Extensive Air Showers and the Cascade Theory

GIUSEPPE COCCONI*
Institute of Physics, University of Catania, Catania, Italy
(Received August 7, 1947)

The barometric effect, the density spectra at different barometric pressures and the variation of frequencies with height above sea level have been calculated for extensive air showers, on the basis of the cascade theory. Good agreement has been found between theoretical results and experimental data.

SECTION 1

THE aim of this work is to study, on the basis of the cascade theory, some phenomena related to the extensive cosmic-ray air showers. In the present state of our knowledge, it is reasonable to think that the extensive showers are due to primary electrons and photons of very high energy (10^{14}–10^{17} ev), which multiply rapidly in the upper atmosphere and give rise in the lower atmosphere to showers consisting of millions of electrons and photons spread over very large surfaces (thousands of square meters), always with the same spatial-distribution law. If the above assumptions are correct, the properties of the extensive showers should depend only on the thickness of air traversed, i.e., on the atmospheric pressure. A given variation Δp in the atmospheric pressure, caused either by barometric change or by a change of height of the recording apparatus above sea level, should cause the same variation in the frequency of the recorded showers.

In the following sections the results of our calculations are collected and compared with the experimental results available to date.

SECTION 2

The barometric effect of the extensive showers has been investigated experimentally by Cosyns[1] and by Auger and Daudin.[2] Their results are rather uncertain, because of the small frequencies of the showers. Nevertheless, all the measurements indicate a high negative barometric effect, i.e., a large increase in the frequencies when the barometric pressure decreases.

Let H be the frequency of the recorded showers, and ΔH be the variation related to a variation Δp of the atmospheric pressure. Then we have

$$\Delta H/H = -\alpha\Delta p. \qquad (1)$$

If Δp is measured in cm Hg, the above experiments indicate $\alpha \approx 0.1$ to 0.2 (10 to 20 percent per cm Hg). The surfaces of the counters used in these experiments were about 200 cm^2 and the distances between them were some meters. Both Cosyns and Auger and Daudin found that α increases when the distance between the counters is increased. It seems that α does not depend on altitude.

SECTION 3

In order to evaluate theoretically the barometric effect of the extensive showers, the density spectrum at different barometric pressures was calculated, following the method described elsewhere.[3] We used again the Molière[4] distribution curve of the particles in the shower, but evaluated the number of particles following Rossi and Greisen,[5] whose formulas are more accurate than Heisenberg's[6] in the low pressure range.

For the primary spectrum we assumed

$$H(E) = 0.05(10^{10}\ \text{ev}/E)^x\ \text{cm}^{-2}\ \text{sec.}^{-1},$$

where $H(E)$ is the number of primary electrons coming isotropically from free space with energies above E, and x is a constant for which we chose the values 1.6 and 1.8.

The calculated barometric effect is practically

* At the present, Research Associate at Cornell University, Ithaca, New York.

[1] M. G. E. Cosyns, Nature **145**, 668 (1940).

[2] P. Auger and J. Daudin, Phys. Rev. **61**, 91 (1942).

[3] G. Cocconi, A. Loverdo, and V. Tongiorgi, Phys. Rev. **70**, 846 (1946).

[4] G. Molière, *Vorträge über Kosmische Strahlung* (Berlin, 1943), p. 24.

[5] B. Rossi and K. Greisen, Rev. Mod. Phys. **13**, 240 (1941).

[6] W. Heisenberg, *Vorträge über Kosmische Strahlung* (Berlin, 1943), p. 10.

independent of the mean barometric pressure, at least in the lower half of the atmosphere, in agreement with the experimental results. The calculations also indicate that the barometric effect should vary noticeably with changes in the area S of the counters' surface. This is caused by the dependence of the mean density of the recorded showers on the counter area.

In Table I are collected the values of α calculated for a threefold coincidence system among three counters (or groups of counters) each of area S, arranged on a horizontal plane 10 or more meters apart in order to record only extensive showers. Nearly the same values should have been obtained for a fourfold coincidence system.

A comparison between our theoretical results and the experimental data cannot be precise, because the recording arrangements used by the above mentioned authors are rather unlike that to which we refer. Nevertheless there is fair agreement between theory and experiment as to the magnitude of the effect. As Table I shows, α does not depend strongly on x, so that it is impossible by this method to determine the shape of the primary spectrum. It appears that a systematic measurement of the barometer effect performed with counters of different areas would be very interesting in order to test the dependence of α on S.

The theory does not predict the dependence of α on the distance between the counters, at least for not too large distances. We shall discuss this point below.

SECTION 4

The theoretical density spectra at various atmospheric pressures, i.e., at various heights above sea level, always have the form

$$H(\Delta)=A(10^{-4}\text{ cm}^{-2}/\Delta)^{\gamma}\text{ sec}^{-1}$$

where $H(\Delta)$ is the frequency of showers of density $\geqslant\Delta$. The calculated values of the constants A and γ, for particular values of the pressure p and of the constant x, are collected in Table II. (In reference 3 the same calculation was made for zero- and 2200-m elevation and $x=1.8$. The values of A and γ found in reference 3 differ a little from the present ones, owing to the use of Heisenberg's formulas instead of those of Rossi and Greisen.) In the last column of Table II are collected the experimental values of A and γ as they result from our experiments[3] at 120 and 2200 m above sea level.

TABLE I. Calculated values of the barometric coefficient α for a threefold coincidence system.

	$S=10$ cm^2	$S=50$ cm^2	$S=200$ cm^2	$S=1000$ cm^2	$S=5000$ cm^2
$x=1.6$	0.075	0.085	0.095	0.11	0.12
$x=1.8$	0.08	0.10	0.11	0.125	0.14

TABLE II. Values of the constants A and γ in the theoretical shower frequency formula.

	$x=1.6$		$x=1.7$		$x=1.8$		Experim. values	
	A	γ	A	γ	A	γ	A	γ
$p=760$-mm Hg (0 m)	0.22	1.22	0.083	1.29	0.032	1.36	0.124	1.46
$p=565$-mm Hg (2300 m)	2.3	1.35	1.07	1.43	0.5	1.50	1.0	1.55
$p=452$-mm Hg (4100 m)	8.9	1.45	4.7	1.55	2.5	1.64		

As for γ, the agreement between theory and experiment is rather good ($x=1.8$), and the theoretical increase of γ with height is confirmed also. It must be observed that other experimenters,[7] on the contrary, have found a reduction of γ with increasing altitude, in disagreement with our experimental and theoretical results. We think that further experiments carried out in order to look into this point might be very interesting. One notes that the variation of γ with altitude is directly related to the variation of α with S in the barometric effect.

As for the constant A, the theoretical results are rather lower than the experimental ones. This may be ascribed, in addition to the uncertainties in the primary spectrum and the cascade theory, also to the fact that the distances between the counters in our experiments were too small (4 m) to eliminate local showers due to non-electronic particles (see the decoherence curve in Molière).[4]

As mentioned above, we think that 10- to 20-m distance between the counters is necessary in order to record only extensive showers. Possibly even the values of γ determined at larger counter distances may differ from those obtained in reference 3. This may also explain why the above

[7] P. Auger and J. Daudin, J. de phys. et rad. 6, 233 (1945).

mentioned experimenters found a barometric effect depending on the distances between the counters.

SECTION 5

From our calculations of the barometric effect one can deduce the manner in which the frequency of recorded showers varies with height above sea level, for the apparatus described in Section 3 (threefold coincidences among identical counters). It follows directly from Eq. (1) that

$$H_s(p) = H_s(p_0)e^{-\alpha\delta p}$$

where $\delta p = p - p_0$ and $H_s(p)$ is the frequency of showers recorded at atmospheric pressure p by counters of area S. If δp is measured in cm Hg, the values of α are those collected in Table I.

Experimental data may be deduced from Hilberry's measurements.[8] Hilberry recorded fourfold coincidences among counters of 200 cm^2 area arranged 2.5 m apart, at elevations between 0 and 4300 m above sea level. From Hilberry's measurements we deduce $\alpha = 0.10$, in agreement with the data of Cosyns and Auger on the barometer effect, and with the theoretical results in Table I. In our opinion this agreement is a good proof of the reliability of the assumptions on which the calculations are based.

SECTION 6

Elsewhere[9,10] the author and co-workers have shown that all extensive showers are associated with penetrating showers (i.e., showers recorded under layers of lead thick enough to eliminate the electronic component), and that the penetrating showers are generated locally in the materials screening the counters. If these results are correct, we can expect that both the barometric effect and the height effect of the penetrating showers associated with extensive ones follow the same laws found for the extensive showers alone.

No experiments have yet been performed in which the barometric effect and the height effect have been measured only for those penetrating showers which are associated with extensive showers. However, all kinds of penetrating showers (regardless of association with extensive showers) have been investigated by Janossy and Rochester,[11] who studied the barometer effect and deduced $\alpha = 0.12$ (with sevenfold coincidences among counters of ≈ 150 cm^2), and by Wataghin,[12] who studied the height effect up to 8500 m and found $\alpha = 0.136$ (fourfold coincidences among counters of 400 cm^2). These results agree with those reached for extensive showers, testifying probably to the local generation of the penetrating showers and their connection with extensive showers. This agreement, of course, might be only chance, since penetrating showers not depending on extensive showers were also recorded in the above mentioned experiments.

In conclusion, we remark that many phenomena pertaining to extensive air showers fit rather well the cascade theory. To the points discussed in this work we should add the agreement between theory and experiments concerning the decoherence curve.[13] Therefore we think that, up to date, many properties of extensive cosmic-ray showers may be described in terms of the cascade theory of electrons and photons.

---

[8] N. Hilberry, Phys. Rev. **60**, 1 (1941).
[9] G. Cocconi, A. Loverdo, and V. Tongiorgi, Phys. Rev. **70**, 852 (1946).
[10] G. Cocconi and C. Festa, Nuovo Cimento **3**, 293 (1946).
[11] L. Janossy and G. D. Rochester, Proc. Roy. Soc. **183**, 186 (1944).
[12] G. Wataghin, Phys. Rev. **71**, 453 (1947).
[13] G. Cocconi, Phys. Rev. **72**, 350 (1947).