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The Infra-Red Spectrum of Heavy Water Vapor

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The vibration-rotation bands ν_2 and ν_3 in the heavy water vapor (D_2O) spectrum have been remeasured. An analysis of the rotational structure has been carried out. From the interpretation of the data the values of the normal frequencies have been determined as well as the values of the reciprocals of inertia effective in these two vibration states. The frequency ν_2 is found to be 1178.30 cm^{-1}, and the frequency ν_3, 2787.23 cm^{-1}. For the state ν_2 the effective reciprocals of inertia are $A(V)=16.60$ cm^{-1}, $B(V)=7.35$ cm^{-1}, $C(V)=4.84$ cm^{-1}. For the state ν_3, $A(V)=14.88$ cm^{-1}, $B(V)=7.24$ cm^{-1}, $C(V)=4.84$ cm^{-1}.

I. INTRODUCTION

THE vibration-rotation spectrum of water vapor has probably been studied more extensively than that of any other molecule. The fine structure is quite extensive since the moments of inertia are small and the rotational energies large. The rotational structure is, moreover, quite irregular since no two moments of inertia of the molecule are equal and thus the molecule is an asymmetric rotator. The spectrum of heavy water vapor is equally as extensive as that of ordinary water vapor. The band centers of the vibration-rotation bands occur at somewhat lower frequencies than the corresponding water vapor bands, however, because the reduced masses of the molecule are increased by the substitution of deuterium for hydrogen.

The far infra-red spectrum of heavy water vapor (D_2O), corresponding to transitions between the rotational energy levels only, has been measured by Fuson, Randall and Dennison[1] and term values have been assigned. Two of the fundamental vibration-rotation bands, ν_2 and ν_3, have been measured by Barker and Sleator.[2] Their resolution of the lines was not good enough to enable them to make any analysis of the rotational structure beyond the identification of a few lines in the band ν_2. The third fundamental vibration band, ν_1, has been observed by Bender[3] as a Raman line.

The incompletely resolved band ν_2 as given by Barker and Sleator has recently been interpreted by King,[4] who has used a punch card system to obtain the band contour which most resembles that obtained experimentally. From the most satisfactory fit he has obtained values for the moments of inertia.

The potential constants of H_2O have been evaluated by Bonner[5] from the observed frequencies and from the dimensions for the molecule given by Mecke.[6] Using this information, Bonner has predicted the fundamental vibration frequencies of D_2O. More recently Darling and Dennison,[7] using more complete data, have evaluated the normal frequencies of H_2O and D_2O together with values for the moments of inertia of the molecule in many of the vibrational states. The recent measurements by Nielsen[8, 9] corroborate essentially the values given by them.

II. EXPERIMENTAL

The absorption spectrum of heavy water vapor was examined with a prism grating spectrometer. The absorption cell which contained the D_2O vapor was 2.54 cm in diameter and 8 cm long. Built onto one side of the cell, and connecting to it, was a small reservoir which contained a sample of liquid D_2O. The windows of the cell were silver chloride. By using this material for the windows there were no etching effects on them due to the heavy water vapor. The windows were sealed onto the cell with red glytal.

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[1] N. Fuson, H. M. Randall, and D. M. Dennison, Phys. Rev. **56**, 982 (1939).

[2] E. F. Barker and W. W. Sleator, J. Chem. Phys. **3**, 660 (1935).

[3] D. Bender, Phys. Rev. **47**, 252 (1935).

[4] G. W. King, J. Chem. Phys. **15**, 85 (1947).

[5] L. G. Bonner, Phys. Rev. **46**, 458 (1934).

[6] R. Mecke, Zeits. f. Physik **81**, 313 (1933).

[7] B. T. Darling and D. M. Dennison, Phys. Rev. **55**, 128 (1940).

[8] H. H. Nielsen, Phys. Rev. **59**, 565 (1941).

[9] H. H. Nielsen, Phys. Rev. **62**, 422 (1942).

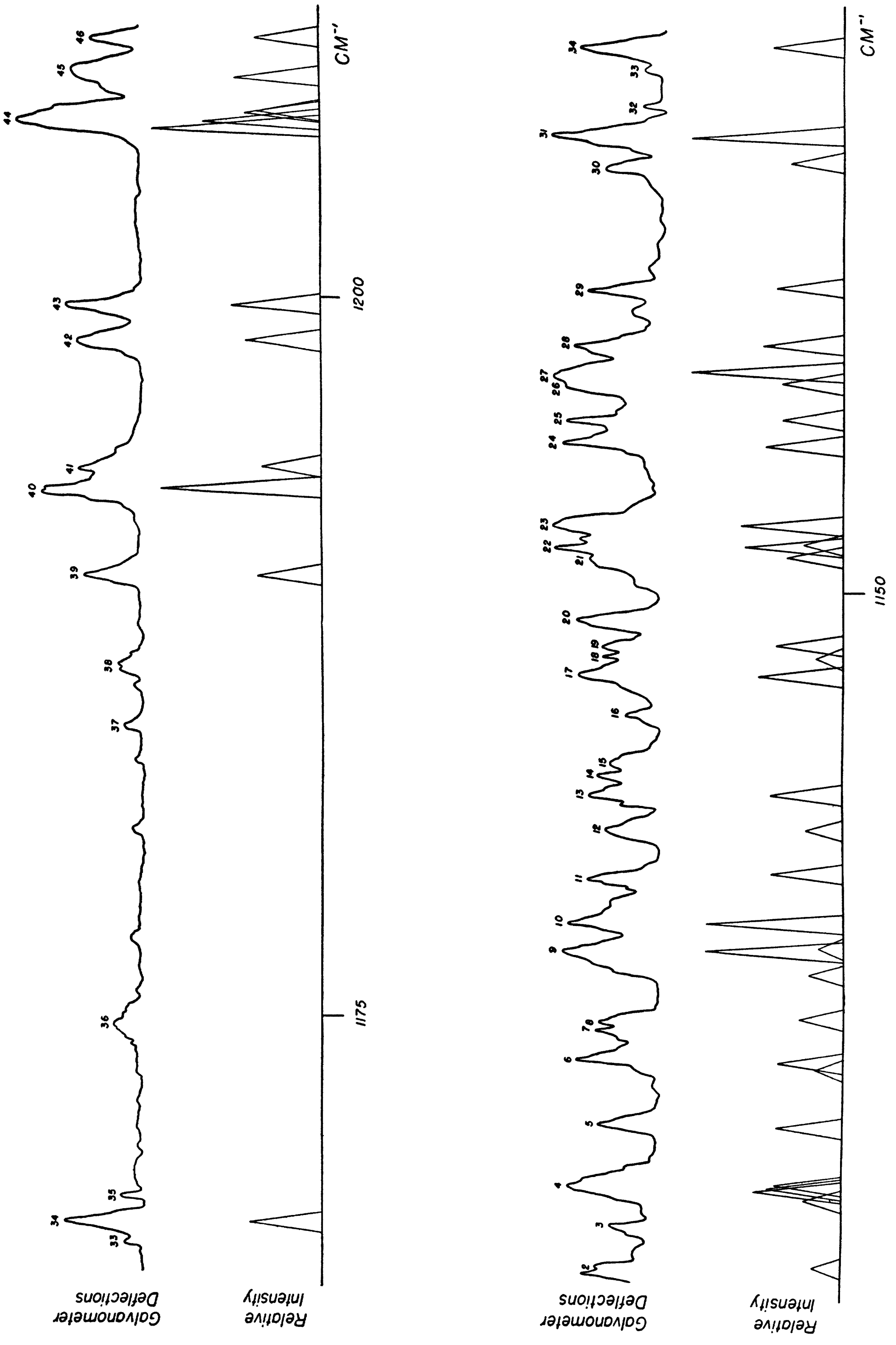

FIG. 1. Absorption band ν_2 for D_2O.

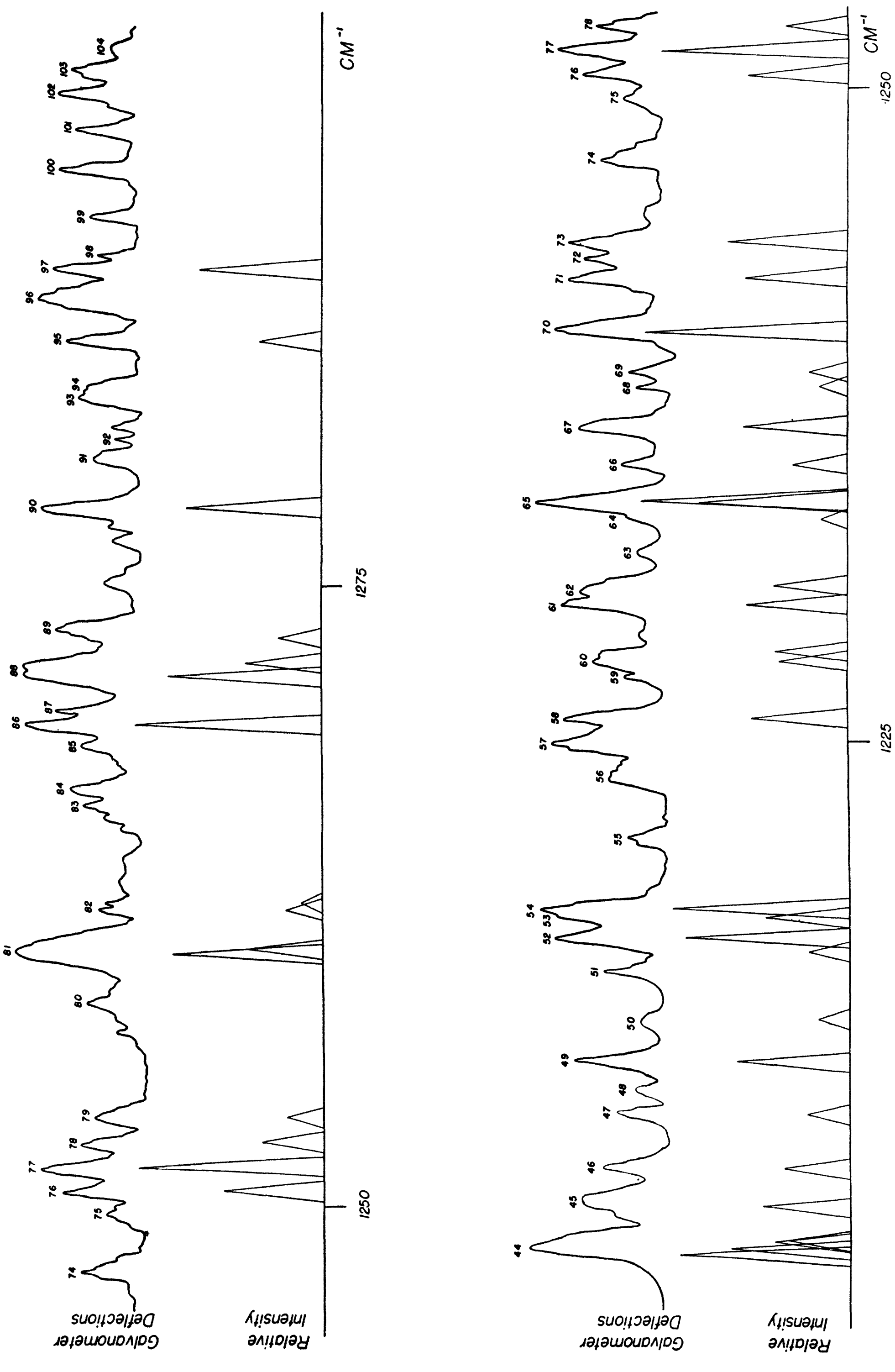

FIG. 2. Absorption band ν_2 for D_2O.

The sample of heavy water used was obtained from the Stuart Oxygen Company and contained a minimum of 99.9 percent D_2O. The sample was introduced into the reservoir of the cell through a stopcock and then frozen by immersing the reservoir in liquid air. The cell was then evacuated and the stopcock closed. When the reservoir was removed from the liquid air, the heavy water melted, and heavy water vapor filled the cell. To prevent the vapor from condensing on the windows, and a resulting loss in the amount of transmitted energy, the windows of the cell were kept slightly warmer than the rest of the cell.

The gratings used for these measurements were replicas made by R. W. Wood. The one used in measuring the band ν_2 was a grating ruled with 3600 lines per inch. The one used in measuring the band ν_3 was ruled with 7200 lines per inch. The grating constants were determined by observing lines of the mercury spectrum in high orders. The position of the lines in wave numbers were reduced to their values in vacuum, and thus the corrected wave numbers are strictly proportional to the oscillation frequencies, the more fundamental figure, since these are not changed when the medium is changed.

The fundamental vibration-rotation band ν_2 was observed in the region between 1125 cm^{-1} and 1300 cm^{-1} with the effective slit width of the spectrometer set to 0.35 cm^{-1}. Readings of the transmitted energy were made and recorded for every five seconds of arc of the grating circle. The results of the measurements may be seen by referring to the upper curves of Figs. 1 and 2. The numbers appearing over the various lines are arbitrary numbers assigned to them. These numbers are useful in quickly picking out any one line. Frequencies in wave numbers, corrected to vacuum, for the lines are given in Table I.

The fundamental vibration-rotation band ν_3 was observed in the region between 2650 cm^{-1} and 2900 cm^{-1}. The effective slit width was 0.35 cm^{-1}, and readings of the transmitted energy were made and recorded for every five seconds of arc of the grating circle. The results of these measurements may be seen by referring to the upper curves of Fig. 3. Frequencies in wave numbers, corrected to vacuum, for these lines are given in Table II.

After these measurements were taken and an analysis of the bands carried out, the automatic recording vacuum spectrometer described by

TABLE I. Frequency positions and identifications of heavy water vapor lines in ν_2.

Line No.	Obs. $\nu_{(vac)}$ (cm^{-1})	Transition
1	1128.66	
2	1128.89	4_2-4_4
3	1130.14	
4	1131.43	$3_{-3}-4_{-3}$ $5_{-4}-5_{-2}$ $6_{-6}-6_{-4}$
5	1133.78	$5_{-2}-5_0$
6	1135.41	$4_{-1}-4_{-3}$ $4_{-1}-5_{-3}$
7	1136.27	
8	1136.54	3_0-3_2
9	1138.78	$3_{-2}-4_{-4}$ 3_1-3_3 $4_{-3}-4_{-1}$
10	1139.72	$2_{-2}-3_{-2}$
11	1141.10	4_0-4_2
12	1142.63	$5_{-5}-5_{-3}$
13	1143.71	$3_{-2}-3_0$
14	1144.31	
15	1144.69	
16	1146.27	
17	1147.54	$6_{-2}-6_0$
18	1148.07	$2_{-1}-2_1$
19	1148.42	$1_{-1}-2_{-1}$
20	1149.09	
21	1151.26	$2_{-1}-3_{-3}$
22	1151.74	3_0-4_{-2} $6_{-4}-6_{-2}$
23	1152.22	$4_{-4}-4_{-2}$
24	1154.94	2_0-2_2
25	1155.65	$5_{-3}-5_{-1}$
26	1156.80	$3_{-1}-3_1$
27	1157.16	$4_{-2}-4_0$
28	1158.10	0_0-1_0
29	1159.91	$3_{-3}-3_{-1}$
30	1163.95	1_0-2_{-2}
31	1164.94	$2_{-2}-2_0$
32	1165.89	
33	1167.10	
34	1167.80	$1_{-1}-1_1$
35	1168.80	
36	1174.56	
37	1184.74	
38	1187.04	
39	1190.29	1_1-1_{-1}
40	1193.28	2_0-2_{-2}
41	1194.06	$2_{-2}-1_0$
42	1198.53	$3_{-1}-3_{-3}$
43	1199.80	1_0-0_0
44	1206.41	$3_{-3}-2_{-1}$ 3_1-3_{-1} $4_{-2}-4_{-4}$ 4_0-4_{-2}
45	1207.93	2_2-2_0
46	1209.38	$2_{-1}-1_{-1}$
47	1211.36	$4_{-2}-3_0$
48	1212.21	
49	1213.26	$6_{-2}-6_{-4}$
50	1214.73	2_1-2_{-1}
51	1216.57	
52	1217.84	$3_{-2}-2_{-2}$ $5_{-3}-5_{-5}$
53	1218.58	3_0-3_{-2}
54	1218.91	$4_{-4}-3_{-2}$
55	1221.46	
56	1223.85	
57	1225.08	
58	1225.95	$4_{-3}-3_{-3}$
59	1227.54	
60	1228.50	$5_{-2}-5_{-4}$ $6_{-4}-6_{-6}$
61	1230.23	$5_{-5}-4_{-3}$
62	1231.16	$5_{-2}-5_{-4}$
63	1232.30	
64	1233.44	3_3-3_1
65	1234.12	2_1-1_1 4_2-4_0 $5_{-4}-4_{-4}$
66	1235.52	3_2-3_0
67	1236.97	2_2-1_0
68	1238.45	$6_{-3}-6_{-5}$
69	1239.02	4_1-4_{-1}
70	1240.65	$6_{-6}-5_{-4}$
71	1242.68	$6_{-5}-5_{-5}$
72	1243.46	
73	1243.98	3_0-2_0
74	1247.24	
75	1249.64	
76	1250.46	$7_{-7}-6_{-5}$
77	1251.47	$7_{-7}-6_{-6}$
78	1252.39	$4_{-1}-3_{-1}$
79	1253.45	3_1-2_{-1}
80	1258.06	
81	1259.98	$8_{-8}-7_{-6}$ $8_{-7}-7_{-7}$
82	1261.90	4_4-4_2 4_4-4_1
83	1266.09	
84	1266.73	
85	1268.40	
86	1269.27	$9_{-9}-8_{-7}$ $9_{-8}-8_{-8}$
87	1270.00	
88	1271.64	3_2-2_2 3_3-2_1
89	1273.29	4_0-3_{-2}
90	1278.13	$10_{-10}-9_{-8}$ $10_{-9}-9_{-9}$
91	1280.31	
92	1281.00	
93	1282.63	
94	1283.20	
95	1285.21	4_1-3_{-3}
96	1287.14	
97	1288.27	4_2-3_0
98	1288.92	
99	1290.57	
100	1292.52	
101	1294.19	
102	1295.81	
103	1296.58	
104	1297.78	

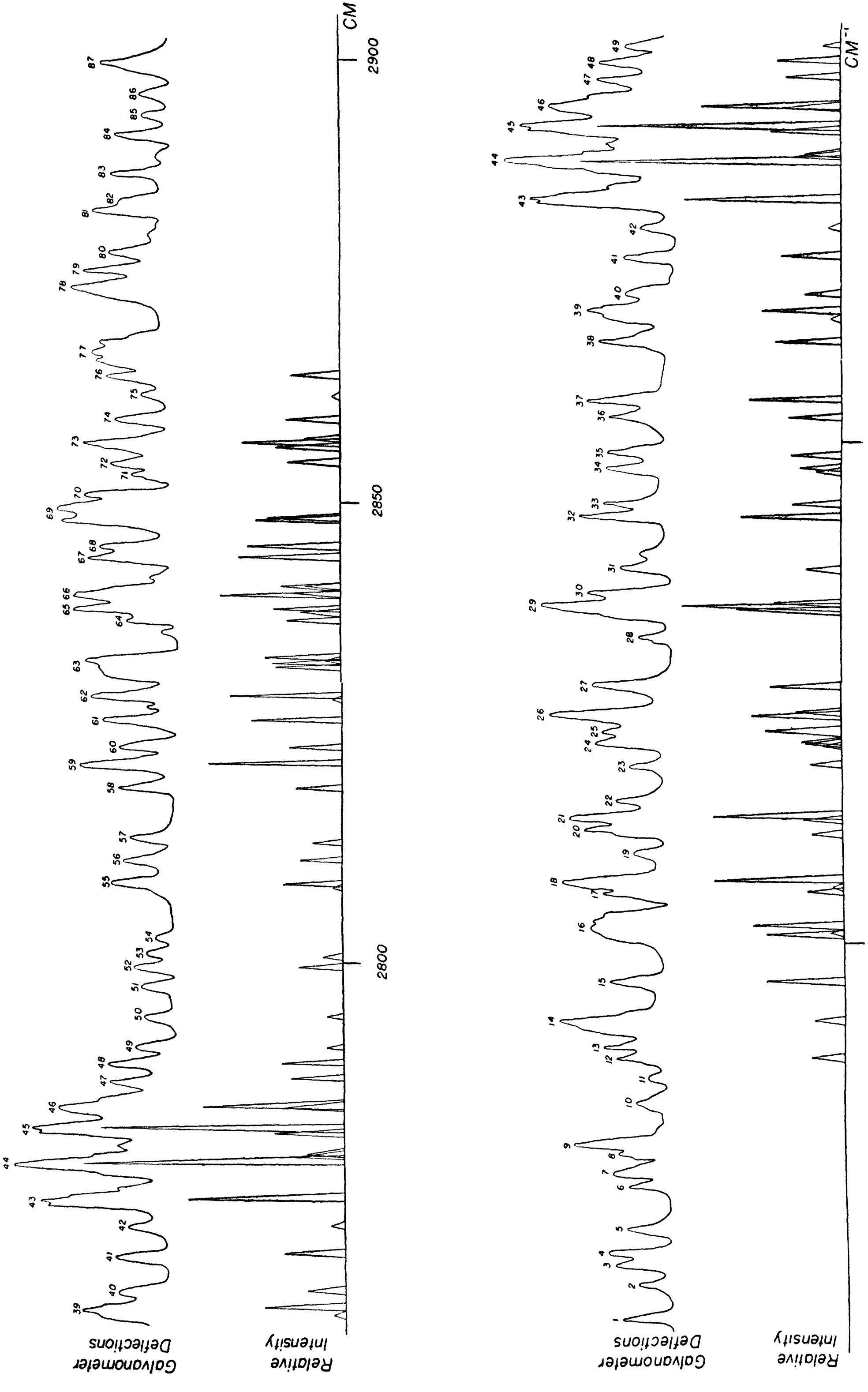

FIG. 3. Absorption band ν_3 for D_2O.

TABLE II. Frequency positions and identifications of heavy water vapor lines in ν_3.

Line No.	Obs. $\nu_{(\text{vac})}$ (cm^{-1})	Transition
1	2663.78	
2	2666.96	
3	2668.80	
4	2669.85	
5	2672.10	
6	2676.33	
7	2677.78	
8	2679.44	
9	2680.47	
10	2684.32	
11	2686.50	
12	2688.60	
13	2689.75	6_0-7_{-1}
14	2692.26	$6_{-2}-7_{-3}$
15	2696.15	5_0-6_{-1} $6_{-1}-7_{-2}$
16	2700.69	$5_{-1}-6_{-2}$ 5_4-6_3 5_5-6_4
	2701.33	5_1-6_0 $6_{-4}-7_{-5}$
	2701.96	
	2702.49	
17	2704.82	5_2-6_1 5_3-6_2
18	2705.88	$5_{-3}-6_{-4}$ $6_{-3}-7_{-4}$
19	2708.65	
20	2710.89	$5_{-2}-6_{-3}$
21	2712.28	$6_{-6}-7_{-7}$ $6_{-5}-5_{-4}$
22	2713.99	
23	2717.41	4_3-5_2 4_4-5_3
24	2719.77	$4_{-2}-5_{-3}$ 4_0-5_{-1} 4_2-5_1
25	2720.95	$4_{-2}-5_{-3}$ 4_1-5_1
26	2722.56	$5_{-5}-6_{-6}$ $5_{-4}-6_{-5}$
27	2725.48	$1_{-1}-2_2$ $4_{-1}-5_{-2}$
28	2730.25	
29	2733.40	$3_{-1}-4_{-2}$ 3_1-4_0 3_2-4_1 3_3-4_2 $4_{-4}-5_{-5}$ $4_{-3}-5_{-4}$
30	2734.48	
	2734.71	
31	2737.32	3_0-4_{-1}
32	2742.35	$3_{-3}-4_{-4}$
33	2743.67	$3_{-2}-4_{-3}$
34	2747.30	2_0-3_{-1} 2_2-2_{-1} $3_{-3}-3_0$
35	2748.90	2_1-3_0
36	2752.82	$2_{-2}-3_{-3}$
37	2754.26	$2_{-1}-3_{-2}$
38	2760.36	1_1-2_0
39	2762.81	$4_{-3}-4_{-2}$
	2763.79	$1_{-1}-2_{-2}$
40	2765.38	1_0-2_{-1}
41	2768.74	6_5-6_6
	2769.30	6_6-6_5
42	2772.11	$3_{-2}-3_{-1}$
43	2774.70	
	2775.15	0_0-1_{-1} 5_4-5_5 5_5-5_4 6_3-6_4 6_4-6_3
	2776.27	
44	2779.21	$2_{-1}-2_0$ $4_{-1}-4_0$ 4_3-4_4 4_4-4_3 5_2-5_3 5_3-5_2 6_1-6_2 6_2-6_1
45	2782.50	
	2782.95	3_0-3_1 3_2-3_3 3_3-3_2 4_1-4_2 4_2-4_1
46	2784.77	1_0-1_1 2_1-2_2 2_2-2_1
47	2787.62	3_1-3_0
48	2789.44	1_1-1_0
49	2791.27	4_0-4_{-1}
50	2794.30	2_0-2_{-1}
51	2797.57	
52	2799.63	$1_{-1}-0_0$
53	2800.90	$3_{-1}-3_{-2}$
54	2802.52	
55	2808.53	$2_{-1}-1_0$ $4_{-2}-4_{-3}$
56	2811.09	$2_{-2}-1_{-1}$
57	2813.41	2_0-1_1
58	2819.13	$3_{-2}-2_{-1}$
59	2821.36	$3_{-3}-2_{-2}$ 3_0-2_1
60	2823.24	2_1-2_{-2} 3_1-2_2
61	2826.16	$3_{-1}-2_0$
62	2828.89	$4_{-3}-3_{-2}$ $4_{-1}-4_{-4}$
63	2830.89	
	2831.13	$4_{-4}-3_{-3}$
	2831.91	4_1-3_2 5_0-4_1
	2832.19	4_2-4_3
	2832.77	
	2832.90	$4_{-1}-3_0$
64	2836.93	$4_{-2}-3_{-1}$
65	2838.36	4_0-3_1 $5_{-4}-4_{-3}$
66	2839.90	$5_{-5}-4_{-4}$
	2840.80	$5_{-2}-4_{-1}$ 5_1-4_2 5_2-4_3 5_3-4_4
67	2844.00	$5_{-3}-4_{-2}$
68	2845.15	$5_{-1}-4_0$
69	2847.90	$6_{-5}-7_{-6}$ $6_{-6}-5_{-5}$
	2849.34	
70	2850.66	
71	2853.07	
72	2854.26	6_1-5_2 6_2-5_3
73	2856.44	$6_{-3}-5_{-2}$ $6_{-1}-5_0$ 6_0-5_1
74	2859.34	$6_{-4}-5_{-3}$
75	2861.40	3_1-2_{-2}
76	2864.06	$6_{-2}-5_{-1}$
77	2865.29	
	2865.88	
	2866.74	
	2867.96	
78	2872.95	
	2874.18	
79	2876.02	
80	2877.79	
81	2882.90	
82	2884.01	
83	2886.85	
84	2891.43	
85	2893.54	
86	2896.25	
87	2899.77	

Bell, Noble, and Nielsen[10] became available at the laboratory, and as a matter of interest, the bands were remeasured on this instrument. The results of these measurements of the band ν_2 are shown in Fig. 4. It should be noted that the gain was increased on the high frequency side of the band. The measurements of the band ν_3 are shown in Fig. 5. Here the gain was reduced over the region occupied by the strong central lines of the Q branch, that is lines 42 to 50, and was somewhat different over the region occupied by lines 51 to 79. A section of the automatic recorded spectrum tracing may be seen in Fig. 6. This includes the region from the center of the band ν_2 to line 67.

III. IDENTIFICATION OF THE OBSERVED LINES

The identification of the lines measured in the two bands was made by following the method used by Nielsen[8, 9] in identifying the lines in the spectrum of water vapor. Each line in one of the bands ν_i is due to the molecules making a transition from a rotation level in the normal vibration state to a rotation level in the vibration state ν_i, and the frequency of such a line will be the difference between a term value in ν_i and one in the normal state. The selection rules governing what transitions may take place have been given by Dennison.[11]

Term values for the normal state have been determined experimentally by Fuson, Randall

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[10] E. E. Bell, R. H. Noble, and H. H. Nielsen, Rev. Sci. Inst. **18**, 48 (1947).

[11] D. M. Dennison, Rev. Mod. Phys. **3**, 280 (1931).

and Dennison.[1] Term values for the state ν_t will be the sum of the vibrational energy plus the rotational energy. Shaffer and Nielsen[12] have given the following expression for the rotational energies of the non-linear triatomic molecule:

$$E(V,J,\tau)=J(J+1)\tfrac{1}{2}(A(V)+B(V)) +\rho_1-J^2(J+1)^2\rho_2+\epsilon_\tau, \quad (1)$$

where ρ_1 and ρ_2 are constants; $A(V)$, $B(V)$, and $C(V)$ are the reciprocals of inertia; V_1, V_2 and V_3 are the vibrational quantum numbers, J is the quantum number of total angular momentum, and ϵ_τ is one of the $2J+1$ roots of the secular determinant which is used to diagonalize the rotational part of the Hamiltonian. Nielsen[8] has expanded this determinant for all values of J up through $J=6$, and given the algebraic relations for ϵ_τ. Constants which occur in (1) may be evaluated by first identifying, by trial and error, enough lines in the spectrum of ν_1 to enable the determination of the term values for $J=0$, $J=1$, and $J=2$. When these term values are known, enough relationships of (1) will be available to permit an evaluation of the constants for the state ν_t. For example, if a line is identified as being due to the transition 1_1-1_{-1}, then the term value for 1_1 in the state ν_t will be equal to the frequency of the line plus the term value for 1_{-1} in the normal state. The correctness of these term values may be checked by using them, together with the term values in the normal state and the selection rules, to see if lines which would occur from such values are actually found in the measured spectrum.

Once the constants of (1) are known, rotational energies and then term values for $J=3$ may be calculated. These term values will give rise to additional spectral lines. A check is then made of the measured spectrum to see if such lines occur. Once these lines are located in the measured spectrum new term values for $J=3$ may be determined from the experimental data. The new term values may be used to improve the values of the constants of (1). The process is then repeated for still higher values of J.

In an attempt to evaluate the centrifugal stretching coefficients from the data of Fuson, Randall, and Dennison[1] by fitting the energy

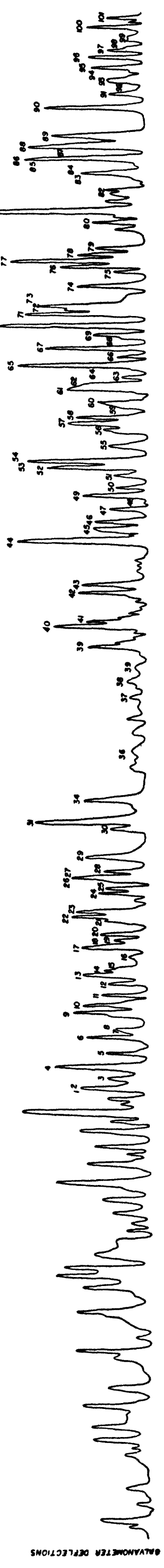

FIG. 4. ν_2 for D_2O measured with automatic recording spectrometer.

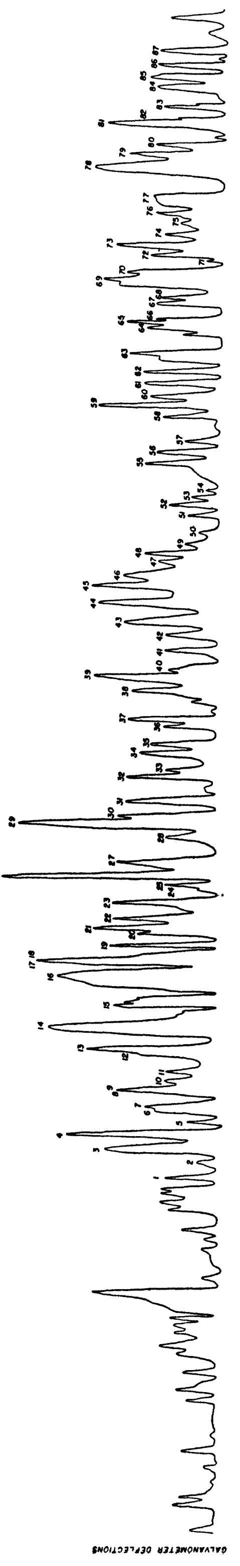

FIG. 5. ν_3 for D_2O measured with automatic recording spectrometer.

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[12] W. H. Shaffer and H. H. Nielsen, Phys. Rev. 56, 188 (1939).

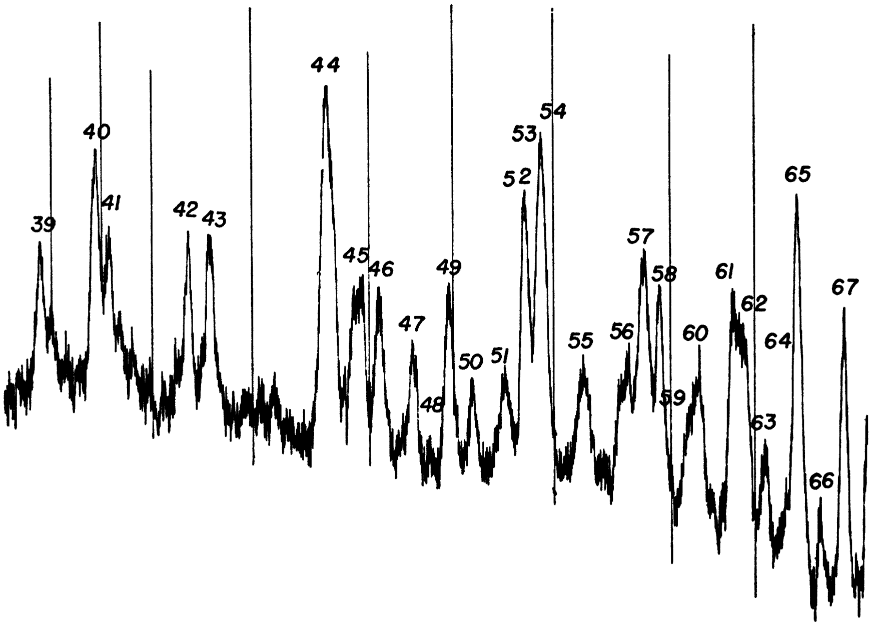

FIG. 6. Section of record of absorption band ν_2 for D_2O.

relations up to $J=6$ to their term values, no consistant values could be obtained. This was interpreted to mean that the effect of the centrifugal stretching on these levels was so small that it was less than the experimental accuracy. Since our evaluation of the moments of inertia of the molecule is made from transitions where J is not greater than 6, the effect of centrifugal stretching has been neglected. It may, in fact, be seen by reference to the relations (39) and (42) of the work of Shaffer and Nielsen[12] that the centrifugal stretching might here be expected to be approximately $\frac{1}{16}$ the magnitude of those for H_2O.

As a further aid in identifying the lines of the spectrum, the relative intensities of the lines have been calculated by using the line strengths given by Cross, Hanier, and King.[13] That is:

$$\sum_{X'Y'Z'} \sum_{M''} \sum_{M'} |(\phi_{Fg}{}^A)_{J''_{\tau''}M'';\, J'_{\tau'}M'}|^2, \tag{2}$$

where ϕ_{Fg} represents the direction cosine element of the asymmetric rotator and $X'Y'Z'$ are the space fixed Cartesian coordinates. Their values are for various parameters of asymmetry. That is:

$$K=\frac{2B(V)-A(V)-C(V)}{A(V)-C(V)}. \tag{3}$$

The values used were those tabulated for $K=-0.5$, since the parameter of asymmetry of D_2O in the normal state is -0.54, in the state $\nu_2-0.57$, and in the state $\nu_3-0.53$.

Term values determined in this manner for the frequencies ν_2 are tabulated in Table III.

TABLE III. Vibration-rotation term values for the frequencies ν_2.

J_τ	ν_2	J_τ	ν_2
0_0	1178.30 cm^{-1}	4_0	1347.51 cm^{-1}
1_1	1202.37	4_{-1}	1341.16
1_0	1199.75	4_{-2}	1321.16
1_{-1}	1190.39	4_{-3}	1296.20
2_2	1257.15	4_{-4}	1293.56
2_1	1256.70	5_{-2}	1401.19
2_0	1229.00	5_{-3}	1386.12
2_{-1}	1221.50	5_{-4}	1348.83
2_{-2}	1214.20	5_{-5}	1348.60
3_3	1345.44	6_{-2}	1492.79
3_2	1345.33	6_{-3}	1471.05
3_1	1295.39	6_4	1460.43
3_0	1293.18	6_{-5}	1411.56
3_{-1}	1268.79	6_{-6}	1410.61
3_{-2}	1253.55	7_{-6}	1483.39
3_{-3}	1248.67	7_{-7}	1483.06
4_4	1468.00	8_{-7}	1565.30
4_3	1468.00	8_{-8}	1565.22
4_2	1398.06	9_{-8}	1657.07
4_1	1397.29	9_{-9}	1657.02

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[13] P. C. Cross, R. M. Hanier, and G. W. King, J. Chem. Phys. 12, 210 (1944).

TABLE IV. Vibration-rotation term values for the frequencies ν_3.

J_τ	ν_3	J_τ	ν_3
0_0	2787.23 cm^{-1}	5_5	3186.33 cm^{-1}
1_1	2809.58	5_4	3186.33
1_0	2807.36	5_3	3110.05
1_{-1}	2799.57	5_2	3110.05
2_2	2859.10	5_1	3046.64
2_1	2858.71	5_0	3037.68
2_0	2836.05	5_{-1}	3009.18
2_{-1}	2828.78	5_{-2}	2998.50
2_{-2}	2823.14	5_{-3}	2985.40
3_3	2939.40	5_{-4}	2955.63
3_2	2939.39	5_{-5}	2954.61
3_1	2897.42	6_6	3351.00
3_0	2895.00	6_5	3351.00
3_{-1}	2875.36	6_4	3260.56
3_{-2}	2861.04	6_3	3260.56
3_{-3}	2857.11	6_2	3185.20
4_4	3048.27	6_1	3184.95
4_3	3048.27	6_0	3126.60
4_2	2988.71	6_{-1}	3122.79
4_1	2988.36	6_{-2}	3094.53
4_0	2949.97	6_{-3}	3073.91
4_{-1}	2943.00	6_{-4}	3065.33
4_{-2}	2925.72	6_{-5}	3017.80
4_{-3}	2903.47	6_{-6}	3017.10
4_{-4}	2901.37	—	——

Term values for the frequencies ν_3 are tabulated in Table IV. The identification of lines is given in the third columns of Tables I and II. Immediately below the measured spectrum shown in Figs. 1, 2, and 3 is shown a spectrum which one would expect using the term values given in Tables III and IV. The lines are drawn as triangles, where the base is twice the equivalent slit width and the height is proportional to the relative intensity, which was determined in the manner described. The term values given for the state ν_2 with values of J higher than 6 were not determined in the manner discussed but rather from the characteristics of the spectrum and the calculated relative intensities.

The term values up to $J=6$ have been employed to calculate the values of the reciprocals of inertia $A(V)$, $B(V)$, and $C(V)$. The values which seem most consistent with the term values given in Tables I and II are stated in Table V. In the same table are stated the values of the corresponding effective moments of inertia I_A, I_B, and I_C for the normal state and the states ν_2 and ν_3.

Darling and Dennison[7] have shown that the quantity $\Delta = I_C - I_A - I_B$ is one which, for planar molecules, is independent of the coefficients of the anharmonic part of the potential energy. Its value may therefore be computed from a knowledge of the normal frequencies and the shape of the molecule. The values of Δ for the states ν_2 and ν_3 computed with the aid of the relation of Darling and Dennison, are also given in Table V, together with the experimentally determined values. It is somewhat disappointing that the computed value of Δ for the state ν_3 seems to differ appreciably from the measured value.

The data of Barker and Sleator[2] has been analyzed by King,[4] using a punch card method developed by them. His values for the moments of inertia, which were originally chosen to obey the Δ condition, vary slightly from the values given by us. The values of the moments of inertia chosen by King are such as to give the best over-all agreement with the contour of the incompletely resolved absorption pattern reported by Barker and Sleator. The values are therefore those which fit the term values of high J values best. No compensation has been made by him for centrifugal stretching. While the centrifugal stretching is much smaller here than in the case of ordinary water vapor, it can probably not be neglected for the higher values of J.

We have, of course, never employed the

TABLE V. Values of $A(V)$, $B(V)$, $C(V)$, I_A, I_B, I_C, and Δ.

	Normal state*	ν_2	ν_3
$A(V)$	15.38 cm^{-1}	16.60 cm^{-1}	14.88 cm^{-1}
$B(V)$	7.25	7.35	7.24
$C(V)$	4.83	4.84	4.84
I_A	1.820×10^{-40} g cm^2	1.686×10^{-40} g cm^2	1.881×10^{-40} g cm^2
I_B	3.860×10^{-40}	3.808×10^{-40}	3.866×10^{-40}
I_C	5.794×10^{-40}	5.782×10^{-40}	5.782×10^{-40}
$\Delta_{obs.}$	0.114×10^{-40}	0.288×10^{-40}	0.035×10^{-40}
$\Delta_{calc.}$	0.105×10^{-40}	0.327×10^{-40}	0.084×10^{-40}

* See reference 1.

method of King and are, therefore, not really qualified to criticize it. We do know, however, that his values of the moments of inertia do not yield lines at positions which fit our data as well as the values of the moments of inertia which we have given in Table V. We believe this to be because his values are chosen so as to give the best agreement at considerable distances from the center of the band where the centrifugal distortion is of considerable significance. They must, therefore, deviate from the values which would give the best fit near the center of the band. In our determination, compensation for centrifugal stretching has also been neglected. The values of the moments of inertia were, however, determined from term values of small J value where the centrifugal stretching is small and may be neglected. Since the lines which have been used to determine these term values are known accurately to about 0.05 cm^{-1}, we believe the values obtained by us for the moments of inertia are somewhat more reliable.

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The Conservation of Momentum in the Beta-Decay of Y^{90}*

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Monolayer sources of yttrium (90) are formed by evaporation in a vacuum of 10^{-7} mm of Hg. The momentum of individual recoil ions is measured by timing their flight in a field free space. The positively charged Zr (90) recoils appear to lose about 6-electron volts energy as they escape from the surface. If a 6-electron volt correction is made to the observed recoil spectra, there is good agreement regarding shape and absolute amplitude with those recoil spectra calculated on the assumption of a neutrino-electron angular correlation function of $(1-\beta\cos\theta)^2$. For this function the most probable angle between the neutrino and the electron is ~135°. These results are in agreement, within experimental error, with earlier data obtained with P^{32}. The exact limits of error due to chemical effects at the surface cannot yet be assigned, so it is not possible to make a definite selection of the form of the neutrino-electron angular correlation function.

1. INTRODUCTION

In earlier experiments[1] the momentum spectra of the recoil ions from monolayer sources of P^{32} ($E_{max}=1.72$ Mev, half-life, 14.3d) were measured. It was concluded that momentum is not conserved between the electron and the recoil nucleus. If one assumes the existence of a neutrino, which is most probably emitted in the opposite hemisphere from the electron, one accounts for the experimentally observed recoil momentum spectra. The experiments reported here measure the recoil momentum spectra of yttrium (90) ($E_{max}=2.16$ Mev, half-life, 62 hr.).

These experiments permit a rather critical test of the previous results, since the Zr^{90} recoil ion from Y^{90} is very different from the S^{32} recoil ion from P^{32}. The recoil ions in these two cases differ appreciably regarding (1) maximum velocity, (2) maximum energy, (3) maximum momentum, (4) mass, (5) ionization potential (S=10.3 volts, Zr=6.92 volts), and (6) the fraction of the recoils that escape in the charged state. If one assumes a $(1-\beta\cos\theta)^2$ neutrino-electron angular correlation function for both cases, about 8 percent of the sulfur, and about 87 percent of the zirconium recoils escape charged.

In addition, the nature of the freshly evaporated foundation surface on which the monolayer is deposited is different. P^{32} was deposited on

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[1] C. W. Sherwin, Phys. Rev. **73**, 216 (1948).