reasonable to consider the physical potentialities of more extended groups, having the Lorentz group as a subgroup, which the four-component spinors are able to represent. Because of the close relationship of the conformal and similarity geometries to the geometry of four-component spinors, it would be of interest to investigate a theory analogous to the present theory but based on spinors rather than tensors. Complex field quantities would enter such a theory automatically. And if the detailed computations bore a sufficient resemblance to those of the present tensorial theory, as is not unlikely, one would expect to find field equations containing not only the fields already obtained but, in addition, pseudovector and pseudoscalar fields, as well as fields pertaining to half-integral values of the spin.

PHYSICAL REVIEW VOLUME 73, NUMBER 9 MAY 1, 1948

The Breakdown of Gases in High Frequency Electrical Fields*

DONALD H. HALE**
Northwestern University, Evanston, Illinois
(Received May 19, 1947)

A theory is proposed to explain the mechanism of breakdown of gases in high frequency electrical fields. It is assumed that breakdown occurs when the electrical field and the frequency are such that an electron acquires the ionizing energy at the end of one mean free path. The field for breakdown is thus a function of the frequency of the applied potential and the ionization potential and pressure of the gas. The fields for breakdown of argon and xenon are calculated and expressed as functions of the frequency and the gas pressure. The calculated potentials are compared with experimental data, and good agreement is found for frequencies greater than 10×10^6 c.p.s.

1. INTRODUCTION

THE breakdown of gases in high frequency electrical fields has been studied by a number of workers.[1] In general, the procedure followed has been to measure the minimum potential for breakdown of the gas as a function of the gas, gas pressure, electrode separation, and frequency of the applied field. The potential for breakdown, V_s, was plotted as a function of the gas pressure, p, for one frequency and gap width. The data, when plotted in this manner, gave V_s-p curves which have the general appearance of the Paschen curves for breakdown in d.c. fields.

Thomson's[2] work was typical. He studied the breakdown of hydrogen in this manner. He used a number of frequencies of the applied potential and experimentally determined a series of V_s-p curves, each of which corresponded to one frequency. As he increased the frequency, from about 2×10^6 to about 40×10^6 c.p.s., V_s took on lower and lower values. However, as the frequency was increased above about 40×10^6 c.p.s., V_s began to increase with increasing frequency. The present paper reports a somewhat different approach to the problem. It is assumed that the breakdown potential for high frequency fields is determined by those electrons in the gas which succeed in acquiring the ionizing energy in one mean free path. Thus, the breakdown potential should be a function of the gas, gas pressure, electrode separation, and frequency of the applied field. Thomson's experimental work showed that for hydrogen V_s was a function of the gas pressure and the frequency of the applied field.

Under this assumption the solution of the equation of motion of the electron which moves in the gas and the high frequency field is not

* This work was sponsored by the U. S. Navy Bureau of Ships.
** Now at the Army Chemical Center, Maryland.
[1] L. B. Loeb, *Fundamental Processes of Electrical Discharge in Gases* (John Wiley and Sons, Inc., New York, 1939), p. 550.
[2] J. Thomson, Phil. Mag. 23, 1 (1937).

difficult since before the gas breaks down the electric field is not distorted by space charge. The motion of the electron under such conditions is given by

$$\ddot{x} = (e/m)E \sin 2\pi ft. \quad (1)$$

The electron is assumed to be at rest when the instantaneous value of the applied field is zero and all collisions which do not result in the formation of an ion pair are disregarded since such collisions cannot lead to the breakdown of the gas. Also it is only those electrons which are at rest when the instantaneous value of the field is zero which will acquire the ionizing potential at the end of one mean free path under the conditions to be imposed. These simplifying assumptions should not greatly alter the results since it is assumed that the electrons which acquire the ionizing energy in one mean free path are those which cause the initial breakdown.

Integration of the equation of motion gives,

$$\dot{x} = v = (e/m)E/2\pi f(1-\cos 2\pi ft), \quad (2)$$

$$x = d = (e/m)E/4\pi^2 f^2(2\pi ft - \sin 2\pi ft). \quad (3)$$

It is assumed that the gas will break down when the values of E and f are such that d will be one electron free path, L_e, for the gas and gas pressure considered, and that the electron will have the ionizing energy at the end of the mean free path. Thus v will be determined by $\frac{1}{2}mv^2 = eV_i$, where V_i is the ionization potential of the gas under consideration. Some value must be assigned to t. This can be done by assuming that the electron is to acquire the ionizing energy and move one mean free path in a time t which is some fractional part of one period of the applied electrical field. Thus, t is expressed in terms of f, and E and f computed for this value of t. This amounts to determining, for the gas and gas pressure considered, the proper values for E and f so that an electron will acquire the ionizing energy, after acceleration during a fraction of a period of the applied field, and be displaced one mean free path. Under such conditions the probability of ionizing collisions should be a maximum. Further, by calculating E and f for several values of t it is possible to plot a curve showing E, the peak value of the field for breakdown in volts per centimeter, as a function of the frequency of the applied field.

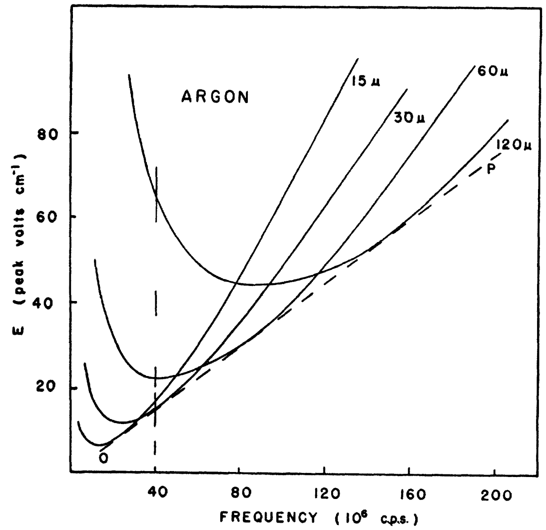

FIG. 1. Calculated values of E, the field for breakdown of A at several pressures, plotted against the frequency.

This has been done. Figure 1 shows calculated values of E, the peak value of the field for breakdown, plotted as a function of frequency for argon at several pressures from 15 to 120 microns of mercury. For any of the curves, the values of E corresponding to values of t less than about $t = 1/4f$ lie to the left of the minimum. This shows that if the electron is to have a high probability of ionizing after being accelerated during a small part of a period of the applied high frequency, E must be large and the frequency comparatively low. If the electron is accelerated over a large portion of one period, t equal to about $1/f$, E is large and the frequency is high for high probability of ionization. The minimum value of E is given by t about equal to $3/8f$. The curves of Fig. 1 show that when the pressure of the gas is increased the minimum value of E increases and lies at greater frequencies. Also the minimum becomes less pronounced for the higher pressures.

Figure 2 shows curves of calculated values of V_s, the peak value of the breakdown potential for a 5-cm gap, for xenon, argon, krypton, and hydrogen, all at a pressure of 30 microns of mercury. These curves show the dependence of E and the frequency, for breakdown, upon the ionization potentials and the electron mean free paths for these gases. The values of E corresponding to $t = 1/4f$ and $t = 3/8f$ are indicated for hydrogen.

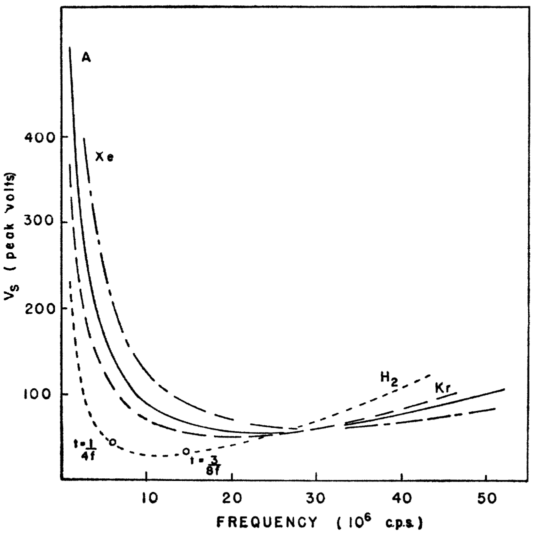

FIG. 2. Theoretical variation of V_s with frequency for a 5-cm gap in A, H_2, Kr, and Xe all at 30μ pressure.

Each of the curves of Fig. 1 is tangent to the line OP at one point. The value of E, for each curve, at the point of tangency is given approximately by $t=7/16f$.

It is possible, using the data of Fig. 1, to derive a set of theoretical V_s-p curves such as Thomson measured experimentally. This is done by reading off, from Fig. 1, the values of E corresponding to the various pressures shown and for one frequency. As an example the following data were taken from Fig. 1 for a frequency of 40×10^6 c.p.s.

p (microns)	V_s (peak volts)
15	17.6
30	15
60	22
120	65

These data were plotted in Fig. 3 as the $E-p$ curve for 40 megacycles. The data, as plotted in Fig. 3, give the peak value of the field for breakdown for a gap in argon as a function of the pressure. Figure 3 also shows similar curves for the breakdown potential at other frequencies. These theoretical curves all have the same general shape as do the Thomson curves for low pressures and high frequencies. That is, both Thomson's experimental curves and the curves of Fig. 3 show E_s minimum decreasing with decreasing frequencies and falling at lower values of the pressure. Thomson's curves for hydrogen do not show E_s increasing as rapidly, to the right of the minimum, as do the curves of Fig. 3. However, the shape of the curves for hydrogen might be expected to be quite different from those for argon. This is shown by a comparison of the curves for hydrogen and argon in Fig. 2.

The minimum value of E_s for any one of the $E-p$ curves of Fig. 3 will be nearly equal to the value of E given by $t=7/16f$ for one of the curves of Fig. 1. This is so because no matter what frequency is chosen in deriving an $E-p$ curve, from a set of the type shown in Fig. 1, the lowest value of E_s for that frequency will always be given by the curve which is tangent to OP at the intersection of OP and the line of constant frequency. Thus it is possible to calculate the minimum value of E_s for any $E-p$ curve and for any gas and frequency. All that need be known is V_i and the frequency applied. A study of Eq. (2) indicates the method. If the velocity and frequency are known and t is set equal to $7/16f$, this equation can be solved for E and this value of E, multiplied by the electrode separation, gives V_s minimum, for the V_s-p curve at the frequency chosen.

It is also possible to estimate the pressure at which the minimum value of V_s will fall for curves of the type shown in Fig. 3. Equation (3) is used in this calculation. The value of E, as calculated above, the frequency under consideration and $t=7/16f$ are substituted in the equation and d is calculated. This gives the electron mean free path in the gas considered and at the pressure for which V_s is a minimum. It is thus possible to estimate the gas pressure.

2. EXPERIMENTAL PROCEDURE

Breakdown potentials were measured for xenon and argon at constant pressure and in the range of frequencies from 5×10^6 to 50×10^6 c.p.s. Two xenon-filled tubes were used. The gas pressure in each was 20 microns of mercury. Two argon tubes were used with gas pressures of 30 and 49 microns of mercury. The pressures were measured at 22°C. The tubes were of Pyrex and were either 22 or 41 mm in diameter. External electrodes were used so that the purity of the gas might be of a high order. The length of all tubes was 22 cm. The external electrodes were made from strips of copper 1 cm wide fitted about the tubes. Electrode separations of 5 and 10 cm were used.

Considerable care was used in filling the tubes. They were attached to the vacuum system and baked out at 400°C over night. The vacuum system had a side bulb in which was fitted a tungsten filament. This filament supported a pellet of barium or calcium. After the gas was admitted the pellet was flashed to act as a getter for contaminating gases. This flashing formed a freshly deposited surface of the getter on the walls of the containing bulb and the gases were allowed to stand in contact with this surface for some time before the discharge tubes were sealed off. After sealing off, the spectra of the gases were examined and showed no trace of contamination. However, this is only a rough check since many discharge phenomena in gases at low pressures can be altered by traces of impurities which cannot be detected by a study of the spectrum of the gas.

Most of the work was done with two oscillators and a few values of V_s, at the lower frequencies, checked with a third oscillator. The two oscillators most used were of the same type but covered different frequency ranges. They were of a modified Hartley type and employed 100 TH tubes rated at 200 watts. The electrodes of the discharge tubes were connected directly across a few turns of the oscillator plate coil. The potential applied to the electrodes was varied by varying the oscillator plate potential. This arrangement made it possible to change the potential on the electrodes continuously and smoothly from about 100 to 1000 volts. A few values of V_s were taken as a check with a Navy type TDE transmitter oscillator. This oscillator had a buffer stage and it was assumed, therefore, that the potential applied to the discharge tubes was nearly free of harmonics. Potentials were measured by a vacuum tube voltmeter. This indicated potentials in peak volts and all values of potential and field mentioned are so expressed.

The procedure followed in determining the breakdown potential was to start with a low value for the applied high frequency potential and then to slowly increase it until the gas broke down. Breakdown was indicated by the appearance of a glow in the body of the gas and also by a slight kick of the voltmeter needle. All

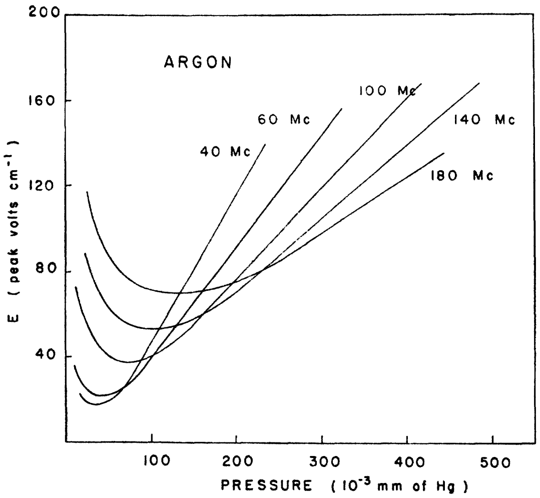

FIG. 3. Calculated E-p curves for breakdown of A at several frequencies.

points were checked a number of times. The breakdown potentials were readily reproducible within the limit of error of reading the voltmeter with the exception of those for the lowest frequencies. For these frequencies there was considerable spread among the readings taken at any one frequency.

3. EXPERIMENTAL RESULTS

The measured values of V_s for argon plotted as a function of frequency are shown in Fig. 4. The values of V_s indicated by circles were taken with the Harley oscillators and those indicated by triangles were taken with the Navy type TDE oscillator. The smooth curves represent the average of the experimental data. The two curves at lower values of V_s were taken using a tube 41 mm in diameter and filled to a pressure of 30 microns of mercury. Electrode separations of 5 and 10 cm were used for the two curves. The values of V_s for the upper curve of Fig. 4 were measured using a tube filled to a pressure of 49 microns of mercury and with a 10-cm electrode separation. This tube was also 41 mm in diameter.

The dotted lines of Fig. 4 are calculated values of V_s for the gap widths and pressures shown. In making such calculations it is necessary to assume some value for the potential drop in the wall of the discharge tube. However, the data of the two lower curves make it possible to do this

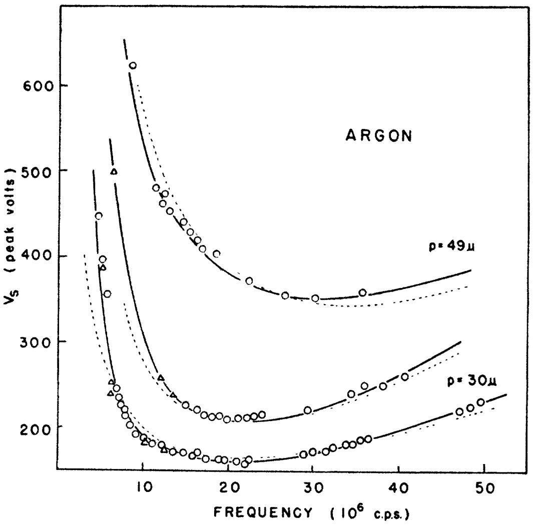

FIG. 4. Measured values of V_s, for breakdown of A, plotted against the frequency. The electrode separation was 5 cm for the bottom curve, and 10 cm for the other two. The dotted curves indicate calculated values.

since it may be assumed that each of the two values of V_s, for any one frequency, is equal to the potential drop in the gas plus the drop in the tube wall, and that the drop in the wall is the same for each. Thus, for any frequency, two equations may be set up and the solution will give the drop in glass and also the field in the gas for breakdown. This calculation was made for several frequencies and gave an average value of 120 v for the potential drop in the tube wall. This value was used in calculating the data shown by the dotted lines.

The experimental data shown in Fig. 4 are in good agreement with the calculated values except at frequencies less than about 10×10^6 c.p.s. The shape of the experimental curves is about that predicted and for the higher frequencies the measured values of V_s differ from the calculated values by no more than the experimental error.

The V_s-f curves of Fig. 5 are for xenon at a pressure of 20 microns. Two discharge tubes were used. The data of the upper curve were taken with a tube 22 cm in diameter and with a 10-cm electrode separation. The data of the lower curve were taken using a tube 41 mm in diameter and with 5-cm electrode separation. The dotted line shows the calculated values of V_s for the larger tube. Here also it was assumed that the potential drop in the tube wall was 120 v. Since measurements were taken for only one electrode separation for the smaller tube it was not possible to calculate the potential drop in the tube wall. Therefore, the calculated value of E was assumed for a frequency of 30×10^6 c.p.s. and the potential drop in the tube wall was calculated upon this assumption. This gave 150 v for the potential drop in the tube wall for this frequency. This value was used in calculating the values of V_s shown by the dotted curve for the smaller tube.

The data of Fig. 5 are not in as good agreement with the calculated values as is the case for argon. In xenon, at this pressure, the minimum value of E should fall at a frequency of about 22×10^6 c.p.s. and should be about 10 v cm^{-1}. However, the discrepancy may be due, in part, to the fact that two tubes of different diameters were used in taking the data. Under such conditions the non-uniformity of the fields in the gas, due to the external electrodes, will be different for the two cases.

The few data taken with the Navy TDE oscillator were used as a check on the data taken with the Hartley type oscillators since these oscillators would be expected to have a considerable harmonic component. Such harmonics would be particularly troublesome in taking values of V_s which lie along the steeply rising part of the V_s-f curve. However, the values of V_s measured with the essentially harmonic free oscillator do not differ greatly from those taken with the Hartley oscillators.

4. DISCUSSION

It was mentioned above that it is possible, on the basis of the proposed theory, to calculate the minimum value of V_s for the V_s-p curves as measured by Thomson and others and also the gas pressure at which the minimum occurs. This can be done for any gas, gap width, and frequency of the applied field.

TABLE I.

Frequency (10^6 c.p.s.)	$(V_s)_{\min}$ Calculated	$(V_s)_{\min}$ Thomson	Pressure Calculated	Pressure Thomson
99.0	112.2 volts	138 volts	177 microns	360 microns
77.1	87.4	104	135	340
65.4	78.8	99	117	300
49.5	56.1	100	70	340
15.6	23.0	112	64	400

This calculation has been made for hydrogen and several frequencies of the applied field. Table I gives the calculated values of V_s, minimum, for a gap width of 2.58 cm as was used by Thomson. Also the table shows the experimentally determined values of Thomson. The agreement is fairly good for the higher frequencies. Thomson also measured the minimum value of V_s for a number of frequencies which were lower than any shown in Table I. For those cases the calculated values of V_s are much lower than the measured values. This may mean that some mechanism, other than the one considered here, determines breakdown at the lower frequencies.

The gas pressure at which V_s is a minimum for the Thomson V_s-p curves has been calculated for hydrogen and for the frequencies used by Thomson. These results, with Thomson's measured values, are shown in Table I. In making the calculations it was assumed that the molecular free path for hydrogen was 18.3×10^{-6} cm at 760 mm of mercury and 0°C. The agreement is poor for all frequencies used by Thomson. However, as shown by Fig. 3, the minima in the V_s-p curves are broad and it would be difficult, experimentally, to determine the pressure at the minimum.

Brasefield[3] measured the conductivity of hydrogen for one frequency and after breakdown, as a function of the pressure. He found that for any one frequency there was a pressure for which the conductivity was a maximum. It has been remarked that the observed maxima, in the values of the conductivity, should occur for those pressures which give the minima in the V_s-p curves for the same gas and frequency.[3,4] It is observed that, for values of the applied potential slightly greater than V_s, discharges in

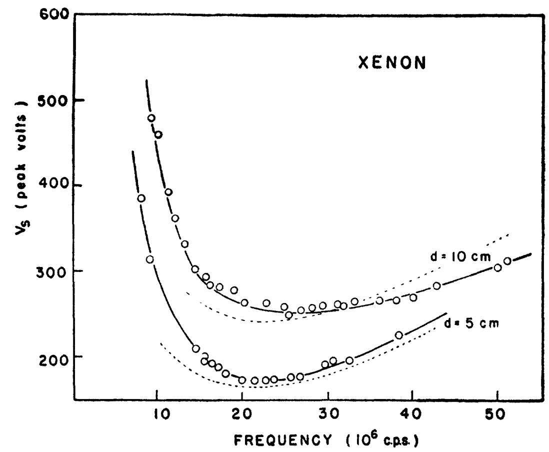

FIG. 5. Measured values of V_s, for breakdown of Xe at 20μ pressure, plotted against the frequency. Electrode separations of 5 and 10 cm were used. The dotted curves indicate calculated values.

argon, krypton, and xenon are brightest when the pressure is about that corresponding to the minimum breakdown potential for the frequency used. If it is assumed that the maximum conductivity and the minimum in the V_s-p curves fall at the same gas pressure for any one frequency of the applied potential, the pressures for maximum conductivity in hydrogen at the frequencies used by Brasefield can be calculated by the method used above in calculating the pressures at minimum breakdown potentials. Table II shows the calculated values and Brasefield's experimental results. Here the agreement is fair for the higher frequencies and is poor for frequencies below about 15×10^6 c.p.s. However, the agreement here is better than was the case for Thomson's data.

The value of the electron mean free path used in calculating E and f was that given by the kinetic theory. This can hardly be correct and the good agreement between the calculated and experimental values, as shown in Figs. 4 and 5, is most likely fortuitous. The kinetic theory yields a value for the mean free path which is independent of the electron energy. The Ramsauer type experiments indicate that the electron mean free path is a function of the electron energy.[5] In the case considered here the electron energy varies from zero to the ionizing energy. It is difficult to estimate the mean free path for this

TABLE II.

Frequency (10^6 c.p.s.)	Pressure Calculated	Brasefield
20	28.6 microns	20 microns
15	21.0	15
12	15.9	30
10	14.0	35
5	7.2	10

[3] C. J. Brasefield, Phys. Rev. **35**, 1073 (1930).
[4] H. Steinhauser, Zeits. f. Physik **54**, 788 (1929).

[5] Reference 1, p. 649.

condition. What is needed is some sort of an "effective" mean free path which would give the average path for an electron which starts from rest, is accelerated in a field which varies in a sinusoidal manner, and then undergoes an ionizing collision. The kinetic theory mean free path used in these calculations can be no more than an approximation to such an "effective" mean free path.

It is implied, in the assumptions made in calculating E and f, that if an electron in the gas acquires the ionizing energy at the end of a mean free path the probability of its ionizing is a maximum. This is also questionable. Smith[6] found that the efficiency of ionization, for electrons in argon, increased with increasing electron energies up to electron energies of several times the ionizing energy. Thus, for argon, the probability of ionization increases quite rapidly with increasing electron energies for energies slightly above the ionizing energy of 15.69 v. This would also be expected in xenon. Thus it may be, for breakdown in high frequency fields, that the large increase in electron density which breakdown demands comes when the electron acquires energy at the end of a mean free path somewhat above that given by the expression $\frac{1}{2}mv^2 = eV_i$. This would mean that the calculated values of E and f, as given here, are too small.

It is assumed, in comparing the calculated and experimental values of V_s, that the electrical field in the gas is uniform under the conditions of this experiment. That is, it is assumed that the field is given by dividing the applied potential by the distance between the electrodes. However, with external electrodes, such as used here, the field cannot be uniform and a distortion of the field along the discharge tube axis must be expected. Thus at some point along the axis the field should be greater than estimated. The breakdown of the gas would then be expected to start at the point where the field is the greatest. Once breakdown starts at any point in the tube, space charge would be expected to cause the rapid propagation of the breakdown throughout the tube.

This assumption of a uniform field makes the measured value of V_s too small. It is likely, since no space charge would be expected before breakdown, that the measured value of V_s is correct to a first-order approximation. Thus it appears that the assumptions that the field is uniform and that breakdown occurs for electron energies given by $\frac{1}{2}mv^2 = eV_i$ may be only approximations, and that the two approximations tend to offset each other and thus make the calculations appear better than they actually are.

The author wishes to acknowledge that the discharge tubes used were filled by Mr. Charles Hendee, and that the Hartley type oscillators were designed and built by Mr. John Frank. Also many helpful discussions were held with Dr. Richard Nelson and Professor Walter S. Huxford during the course of the experiments.

[6] P. T. Smith, Phys. Rev. 36, 1293 (1930).