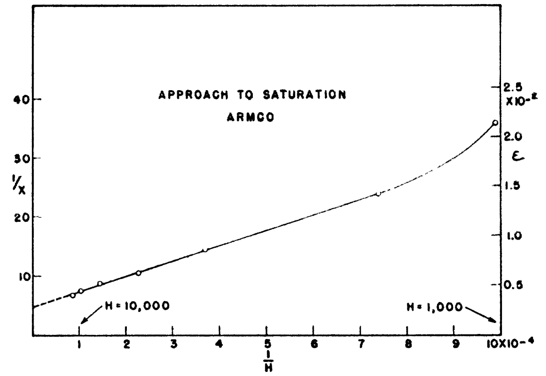

FIG. 9. The directly measured quantity, $1/x$, and the departure from magnetic saturation, ϵ, which is proportional to it, as a function of H.

in terms of a change in the exponent of H to even lower values as H is increased, such as to a $1/H^{\frac{1}{2}}$ behavior. Figure 10 is a similar plot of the measurements made with the 6-cm block of cold-rolled steel and it shows the same dependence on $1/H$ and also the same behavior at very high fields; that is, there is again evidence for a change in the $1/H$ law for high fields. W. F. Brown[21] has given a theoretical basis for laws of approach to saturation other than $1/H^2$ in terms of dislocations.

In the work of Bloch, Hammermesh, and Staub described above, the values of ϵ were determined by measuring changes in the magnetization by standard magnetic methods, which gave only changes in ϵ with changes in H rather than the actual values of ϵ. Values of ϵ were obtained by assuming that the data, which fitted a $1/H^2$ law for the region in which the magnetic measurements were made, continued to obey this law to saturation. If the findings of the present work are correct, it can be concluded that Bloch's method of getting the actual values of ϵ would be subject to error, and in a direction such that his assumed values of ϵ would be too small. The effect of such an error in ϵ would be that the resulting p value would be too small.

In order to use the polarization measurements as a basis for the determination of the actual magnitude of ϵ it is, of course, necessary to know λ. Values of λ (half the grain size) have been determined by metallographic examination of the blocks by Mr. H. Paine of the Argonne

[20] R. Becker and W. Doring, *Ferromagnetismus* (Verlag Julius Springer, Berlin, Germany, 1939) p. 167.

[21] W. F. Brown, Phys. Rev. 60, 139 (1941).

Laboratory and by Dr. J. Berger of the University of Pittsburgh. While the grain size determinations are necessarily somewhat rough in nature, they do allow an estimate of the absolute values of ϵ for the materials studied. The λ's obtained were 3.55×10^{-3} cm for Armco and 1.7×10^{-3} cm for cold-rolled steel. From these values it is then possible to calculate the actual values of λ which is done for Figs. 9 and 10 by placing a scale of values for ϵ on each figure. Thus, within the accuracy of the grain size determinations, these figures show the actual values of the departure from magnetic saturation as a function of applied field. The superiority of cold-rolled steel over Armco in producing polarized is shown in Figs. 9 and 10 to be caused by the much smaller ϵ for the former, which more than compensates for the larger grain size of the Armco.

VII. GRAIN SIZE

The quantity λ in the expression for $f(x)$ is related to the grain size of the iron blocks. However, the relationship is not unique for a given value of λ may correspond to either of two values of the actual grain size δ. The relationship between λ and δ changes when the grain size passes a critical size, l, which is twice the distance traversed by a neutron in one radian of the Larmor precession. The critical grain size, l, is about 7×10^{-4} cm for thermal neutrons. If δ is larger than l, then $\lambda\cong\delta/2$. If δ is much less than l then $\lambda\cong l^2/\delta$, and near the critical size λ is about equal to δ. These relationships show that λ increases as δ departs from the critical size in either direction and an ambiguity results

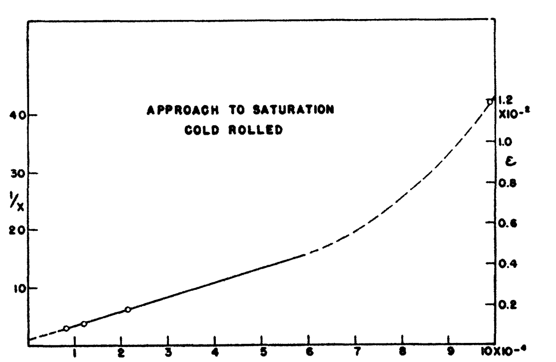

FIG. 10. The same material as Fig. 9, for cold-rolled steel.

when it is attempted to estimate the grain size from a value of λ. Bloch, Hammermesh, and Staub obtained an experimental value of λ and, assuming that δ was less than the critical size, calculated δ as 1.4×10^{-4} cm.

The quantity δ considered here is the average length of the regions in the iron in which the magnetization is constant. For highly magnetized iron δ is not the size of the usual magnetic domain but is to be interpreted as the grain size. The magnetization in the polarization experiments is so nearly complete that the region of the B-H curve in which the usual magnetic domains are evident (the region of the Barkhausen effect) has been passed, and further increase of the magnetization is caused only by rotation of the magnetic vector (uniform throughout each crystal grain), toward the direction of the applied field. If this view is correct, then it should be possible to relate the values of δ as determined in the polarization experiments with the grain size as determined metallurgically. With the object of correlating the two methods of grain size investigation, several experiments with polarized neutrons were performed and the results compared with standard grain size determinations.

Preliminary grain size determinations based on the polarization experiments gave results that were above the critical grain size, in disagreement with the findings of Bloch. In order to clear up this discrepancy it was decided to perform an experiment designed to discriminate between the small and large grain size possibilities. Use was made of the fact already discussed that for large grain sizes λ is independent of velocity, while for small grain sizes λ is proportional to the square of the velocity. It follows from the shape of the $f(x)$ curve that if ϵ is changed by a certain amount for two velocity groups, then the fractional change in $f(x)$ will be much greater for the group of lower velocity if the grain size is small. If the grain size is large, then the fractional change in $f(x)$ will, of course, be independent of velocity.

The two velocity groups used were the Maxwell distribution (about 2250 m/sec.) and a group on the last Debye-Scherrer ring (about 1140 m/sec.) obtained with the chopper. These two groups differ by a factor of four in v^2. The single transmission effect for a 1.3-cm block of cold-rolled steel was measured for both groups at a high H value (13,000 oersteds), then H was changed by a sufficient amount (to 4000 oersteds) to give a factor of 2.58 in x. Under such conditions the expected change in $f(x)$ or in the single transmission effect, for the high velocity group, is a decrease to 76 percent of the high field value (calculated from the curves of Figs. 5 and 10) whereas the expected change for the low velocity group would be to 41 percent if the grains are small, and again to 76 percent if the grains are large. The resulting values of the single transmission effect for the low velocity group are shown in Fig. 11. The average change for the points shown is a reduction to 77 percent of the high field value while the change for the thermal group is to 76.5 percent (both results good to about 8 percent). This agreement shows that the grain size is above the critical size because the change for the low velocity group is much less then would have been expected if the grain size were small.

As described in Section VI the grain sizes of the Armco as well as the cold-rolled blocks were measured metallographically by Mr. Paine and Dr. Berger. These direct measurements also show that the grain sizes are above the critical size. As the polarization gives only the ratio λ/ϵ it would be necessary to obtain ϵ by a separate magnetic measurement in order to use neutrons for determination of grain size. No measurements of ϵ have been made in this work and it is not possible to say at present whether the single transmission effect alone will be a useful tool for the determination of grain size in ferromagnetic materials. Preliminary attempts to investigate changes in grain size with metallurgical operations such as rolling and annealing have been difficult to interpret because of the fact that both ϵ and λ change with metallurgical treatment.

VIII. CONCLUSION

In view of the fact that the best polarizer of neutrons turned out to be a material which was selected by chance, a survey of various types of iron was made in an attempt to obtain still better polarizers. A variety of irons and steels was tested for polarization, but it was found that none of these was better than the sample of cold-rolled steel which was tried early in the experiments (1020 steel). The efficacy of a material in neutron polarization is determined by the ratio λ/ϵ, and because of the rather complicated dependence of ϵ on mechanical or heat treatment, it is very difficult to estimate in advance what type of treatment will give the maximum polarization. Thus increasing the grain size when it is above the maximum will certainly increase λ, but if in the process the value of ϵ for a given H is increased, then the advantage of the larger grain size might be overcompensated. The determination of the best polarizing material is, of course, intimately related to the study of the approach to saturation of Section VI, and it is intended to extend that study at a later date.

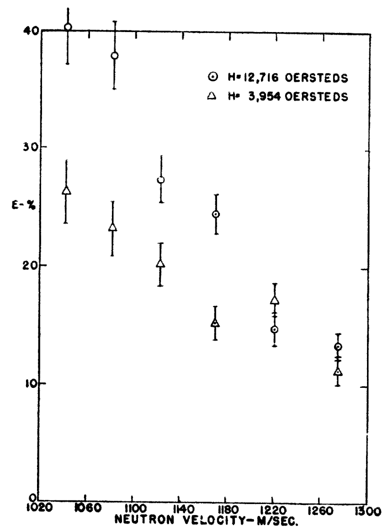

FIG. 11. The single transmission effect for two values of H at velocities just above the cut-off.

It was found possible to obtain single transmission effects in nickel and the values lead to an estimated p for nickel of about 1 b. This value of p is roughly consistant with the fact that the saturated magnetization of nickel is about $\frac{1}{3}$ the value in iron. Various samples of Permalloy showed single transmission effects which were intermediate to those of nickel and steel. Although the single transmission effects were smaller than those for steel, it was found that E_s could be obtained with very small values of magnetizing field, H, as would be expected from the magnetic properties of Permalloy.

During the course of the work described here a new effect was discovered which interfered with measurements of single transmission for some time. The new effect, which causes large single transmission results when extremely good geometry prevails, is now known to be a small angle scattering (extending to about 1°) which exists for unmagnetized iron, but which disappears when the iron is magnetized. In other words, it is some kind of small angle, magnetic scattering which disappears when the domains (not grains in this case) disappear by alignment. For the present investigation the scattering was investigated sufficiently to insure that it did not affect the results. A much more complete investigation is of course now under way.

Some of the topics investigated in the course of these measurements are worthy of much more extensive effort and it is intended to pursue them further as time permits. At the present time the "double transmission" effect (use of polarizer and analyzer magnets) is being studied, as well as the depolarization of neutron beams by passage through magnetic fields and through thin sheets of unmagnetized iron.

It is a pleasure to acknowledge our indebtedness to the many persons who have been of assistance during these experiments. Professors Bloch, Fermi, Halpern, Sachs, Teller, and Wick have contributed many invaluable discussions. Mr. T. Brill is responsible for the design and construction of all the electronic equipment we have used. Mr. H. Paine and Dr. J. A. Berger have helped greatly in the preparation and metallographic study of specimens. The cross-section curve for iron was obtained with the assistance of Mr. G. Arnold, and Mr. M. Burgy has aided in the later stages of the work.

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Proton-Proton Scattering at 7 Mev*

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Scattering experiments have been performed with protons from the cyclotron scattered in hydrogen gas. Photographic plates were used in a camera which recorded simultaneously individual protons scattered at angles from 10.5° to 45°. 5000 tracks were counted in each of nine narrow angular ranges to give good statistical accuracy. An experimentally determined correction was necessary due to proton penetration of the edges of the slits which admitted scattered protons to the photographic plates. The energy of the protons was determined by magnetic deflection and from ionization ranges. Analysis of the results shows an s phase shift somewhat lower than that predicted by extrapolation of the earlier low energy data as analyzed in terms of square well, Gauss, and meson potentials. On the assumption of central p wave scattering the results are fitted by a p phase shift of $-0.22°$ which corresponds to scattering from a repulsive square well with interior Coulomb potential, range e^2/mc^2 and height 2.2 Mev. Extending the Rarita-Schwinger neutron-proton interactions to the proton-proton system for a prediction of tensor ^3P wave scattering, we find that the experimental results are reconcilable with the "symmetrical" theory and with the "charged" theory as formally constructed by them. The "neutral" theory gives ^3P scattering in definite disagreement with the experimental results.

I. INTRODUCTION

EXPERIMENTS on the anomalous scattering of protons by protons have been reported in the energy range from 200 to 2400 kev[1–3] with results consistent with nuclear distortion of Mott

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