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On the Hyperfine Structure of Deuterium

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THE unexpected effects revealed by the very accurate measurements of the hyperfine structure of hydrogen and deuterium[1] have, as is well known, given rise to a re-examination of the consequences of quantum electrodynamics which has offered a convincing explanation of the main features of the phenomenon.[2] Still, a discrepancy so far remains between theory and experiment as regards the ratio between the HFS splittings in H and D. Assuming that the nuclei can be represented by charged mass points endowed with spin and magnetic moments, this ratio should be given by[3]

$$\kappa = \tfrac{3}{4}(\mu_D/\mu_H)\cdot(m_D/m_H)^3, \tag{1}$$

where μ_D and μ_H are the magnetic moments of the deuteron and the proton, respectively, while m_D and m_H are the reduced electronic masses in the two atoms. The measurements, however, give a value for κ which is larger by 0.017 percent than that given by (1), whereas the estimated experimental uncertainty is only about 1:100,000.[4]

By taking the compound structure of the deuteron into account, it appears possible, however, to explain the major part of this discrepancy, at any rate. In fact, because the electron is bound by the charge of the proton, there will be an essential asymmetry between the magnetic interaction of the electron, on the one hand with the proton, and on the other hand with the neutron, of which the deuteron is composed. Thus, in the interior part of the atom, the electron "moves" very fast compared with the nucleons and in this region, therefore, the motion of the proton will only adiabatically influence the electron binding, the characteristics of which will be approximately stationary with respect to the position of the proton. Whereas the influence on the energy state of the atom of the magnetic moment of the proton will correspond with high accuracy to that of a central magnetic dipole, the effect of the neutron magnetic moment will consequently equal that of a spatial magnetization distributed with a density given by the wave function describing the motion of the neutron relative to the proton. Because of the symmetry of this distribution, the effective magnetic field of the neutron largely cancels within distances comparable to the deuteron radius. Since the magnetic moment of the neutron is negative, one should expect a relative increase in the HFS of D corresponding to

$$\epsilon = \Delta\kappa/\kappa \approx (\mu_N/\mu_D)\cdot(d/a), \tag{2}$$

where μ_N is the magnetic moment of the neutron while d and a represent the radii of the deuteron and the deuterium atom, respectively. In fact, because of the rapid increase of the magnetic field with decreasing distance from the nucleus, the contribution to the HFS of any spherical shell of the atom, of given thickness, is approximately independent of the radius. Since $d/a \approx 10^{-4}$, the estimate (2) will be seen to give a value of ϵ which is just of the same order of magnitude as the observed effect.

The fact that, in the interior part of the atom, the proton, rather than the deuteron, responds to the electronic motion implies, of course, a certain modification of the correction factor in (1), depending on the reduced mass of the atom. The effect would, however, hardly seem to exceed the experimental uncertainty in κ, since the part of the atom in which the adiabatic adjustment holds extends only to distances from the nucleus of the order

$$\rho = d(M/m)^{\frac{1}{2}}, \tag{3}$$

where m and M are the electron and nucleon

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[1] J. E. Nafe, E. B. Nelson, and I. I. Rabi, Phys. Rev. **71**, 914 (1947); D. E. Nagle, R. S. Julian, and J. R. Zacharias, Phys. Rev. **72**, 971 (1947).
[2] Julian Schwinger, Phys. Rev. **73**, 416 (1948).
[3] G. Breit and E. R. Meyerott, Phys. Rev. **72**, 1023 (1947).
[4] J. E. Nafe and E. B. Nelson, Phys. Rev. **73**, 718 (1948). I am indebted to Professor Rabi for kindly informing me of the experimental results and for illuminating discussions regarding their theoretical interpretation.

mass, respectively. This result may be obtained from well-known arguments regarding the influence of molecular vibrations on electron binding states; and it may be noted that the distance ρ just corresponds to the atomic region where the restitutional frequency, $\hbar/m\rho^2$, of the electron configuration is comparable with the frequency, $\hbar/Md^2$, of the intrinsic nuclear motion.

Although very small compared with a, the value of ρ is still large compared with d, and in the inner atomic region where effects depending on the nuclear constitution become significant, one thus obtains a high approximation by assuming the electron binding to adjust itself to the momentary position of the proton. It may be added that this picture should not be essentially affected by exchange effects between the proton and the neutron, the frequency of which is also small compared with the local electron frequency. The contribution of the central part of the atom to the electron-neutron magnetic interaction will thus depend on the integral

$$\mathfrak{F}=\int\int\varphi^*(r)\chi^*(R) \times\{-(e/c)\boldsymbol{\alpha}\mathbf{A}\}\varphi(r)\chi(R)d\tau_r d\tau_R, \quad (4)$$

where $\varphi(r)$ and $\chi(R)$ are the wave functions representing the motion of the electron and the neutron relative to the proton. The operator $\boldsymbol{\alpha}$ is the Dirac vector, and

$$\mathbf{A}=\boldsymbol{\mu}_\mathrm{N}\times\nabla_R\frac{1}{|\mathbf{r}-\mathbf{R}|} \quad (5)$$

is the vector potential due to the magnetic moment of the neutron. In the expression (4) the spinor parts of the wave functions have been left out since these play the same part as in usual HFS calculations where the deuteron is considered as a single particle. In fact, because of the very strong coupling between the spins of the neutron and the proton, we may, even in the region around the nucleus, with high approximation disregard combinations of the electron spin with the spins of the individual nuclear particles.

The integral (4) may be conveniently written

$$\mathfrak{F}=\int\varphi^*(r)\{(e/c)\boldsymbol{\alpha}\cdot(\boldsymbol{\mu}_\mathrm{N}\times\nabla_r)P(r)\}\varphi(r)d\tau_r, \quad (6)$$

where

$$P(r)=\int\chi^*(R)\frac{1}{|\mathbf{r}-\mathbf{R}|}\chi(R)d\tau_R \quad (7)$$

involves the structure of the deuteron. This latter integral is easily estimated if, to a first approximation, we neglect the range of the nuclear forces compared with the deuteron radius. In fact, the deuteron wave function will then be independent of specific assumptions regarding these forces and may be written

$$\chi(R)\sim(1/R)e^{-R/d}, \quad (8)$$

with

$$d=\hbar/(MW)^{\frac{1}{2}}, \quad (9)$$

where W is binding energy of the deuteron. From (7) we thus get

$$\nabla P(r)\sim-(\mathbf{r}/r^3)(1-e^{-2r/d}), \quad (10)$$

which shows that the finite size of the deuteron simply implies an extra factor $(1-e^{-2r/d})$ in the radial part of the integral (6). In the case of the 1s state for which $\varphi(r)\sim e^{-r/a}$, the normal HFS involves the radial integral

$$\int_0^\infty e^{-2r/a}dr=a/2. \quad (11)$$

Therefore, it follows immediately that, since $a\gg\rho\gg d$, the relative decrease of the contribution of the neutron magnetic moment to the HFS is given by

$$(2/a)\int_0^\infty e^{-2r/d}dr=d/a, \quad (12)$$

corresponding to a relative correction in κ equal to

$$\epsilon=(\mu_\mathrm{N}/\mu_\mathrm{D})\cdot(d/a)=1.84\cdot10^{-4}, \quad (13)$$

which conforms approximately with the observed effect.

For the estimate (13), the deuteron radius d was regarded as large compared with the range λ of the nuclear forces. Since, however, λ/d is assumed to be about $\frac{1}{3}$, it is significant, in view of the accuracy of the measurements, to consider also the influence of the finite value of λ. In this connection it would seem necessary to take into account two different types of effects.

First, the deuteron wave function must be expected to deviate essentially from (6) at distances comparable to or smaller than λ. Since the density within such distances will be smaller than according to (8), the effect will be to increase the average distance between the nucleon and thus also the value of ϵ. Using a wave function corresponding to a rectangular potential well type of nucleon interaction of range λ, one finds, from (6) and (7), by neglecting terms of relative order $(\lambda/d)^2$,

$$\epsilon' = (\lambda/d)\epsilon \tag{14}$$

for the increase in ϵ.

A second correction to κ may result from possible deviations of the magnetic field of the nucleons from simple dipole fields, within distances comparable with the range of nuclear forces. The order of magnitude of this effect may be simply estimated by assuming a model in which the anomalous part of the nucleon magnetic moments, or approximately this part, is the result of a spatial magnetization extending over distances comparable with λ. Thus, in analogy to the estimate (12), the HFS of H would be decreased by a fraction

$$\delta_H = (\lambda'/a)[(\mu_H - 1)/\mu_H], \tag{15}$$

where λ' is some suitably chosen distance of the order of λ. Since, however, according to (4), the neutron magnetic moment is already effectively distributed over distances d, a deviation of the neutron from a point dipole within distances comparable with λ should give effects only of the relative order $(\lambda/d)^2$. One thus finds, corresponding to (15),

$$\delta_D = (\lambda'/a)[(\mu_H - 1)/\mu_D]. \tag{16}$$

This yields, by means of (13),

$$\epsilon'' = -(\delta_D - \delta_H) \approx -(\lambda'/d)\epsilon \tag{17}$$

for the resulting correction to ϵ.

It need hardly be added that such estimates of ϵ' and ϵ'' may depend considerably on the specific models chosen for the problems in question. Still, from a very general point of view it would seem that the sign of the estimates should be reliable and the close agreement of (13) with the observed value for $\Delta\kappa$ may perhaps therefore be related to the circumstance that the two corrective effects tend to cancel each other. If the estimate of ϵ' gives the right order of magnitude, it would indeed seem that the phenomenon gives a direct indication of the existence of an effect of the type corresponding to ϵ''. Together with evidence on the deuteron constitution, the hyperfine structure phenomenon might therefore offer some information regarding the nature of the anomalous magnetic moment of the proton.

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