The energy balance of the spectrum is shown in Fig. 2:

(a) $S(n)$ the turbulent energy transport,
(b) $d\epsilon v/dn$ the rate of viscous dissipation per frequency (wave number),
(c) $n(d\epsilon v/dn)$ the rate of viscous dissipation per turbulent "cascade-stage" (e.g., per octave).

Other formulae have been obtained connecting ϵ_0, the total dissipation of energy, with F_0, n_0 and, similarly, with R_λ, the Reynolds number of the microscale.

Disregarding the lower frequency part of the spectrum, we can compute correlation functions and other statistical properties that could be measured by cutting off the low frequency component.

A more detailed account will be published soon.

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** A. Obukhoff, Comptes Rendus (Doklady) U.S.S.R. **32**, No. 1; L. Onsager, Phys. Rev. **68**, 286 (1945); C. F. von Weizsacker (unpublished, 1945).

*** This assumption is based on the fact that the energy transport is due to the non-linear "inertia terms" in the Navier-Stokes equation. Since the product of two Fourier components of differing frequencies gives zero time average, the different stages of the cascade are uncorrelated and it is likely that no contribution is made to the energy transport by other frequencies.

† The viscous terms are linear in the Navier-Stokes equation, and the non-linear terms are not affected by viscosity. Therefore, it is natural to assume similarity in the energy transport mechanism.

†† L. F. G. Simons, Proc. Roy. Soc. **165** (1938).

On the Quantum Theory of Ferromagnetic Resonance

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A FERROMAGNETIC material, magnetized up to its saturation value M_0 by means of a constant external field H_0, shows interesting resonance phenomena when small additional high frequency magnetic fields are applied. Kittel[1] has been able to give an explanation of the effect by using classical equations of motion for the magnetic dipoles. In the special case of a non-conducting ellipsoidal specimen, magnetized in the direction of one of its principal axes (z axis), it is easy to derive an expression for the induced alternating magnetization m when a small external alternating field $h_z \exp j\omega t$ is applied, provided that the dimensions of the ellipsoid are small in comparison with the wave-lengths of the electromagnetic waves inside the material. Neglecting damping terms and anisotropy effects one finds[2] exponential time factors being omitted):

$$[\gamma^2\{H_0+(N_x-N_z)M_0\}\{H_0+(N_y-N_z)M_0\}-\omega^2]m_x = \gamma^2 M_0\{H_0+(N_y-N_z)M_0\}h_x,$$

$$[\gamma^2\{H_0+(N_x-N_z)M_0\}\{H_0+(N_y-N_z)M_0\}-\omega^2]m_y = j\omega\gamma M_0 h_x,$$

where N_x, N_y are the demagnetization factors in the x, y directions, and so on, and where γ should be equal to the gyromagnetic ratio of the spins.

In order to justify the use of classical equations of motion, it is our aim to derive these expressions from quantum-mechanical considerations. We use as a starting point a Hamiltonian containing terms describing the interaction of the spins with the external magnetic fields, an isotropic exchange interaction between nearest neighbors, and the magnetic dipole-dipole interaction between the spins in the small ellipsoid. When the constant external field is sufficiently high and the temperature is sufficiently low only those stationary states of Schrödinger's equation will play a role for which there is nearly complete alignment of the spins. Holstein and Primakoff[3] have developed an approximate method for dealing with these states and have determined the corresponding eigenvalues of the energy in a first approximation. Their method can be considered as an improved Bloch spin-wave treatment of ferromagnetism; the improvement consists in including the dipole-dipole terms.

When we now consider the additional alternating field as a small perturbation, we can determine m_x and m_y by means of a method analogous to that used in the theory of coherent scattering. Resonance will occur at frequencies for which $\hbar\omega$ is equal to the difference of the energies of two stationary states between which the alternating magnetic field can cause transitions. We find that in H and P's approximation the energy difference between any two of these states is always equal to $2\beta[\{H_0+(N_x-N_z)M_0\}\times\{H_0+(N_y-N_z)M_0\}]^{\frac{1}{2}}$ and that the approximation eventually leads to expressions for m_x and m_y which are identical with the classical expressions if we put $\gamma=2\beta/\hbar$. Here β is the magnetic moment of the spins, and thus γ is equal to the gyromagnetic ratio in the case of free spins. By taking into account terms in the Hamiltonian, which are neglected in H and P's approximation, it should also be possible to estimate the width of the resonance line.

Our quantum-mechanical consideration suggests a possible explanation for the observed discrepancy[4] between the value of γ determined by resonance experiments and the gyromagnetic ratio. When we assume that spin-orbit interaction causes a deviation of the effective magnetic moment β of the spins from the spin only value μ_B, let us say $\beta>\mu_B$, then $\gamma>2\mu_B/\hbar$. At the same time the spin-orbit interaction will cause a still larger deviation of the effective angular momentum of the spins from the spin only value $\frac{1}{2}\hbar$. Therefore the gyromagnetic ratio may be expected to be smaller than $2\mu_B/\hbar$. It would be interesting if this connection between the deviations of the angular momentum and γ from spin only values could be verified experimentally. A fuller account of our calculations will be given shortly.

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[2] C. Kittel, Phys. Rev. **73**, 155 (1948).
[3] T. Holstein and H. Primakoff, Phys. Rev. **58**, 1098 (1940).
[4] W. A. Yager and R. M. Bozorth, Phys. Rev. **72**, 80 (1947).