THE PHYSICAL REVIEW

A journal of experimental and theoretical physics established by E. L. Nichols in 1893

SECOND SERIES, VOL. 73, No. 1 — JANUARY 1, 1948

A Phase-Shift Analysis of the Scattering of Protons by Deuterons

CHARLES L. CRITCHFIELD*

Los Alamos Scientific Laboratory, Los Alamos, New Mexico

(Received September 22, 1947)

Numerical values for the S-wave and P-wave phase shifts are deduced from the observed angular distribution of protons scattered by deuterons. The experimental results used are those obtained at Los Alamos in the energy range, 1.5 to 3.5 Mev. In reducing the results to phase shifts the theoretical work on neutron-deuteron scattering, as presented by Buckingham and Massey, was used as a guide. It is found that the p-d phase shifts are qualitatively similar to the calculated n-d phase shifts, for the case of exchange forces between nucleons, except that the doublet P-wave is much more strongly refracted in the p-d results. The discrepancy may be evidence for a difference between the actual nuclear forces and the type of central, exchange forces assumed in the calculations. The results are definitely not similar to those for ordinary forces, however, and such forces would appear to be ruled out. Phase shifts in S- and P-waves alone are not sufficient to represent the p-d results in the range of energies used, but it is necessary to include interference with D-waves.

I. INTRODUCTION

THE angular distribution of protons, as scattered by deuterium gas, has been determined at the Los Alamos Laboratory.[1] Very complete and accurate results were obtained for five energies of incident protons, ranging from 1.5 Mev to 3.5 Mev. The scattered intensity was measured at fourteen angles of scattering, running from 22.5° to 164.5° in the center of gravity coordinate system. It is convenient, for the purposes of analysis, to represent this data by plotting $k^2\sigma(\theta)$ against $\cos\theta$, where θ is the angle of deflection of the proton in the center of gravity system, $\sigma(\theta)$ is the usual differential cross section, and

$$k=\frac{2\pi}{\lambda}=\frac{2}{3}\frac{Mv}{\hbar}=1.463(E_{\mathrm{Mev}})^{\frac{1}{2}}\times 10^{12}\ \mathrm{cm}^{-1} \quad (1)$$

for protons of mass M and relative velocity v (energy E_{Mev} in the laboratory system). The experimental results are plotted in these terms in Fig. 1.

The experimental results are characterized by very strong scattering at small angles ($\cos\theta\simeq 1$), i.e., in the forward direction of the protons, and this is an obvious effect of the Coulomb repulsion between the proton and the deuteron. The pure Coulomb field differential cross section (times k^2) is shown as a dotted curve in Fig. 1, for $E=2.53$ Mev, just for comparison. The more remarkable feature of the results is that the scattering at angles near 180°, i.e., in the backward direction, becomes very strong also, especially for the higher energies of bombardment.

Strong backward scattering, in the energy range concerned, has been found in the theoretical work of Buckingham and Massey[2] on the

* On summer leave from the University of Minnesota, Minneapolis, Minnesota.

[1] R. Sherr, J. M. Blair, H. R. Kratz, C. L. Bailey, and R. F. Taschek, Phys. Rev. **72**, 662 (1947).

[2] R. A. Buckingham and H. S. W. Massey, Proc. Roy. Soc. **A179**, 123 (1942); Phys. Rev. **71**, 558, 829 (1947).

scattering of *neutrons* by deuterium. These authors use the resonating group-structure theory of Wheeler[3] and thus allow for the possibility that the incoming neutron may form a new deuteron with the proton and liberate the neutron that was originally held by the proton. It is plausible, just from the persistence of momentum of the impinging neutron, that the newly formed deuteron will move predominantly in the forward direction so that the liberated neutron will fly backward. Greater detail in this mechanism can be followed through the integro-differential wave equation set up and solved in reference 2. The strong scattering near 180° is thus *not* direct evidence of the exchange nature of nuclear forces but rather arises as an exchange of partners for the proton in forming a deuteron. On the contrary, it was found that if the nuclear forces are of an exchange nature they partially defeat the exchange of partners so that ordinary forces should give much more backward scattering, in this energy range, than exchange forces.

The calculations of neutron-deuteron scattering were made on the basis of various assumptions concerning the nuclear forces. Since nuclear forces are believed to be the same between two protons as between two neutrons, the qualitative features of these calculations should be applicable also to the proton-deuteron scattering. Hence, in the present paper, we shall depend upon the calculations by Buckingham and Massey for guidance in reducing the proton-deuteron results in terms of phase shifts. The results of calculation and of experiment cannot be expected to be quantitatively exactly the same, however, since the Coulomb repulsion in the p-d case will affect the amplitudes of the waves to some extent and also the calculations have been made assuming central forces with rather arbitrarily chosen exchange characters. The actual forces between nucleons are known to be partly non-central and may have a somewhat different exchange nature than those in the particular examples taken for calculation. No attempt has been made to interpret the differences between n-d calculations and p-d experimental results, as found below, except that it appears that nuclear forces are certainly not purely ordinary forces. This fact is known already from the saturation character of forces in nuclei.

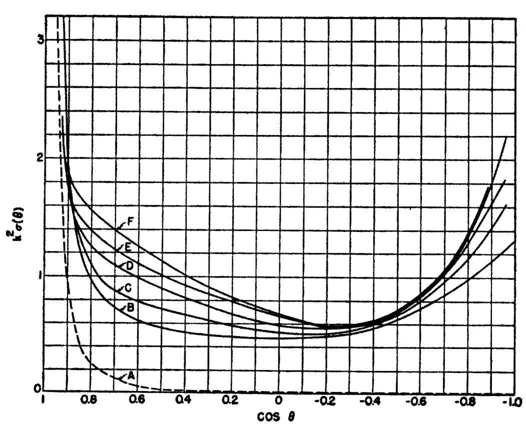

FIG. 1. Experimental results on angular distribution of protons scattered by deuterons.

A—Coulomb effect only ($E=2.53$)
B—1.51 Mev
C—2.08 Mev
D—2.53 Mev
E—3.00 Mev
F—3.49 Mev

In Fig. 2 is summarized the results of calculation for S-wave and P-wave phase shifts in quartet and doublet collisions of neutron and deuteron. For convenience, the phase shifts are represented in the first and fourth quadrants. Figure 2a is for ordinary forces and Fig 2b for a particular combination of exchange forces (and ordinary forces) as assumed in reference 2. It is evident from these figures that the S-wave shifts are practically independent of spin orientation.** Considering the very different nature of the S-wave solutions for quartet and doublet collisions (there is a region of repulsion in the quartet state, none in the doublet) it would seem that the independence of spin is accidental. In any case, for the analysis of the p-d results we shall assume that this feature of the n-d calculation carries over and adopt a single phase shift, K_0, for both S-waves. We assume also, of course, that this K_0 lies in the fourth quadrant.

Because of the effect of changing partners (in the p-d collisions the protons exchange) the higher partial waves, P, D, F, etc., may become affected when λ becomes of the order of the radius of the deuteron, i.e., at 2 Mev in the

[3] J. A. Wheeler, Phys. Rev. 52, 1107 (1937).

** The calculations actually predict the quartet and doublet phase shifts to differ very nearly by 180°. Since, however, scattering experiments do not distinguish phase shifts differing by 180°, the two are plotted in the same quadrant.

center of gravity coordinates or 3 Mev in the laboratory system. Since the Coulomb repulsion at this radius is rather small compared with the bombarding energy, the extent to which the waves of higher orbital angular momenta, l, are affected will diminish rapidly with l, as in the collision of neutral particles. Hence, in the energy range of the experiments on *p-d* scattering we are justified in considering the *S*- and *P*-waves only as strongly shifted in phase with small shifts of diminishing importance in *D* and higher waves. The *n-d* results, Fig. 2, show that the *P*-waves are indeed strongly refracted and that there is a considerable difference in phase shift between ^2P and ^4P waves. For both ordinary and exchange forces, however, the ^4P phase shift lies above the ^2P and in the first quadrant. In analyzing the *p-d* data we shall assume, therefore, that the ^4P phase shift lies in the first quadrant.

The formula for $k^2\sigma(\theta)$, which forms the basis of the *p-d* analysis below, is written most conveniently for the simplified case of no spin-dependence of scattering

$$k^2\sigma(\theta) = \left| -\frac{\eta}{1-\cos\theta} e^{i\eta \ln 2/1-\cos\theta} + e^{iK_0}\sin K_0 + 3\cos\theta e^{i(\bar{K}_1+\phi_1)}\sin\bar{K}_1 + \frac{5}{2}(3\cos^2\theta-1)e^{i(\bar{K}_2+\phi_2)}\sin\bar{K}_2\cdots \right|^2, \quad (2)$$

$$\eta = \frac{e^2}{hv}, \quad \phi_\lambda = \arg\frac{1+i\eta}{1-i\eta},$$

with K_0, $\bar{K}_1$, $\bar{K}_2$ the phase shifts in *S*, *P*, *D*, waves. In the energy range of interest, η is of the order 0.1, so that $\phi_1 \simeq 2\eta$ and the $\bar{K}_2$ involved are so small that

$$e^{i(\bar{K}_2+\phi_2)}\sin\bar{K}_2 \simeq \bar{K}_2. \quad (3)$$

II. ESTIMATES OF *S*- AND *P*-WAVE SHIFTS

The first step in a preliminary reduction of the experimental data is to estimate the phase shifts for the *S*-waves. As remarked above, we shall assume that *D*-waves and higher partial waves have very small phase shifts and also that the ^4S and ^2S waves have the same phase shift, K_0. We may then use Eq. (2), setting $\bar{K}_2, \cdots$ equal to zero, and avoid possible contributions of the

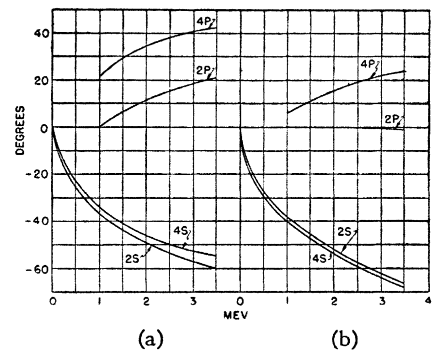

FIG. 2. Calculated phase shifts of *S*-waves and *P*-waves in the scattering of neutrons by deuterons. (a) for ordinary forces; (b) for the "mixed exchange" forces of reference 2.

P-waves by using the experimental data at $\theta = 90°$. The formula for $k^2\sigma(90°)$ then becomes

$$k^2\sigma(90°) = \eta^2 - 2\eta\sin K_0\cos(K_0 - \eta\ln 2) + \sin^2 K_0. \quad (4)$$

By the neglect of terms of the order η^3 (≤ 0.004 in the range of the experiments) Eq. (4) may be solved for $\cos(2K_0 - 2\eta)$ in terms of η and the measured values of $k^2\sigma(90°)$, *viz.*:

$$\cos(2K_0 - 2\eta) = \{1 - 2\eta^2(1-\ln 2)\}\{1 - 2k^2\sigma(90°)\}. \quad (5)$$

Equation (5) then leads to two possible values of K_0 for each energy represented in Fig. 1. In accordance with the *n-d* predictions we choose the solutions for K_0 that lie in the fourth quadrant. The deductions of K_0 from the experimental curves are given in Table I. Comparing these results with those in the *n-d* calculations, Fig. 2, we see that the *p-d* phase shifts are about $\frac{3}{4}$ those predicted for exchange forces and about $\frac{5}{6}$ those for ordinary forces. The smaller *S*-wave shift in *p-d* collisions may be due to the Coulomb repulsion, and it is not possible to conclude from this result alone whether ordinary or exchange forces are operative.

The second step is to estimate the *P*-wave

TABLE I. *S*-wave phase shifts.

E Mev	η	$k^2\sigma(90°)_{obs}$	K_0
1.51	0.129	0.48	−36.5°
2.08	0.110	0.54	−41.0°
2.53	0.100	0.58	−43.9°
3.00	0.092	0.64	−47.8°
3.49	0.085	0.68	−50.6°

TABLE II. P-wave phase shifts.

E	s	d	K_1	k_1
1.51	2.781	−0.054	15.2°	−6.0°
2.08	2.640	−0.147	16.9	−11.8
2.53	2.508	−0.139	17.2	−17.6
3.00	2.484	−0.098	16.1	−19.9
3.49	2.370	−0.064	17.2	−23.0

TABLE III. D-wave phase shifts.

E Mev	$5(\sin 2K_1 - \sin 2k_1)$	$\lvert\delta\rvert$	K_2 rad.	K_2 deg.	k_2 rad.	k_2 deg.
1.51	3.58	0.13	−0.036	−2.1	0.072	4.2
2.08	4.78	0.25	−0.052	−3.0	0.104	6.0
2.53	5.70	0.35	−0.061	−3.6	0.122	7.2
3.00	5.86	0.44	−0.075	−4.4	0.150	8.8
3.49	6.42	0.42	−0.065	−3.8	0.130	7.6

shifts. For the time being let us assume that there is no spin dependence, so that we may again use Eq. (2) with $\bar{K}_2=0$, etc., and substitute the values of K_0 already obtained. The formula for $k^2\sigma(\theta)$ then becomes:

$$\begin{aligned}k^2\sigma(\theta)=\tfrac{1}{4}\eta^2\csc^4\tfrac{1}{2}\theta-\eta\csc^2\tfrac{1}{2}\theta[&\sin K_0\cos(K_0-\beta)\\&+3\cos\theta\sin\bar{K}_1\cos(\bar{K}_1+\phi_1-\beta)]\\&+\sin^2K_0+9\cos^2\theta\sin^2\bar{K}_1\\&+6\cos\theta\sin K_0\sin\bar{K}_1\cos(K_0-\bar{K}_1-\phi_1),\end{aligned}$$

with

$$\beta=\eta\ln\frac{2}{1-\cos\theta}.$$

By the neglect of terms of the order η^3, this may be written in the approximate form:

$$\begin{aligned}k^2\sigma(\theta)=\tfrac{1}{4}\eta^2\csc^4\tfrac{1}{2}\theta-\eta\csc^2\tfrac{1}{2}\theta[&\sin K_0\cos K_0\\&+\beta\sin^2K_0+3\sin\bar{K}_1\cos\bar{K}_1\\&-3\sin^2\bar{K}_1(2\eta-\beta)]+6\eta\sin\bar{K}_1\\&\times\cos\bar{K}_1-6\eta(2\eta-\beta)\sin^2\bar{K}_1\\&+\sin^2K_0+9\cos^2\theta\sin^2\bar{K}_1\\&+6\cos\theta\sin K_0\sin\bar{K}_1[(1-2\eta^2)\\&\times\cos(K_0-\bar{K}_1)+2\eta\sin(K_0-\bar{K}_1)].\end{aligned}\quad(6)$$

In order to effect the best comparison of Eq. (6) with the experimental results we use the data (Fig. 1) for those angles θ_1 and θ_2 for which the D-wave vanishes, *viz.*,

$$\theta_1\equiv\cos^{-1}(3^{-\frac{1}{2}})=54.7°,\quad\theta_2\equiv\cos^{-1}(-3^{-\frac{1}{2}})=125.3°.$$

Then

$$\begin{aligned}k^2\sigma(\theta_{1,2})=\tfrac{1}{4}\eta^2(12\pm6\sqrt{3})-\eta(3\pm\sqrt{3})\{&\sin K_0\cos K_0\\&+\beta_{1,2}\sin^2K_0+3\sin\bar{K}_1\cos\bar{K}_1\\&-3(2\eta-\beta_{1,2})\sin^2\bar{K}_1\}+6\eta\sin\bar{K}_1\cos\bar{K}_1\\&-6\eta\sin^2\bar{K}_1(2\eta-\beta_{1,2})+\sin^2K_0\\&+3\sin^2\bar{K}_1\pm2\sqrt{3}\sin K_0\sin K_1[(1-2\eta^2)\\&\times\cos(K_0-K_1)+2\eta\sin(K_0-K_1)],\end{aligned}$$

the upper signs going with the subscripts 1 and the lower signs with the subscripts 2. It is convenient to take the sum and difference of $k^2\sigma(\theta)$ at θ_1 and θ_2. It is convenient also to approximate β_1 and β_2 in terms that neglect $\eta^2/10$ compared with unity, *viz.*,

$$\begin{aligned}\tfrac{1}{2}(\beta_1+\beta_2)&=0.90\eta\simeq\eta,\\\tfrac{1}{2}(\beta_1-\beta_2)&=0.66\eta\simeq3^{-\frac{1}{2}}\eta.\end{aligned}\quad(7)$$

Reducing the expression for

$$\Sigma\equiv k^2[\sigma(\theta_1)+\sigma(\theta_2)]$$

in a manner similar to that used previously for $k^2\sigma(90°)$, we find

$$3\cos(2\bar{K}_1-\eta)+\cos(2K_0-3\eta)=(1-\tfrac{1}{2}\eta^2)(4-\Sigma).\quad(8)$$

An alternative method of determining $\bar{K}_1$ would be to calculate the formula for the difference

$$\Delta\equiv k^2[\sigma(\theta_1)-\sigma(\theta_2)]$$

which gives, to the same approximation, i.e., using (7),

$$\begin{aligned}3^{-\frac{1}{2}}\Delta=[1+\tfrac{1}{2}\eta^2-\cos(2K_0-3\eta)]&\\\times[1+\tfrac{1}{2}\eta^2-\cos(2\bar{K}_1-\eta)]&\\+\sin(2K_0-3\eta)\sin(2\bar{K}_1-\eta).&\end{aligned}\quad(9)$$

However, as anticipated in the introduction, there are no single values of $\bar{K}_1$ that satisfy (8) and (9) simultaneously, because there is a pronounced spin-dependence in the P-wave shifts.

To adapt the formulas just derived for Σ and Δ to the case of spin-dependence (in P-waves) we have to introduce a quartet P-wave shift, K_1, and a doublet P-wave shift, k_1, and replace $3\cos(2\bar{K}_1-\eta)$ by $2\cos(2K_1-\eta)+\cos(2k_1-\eta)$ and replace $3\sin(2\bar{K}_1-\eta)$ by $2\sin(2K_1-\eta)+\sin(2k_1-\eta)$. We may eliminate the cosine terms in Δ also, in terms of Σ, and arrive at the convenient relations for K_1 and k_1:

$$\begin{aligned}&2\cos(2K_1-\eta)+\cos(2k_1-\eta)\\&\quad=(1-\tfrac{1}{2}\eta^2)(4-\Sigma)-\cos(2K_0-3\eta)\equiv s,\\&2\sin(2K_1-\eta)+\sin(2k_1-\eta)\\&\quad=\{3^{\frac{1}{2}}\Delta-3[1+\tfrac{1}{2}\eta^2-\cos(2K_0-3\eta)]\\&\qquad\times[1+\tfrac{1}{2}\eta^2-(s/3)]\}\csc(2K_0-3\eta)\\&\qquad\equiv3d\csc(2K_0-3\eta).\end{aligned}\quad(10)$$

We now obtain the observed values of Σ and Δ from Fig. 1, use the previously determined values of K_0, and compute the quantities on the right-hand side of the two equations (10). These are then simultaneous equations for K_1 and k_1. Again, there will be two roots for each, but we choose that set of roots in which K_1 lies in the first quadrant as indicated by the *n-d* calculations. The results appear in Table II.

Comparing the values thus obtained with the *n-d* calculations, Fig. 2, we see that the ^4P-shift, K_1, is in general agreement with predictions for exchange forces but that the ^2P-shift is very different. Instead of remaining close to zero, the ^2P-shift grows rapidly negative. There is no clear indication of what this may mean, assuming the analysis is valid, in terms of the exchange character or of the non-central character of nuclear forces. This behavior, however, is so different from that of ordinary, central forces that it suggests very strongly that the nuclear forces are of an exchange nature.

III. EFFECT OF HIGHER ANGULAR MOMENTA

In Figs. 3, 4, 5, 6, and 7 are shown the departures of the experimentally determined cross sections (times k^2) from the calculations with formula (6), modified to take account of spin-dependence in the P collisions and using the values of K_0 from Table I and K_1, k_1 from Table II. By virtue of the method of estimating K_0, K_1, and k_1 these curves pass through zero at

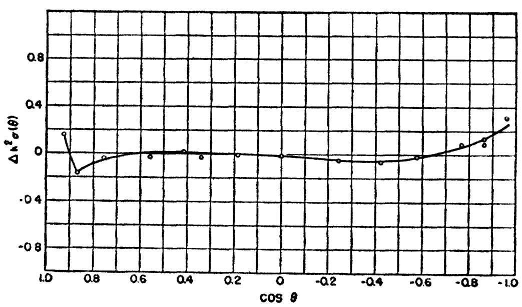

FIG. 3.

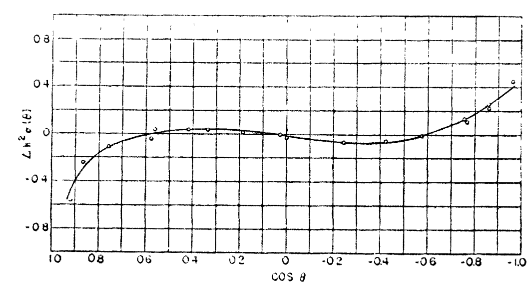

FIG. 4.

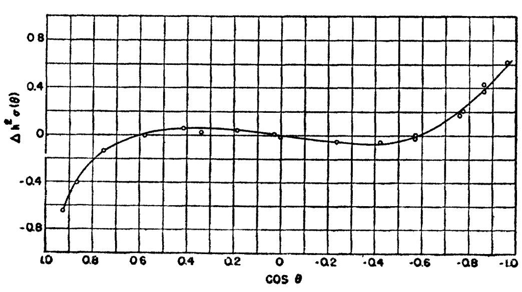

FIG. 5.

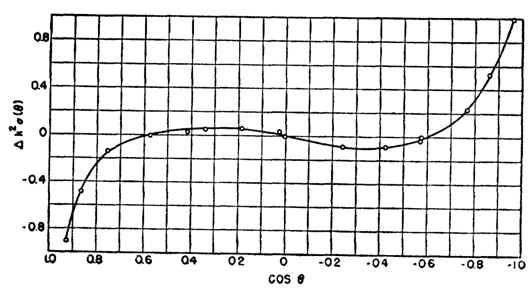

FIG. 6.

FIGS. 3–7. Deviations of experimental points (open circles) from angular distribution predicted on basis of S-wave and P-wave shifts alone. (Fig. 3, $E=1.51$ Mev; Fig. 4, $E=2.08$ Mev; Fig. 5, $E=2.52$ Mev; Fig. 6, $E=3.00$ Mev; Fig. 7, $E=3.49$ Mev.)

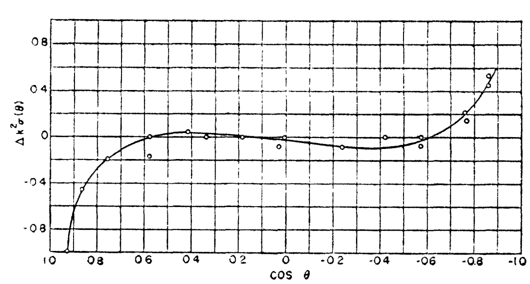

FIG. 7.

$\cos\theta=0.58$, 0, -0.58 (the agreement is not quite perfect for $E=3.49$ because the spread of the experimental points permitted a certain arbitrariness in selecting values for Σ and Δ). The curves in Figs. 3–7 are antisymmetrical about the origin, within the errors of experiment. This indicates that the departure contains very little S–D interference, and since the D-wave shifts, K_2 for the quartet and k_2 for the doublet, are expected to be small, we may deduce at once that

$$2K_2+k_2=0, \tag{11}$$

within the accuracy of the determinations. Condition (11) is based on the fact that the S–D interference would contribute to $k^2\sigma(\theta)$ a term (cf. Eq. (2))

$$\begin{aligned}5(3\cos^2\theta-1)\sin K_0\{\tfrac{2}{3}\sin K_2\cos(K_0-K_2-\phi_2)\\+\tfrac{1}{3}\sin k_2\cos(K_0-k_2-\phi_2)\}\\\simeq\tfrac{5}{6}(3\cos^2\theta-1)\sin 2K_0(2K_2+k_2),\end{aligned}$$

and it simultaneously eliminates the D-Coulomb interference. It follows that the departures represented in Figs. 3, 4, 5, 6, and 7 are probably almost pure D–P interference effects. Again, from Eq. (2), and allowing for spin-dependence, the P–D interference term is of the form

$$\begin{aligned}\Delta k^2\sigma(\theta)&=5\cos\theta(3\cos^2\theta-1)\\&\times[2\sin K_1\sin K_2\cos(K_1-K_2-\eta)\\&+\sin k_1\sin k_2\cos(k_1-k_2-\eta)]\\&\simeq 5\cos\theta(3\cos^2\theta-1)K_2(\sin 2K_1-\sin 2k_1),\end{aligned} \tag{12}$$

using relation (11).

We shall use the mean departures at $\cos\theta=-0.852$ and $\cos\theta=0.852$, at which angles $\cos\theta(3\cos^2\theta-1)=\mp1$, and call the average absolute values $|\delta|$. The phase shift K_2 is then determined by the relation

$$K_2=-\frac{|\delta|}{5(\sin 2K_1-\sin 2k_1)}.$$

The results for K_2 and k_2 (from Eq. (11)) are shown in Table III.

It is self-evident that using the values of K_2 and k_2 determined in this way will reproduce the curves in Figs. 3, 4, 5, 6, and 7 reasonably well. This comparison will not be carried out in detail as it appears to be pushing the data farther than justifiable. If further refinement were attempted, one should next include the possibility that the quartet and doublet S-waves are not shifted by exactly the same amount. This would lead principally to an antisymmetric contribution through interference with the P-waves (since these are of opposite sign). A further antisymmetric term that becomes of increasing importance as the energy increases will be the S–F interference. Refined analysis would also detect symmetrical contributions such as the S–D interference, the D^2-terms and, finally, the possibility of spin-orbit coupling which gives waves that do not interfere with the other partial waves. Thus, the estimates of D-wave shifts presented in Table III characterize the limits of the experimental data in yielding to phase-shift analysis rather better than a final analysis of the phase shifts in the proton-deuteron scattering.

This paper is based on work performed at Los Alamos Scientific Laboratory of the University of California under Government Contract No. W-7405-eng-36, and the information contained therein will appear in Division V of the National Nuclear Energy Series (Manhattan Project Technical Section), as part of the contribution of the Los Alamos Laboratory.

ACKNOWLEDGMENTS

I should like to thank Dr. Rubby Sherr of the P Division at Los Alamos for access to the experimental results before publication, and Dr. Bengt Carlson for the assistance by his group in computing the angular distributions on which Figs. 3, 4, 5, 6, and 7 are based, and Mr. Louis Jacot for preparation of the figures. I am indebted also to Professor Edward Teller for many interesting discussions of the exchange effects in this scattering problem.