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On the Magnetic Moment of the Deuteron*

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An attempt is made to calculate the relativistic correction to the magnetic moment of the deuteron in order to determine the D state probability from the accurately determined proton, deuteron, and neutron moments. It is found that the resulting corrections depend strongly on the transformation properties of the nuclear fields. If a scalar field is assumed the D state probability is estimated at 4.8 percent, while for a vector field the corresponding estimate is 3.8 percent. The effect of a non-central tensor interaction is included in these estimates. It is concluded that the magnetic moment cannot be used for a precision determination of the amplitude of the D function. The results also indicate that little can be said at this time concerning the question of the additivity of the intrinsic proton and neutron moments.

1. INTRODUCTION

An estimate of the amount of D state required to account for the quadrupole moment of the deuteron has been made by Rarita and Schwinger.[1] They found the D state probability to be about 3.9 percent. This result depends to some extent on the assumptions made concerning the nuclear forces, since the quadrupole moment depends on the radial distribution of charge as well as the angular distribution. For example, Bethe[2] found a D state probability of 6.7 percent on the basis of different assumptions concerning the neutron-proton interaction.

A more direct method for determining the amount of D state would be to make use of the recent precision measurement[3] of the magnetic moment of the neutron along with the previous determination[4] of the moments of the deuteron and proton. In ordinary Schroedinger theory these measurements determine the D state probability uniquely without any reference to the nuclear forces. However, it has been pointed out by Margenau[5] and Caldirola[6] that relativistic corrections to the magnetic moment of the deuteron which are not included in the ordinary Schroedinger theory may be of the same order as the D state effect.

In their calculations, Margenau and Caldirola assumed that the nuclear particles move in a central field of force with no interaction between them. They also tacitly assumed that the central forces arise from a vector field. Since one would like, if possible, to make a precise determination of the relativistic correction of the same order of precision as the magnetic moment measure-

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[1] W. Rarita and J. Schwinger, Phys. Rev. **59**, 436 (1941).

[2] H. A. Bethe, Phys. Rev. **57**, 390 (1940).

[3] W. R. Arnold and A. Roberts, Phys. Rev. **70**, 766 (1946).

[4] J. M. Kellogg, I. I. Rabi, M. F. Ramsey and J. R. Zacharias, Phys. Rev. **56**, 728 (1939).

[5] H. Margenau, Phys. Rev. **57**, 383 (1940).

[6] P. Caldirola, Phys. Rev. **69**, 608 (1946).

ment, it seems worth while to consider this correction in somewhat more detail. There are several possible deviations from the results of Margenau and Caldirola. In the first place, the force field acting on one particle arises from the other particle. Since the source of the field is moving with a velocity comparable to the particle, one would expect vector potential terms to contribute to the magnetic moment. These do not appear in the Margenau and Caldirola theories, because they have assumed that the field arises from a stationary center. In addition, it is known that the forces acting between the particles are not central in character. It is just this non-central nature of the forces which is required to lead to admixture of the D state.[1] It has been estimated by Rarita and Schwinger that the strength of the non-central forces is approximately $\frac{3}{4}$ of that of the central force. There are two effects of the non-central field, the first of which is the effect on the wave function which is to be used in computing the relativistic correction. The other is a change in the form of the relativistic correction terms.

In addition to the above modifications to the Margenau and Caldirola theories, one might expect that the relativistic corrections would depend on the nature of the nuclear fields; i.e., whether the fields arise from 4-vector potentials or from scalar potentials.

The purpose here is to attempt to carry through to order v^2/c^2 a two-body calculation taking into account some of the effects described above. For this purpose it is desirable to avoid perturbations of the magnetic moment due to the meson field since we would like to see how well the observed results can be accounted for without introducing such perturbations. Therefore, the problem will be treated using classical fields. It is known that a satisfactory relativistic quantum theory of two interacting bodies cannot be written down in this case. It is possible, however, to construct an essentially classical Hamiltonian which gives the appearance of relativistic co-variance. This Hamiltonian can be expanded to terms of order v^2/c^2, and then the correspondence principle may be introduced to give the appropriate relativistic formulation of the problem. In this theory the anomalous moments of the neutron and proton will be introduced by adding the Pauli[7] type interaction with the electromagnetic field to the Hamiltonian.

The one-body problem in a scalar field will first be repeated using the methods indicated above in order to indicate what deviations from the results of Margenau and Caldirola are to be expected. Then the two-body problem will be treated using both vector and scalar fields without introducing the non-central part of the field. Finally, the effect of non-central tensor forces will be considered.

The results of these calculations indicate that the answer is not unique but that it depends strongly on the nature of the fields. As a consequence, the amount of D state as determined from the magnetic moment may be anywhere between about 3.8 percent and 4.8 percent. It can be hoped that a more accurate knowledge of the neutron-proton potential combined with a knowledge of the quadrupole moment will make it possible to determine the D state probability with some accuracy and thereby gain information concerning the nature of the nuclear fields. For this purpose it would be assumed that the perturbation of the intrinsic moments of the neutron and proton by their interaction is smaller than the effects under discussion. There is no available evidence as to the correctness of such an assumption.

2. PARTICLE IN A CENTRAL FIELD

The treatment of the one particle problem can be handled as indicated by Pauli.[8] The Dirac equation modified by the addition of the Pauli term is written in the form

$$(E-e\phi_0)\varphi=c(\boldsymbol{\sigma}\cdot\boldsymbol{\pi})\chi-\lambda(\boldsymbol{\sigma}\cdot\mathbf{H})\varphi, \quad (1)$$

$$(E-e\phi_0+2mc^2)\chi=c(\boldsymbol{\sigma}\cdot\boldsymbol{\pi})\varphi+\lambda(\boldsymbol{\sigma}\cdot\mathbf{H})\chi, \quad (2)$$

where

$$\boldsymbol{\pi}=\frac{\hbar}{i}\,\mathrm{grad}-\frac{e}{c}\boldsymbol{\phi}, \quad (2')$$

ϕ_0 is the scalar potential, and $\boldsymbol{\phi}$ the vector potential of the nuclear force field, $\mathbf{H}$ is the magnetic field vector, λ is the anomaly in the magnetic

[7] W. Pauli, *Handbuch der Physik* (Julius Springer, Berlin) 24/1.

[8] W. Pauli, reference 7. It is to be noted that there is a typographical error in the sign of the $(\mathcal{E}\cdot\boldsymbol{\pi})$ term in his Eq. (89), p. 237.

moment, $\boldsymbol{\sigma}$ is the vector whose components are the Pauli spin operators, and φ and χ are the 2-component wave functions. The wave function χ is of the order of $v\varphi/c$ when the velocities are small. To order v^2/c^2 these equations can be combined to give a modified Schroedinger equation for the function φ. If we write $\mathcal{H}$ for the classical Hamiltonian in the presence of a magnetic field, H, then the equation for φ becomes

$$E\varphi=\mathcal{H}\varphi-\frac{1}{(2mc)^2}(\boldsymbol{\sigma}\cdot\boldsymbol{\pi})(E_0-e\phi_0)(\boldsymbol{\sigma}\cdot\boldsymbol{\pi})\varphi+\frac{\lambda}{(2mc)^2}(\boldsymbol{\sigma}\cdot\boldsymbol{\pi})(\boldsymbol{\sigma}\cdot\mathbf{H})(\boldsymbol{\sigma}\cdot\boldsymbol{\pi}). \quad (3)$$

Here

$$\mathcal{H}=\frac{\pi^2}{2m}-\left(\frac{e\hbar}{2mc}+\lambda\right)(\boldsymbol{\sigma}\cdot\mathbf{H})+e\phi_0,$$

and E_0 is the energy value for the zero-order equation $\mathcal{H}\varphi_0=E_0\varphi_0$. In order to determine the magnetic moment we first seek the solutions to Eq. (3) when $H=0$ and then determine the correction to the energy caused by the presence of the magnetic field by means of perturbation theory. The coefficient of H in the expression for the energy is then the magnetic moment. The second term in Eq. (3) can be simplified by commuting the factor $(\boldsymbol{\sigma}\cdot\boldsymbol{\pi})$ on the right through $(E_0-e\phi_0)$. The resulting terms in the energy which contain the magnetic field are given by the average over the wave function of the quantity

$$\begin{aligned}\epsilon=&-\frac{e\hbar}{2mc}[(\mathbf{L}+\boldsymbol{\sigma})\cdot\mathbf{H}]-\lambda(\boldsymbol{\sigma}\cdot\mathbf{H})\\&+\frac{T}{mc^2}\frac{e\hbar}{2mc}[(\mathbf{L}+\boldsymbol{\sigma})\cdot\mathbf{H}]+\frac{e\hbar}{2mc}\cdot\frac{e}{4mc^2}\\&\times\left\{\frac{1}{i}(\mathcal{E}\times\mathbf{r}\cdot\mathbf{H})-[(\boldsymbol{\sigma}\cdot\mathbf{r})(\mathcal{E}\cdot\mathbf{H})\right.\\&\left.-(\boldsymbol{\sigma}\cdot\mathbf{H})(\mathcal{E}\cdot\mathbf{r})]\right\}+\frac{\lambda}{2m^2c^2}(\boldsymbol{\sigma}\cdot\mathbf{p})(\mathbf{p}\cdot\mathbf{H}) \quad (4)\end{aligned}$$

where $\mathbf{L}$ is the (dimensionless) orbital angular momentum operator, $\mathbf{r}$ is the position vector, $\mathbf{p}$ is the momentum of the particle, T is the kinetic energy of the particle, and $\mathcal{E}$ is the "electric" field, i.e. $\mathcal{E}=-\mathrm{grad}\phi_0$. The first term is the usual expression, and all of the other terms are corrections of the order v^2/c^2. In the absence of non-central fields, the term $(\mathcal{E}\times\mathbf{r}\cdot\mathbf{H})$ vanishes and the other terms may be calculated without reference to the shape of the potential. The result is found to agree with that obtained by Caldirola. If the field is non-central there are the two effects suggested above; namely, the $(\mathcal{E}\times\mathbf{r}\cdot\mathbf{H})$ term may not vanish, and the average value of the other terms depends on the amount of admixture of states in the wave function. Since the recoil of the source of the field has been neglected here, the result does not contain the specifically nuclear vector potential which is of the order of $v\phi_0/c$ where v is the velocity of the source of the field.

It will be noted that, according to Eq. (4), the anomalous part of the moment does not depend on the nature of the nuclear forces, except through the wave function to be used in the averaging. It will be found that this result obtains in general.

3. TWO-BODY PROBLEM WITH SCALAR FIELD

Because of its relative simplicity we first treat the problem of the scalar field without tensor forces. Designating the field acting on particle 1 (the proton) resulting from particle 2 as[8a] U_1, it is assumed that U_1 may be obtained from the field equation

$$\square_1 U_1-\kappa^2U_1=-i(\gamma_2\cdot\mathcal{S}_2), \quad (5)$$

where $\square_1$ is the D'Alembertian with respect to the first particle, κ characterizes the range of the nuclear forces, $\mathcal{S}_2$ is the current density 4-vector for particle 2 (the neutron) and γ_2 is a 4-vector which will later be identified with that which can be obtained from the Dirac matrices of the second particle; namely,[7]

$$\gamma_2{}^k=-i\beta_2\alpha_2{}^k$$
$$\gamma_2{}^4=\beta_2.$$

All that will be required for our purpose is the solution of Eq. (5) to zero order in v/c. To that

[8a] U_1 represents half of the interaction potential and U_2 represents the other half. This division is required in order to separate the roles of the two particles in the relativistic equation, Eq. (8). It will be seen below that the interaction potential is related to the sum of U_1 and U_2.

approximation, Eq. (5) reduces to

$$\Delta_1 U_1^0 - \kappa^2 U_1^0 = \beta_2 \rho_2 \tag{6}$$

where ρ_2 is the charge density of the second particle. The solution to this equation is of the form

$$U_1^0 = \beta_2 V_1(|\mathbf{r}_1 - \mathbf{r}_2|) \tag{7}$$

where V_1 is a Yukawa type potential. A similar expression can be found for the potential acting on the neutron caused by the proton. Making use of these potentials we may now write the Hamiltonian for the two interacting particles

$$\begin{aligned}\mathbf{E} + Mc^2 = m_1c^2\beta_1 + m_2c^2\beta_2 + c(\boldsymbol{\alpha}_1\cdot\boldsymbol{\pi}_1)\\ + c(\boldsymbol{\alpha}_2\cdot\boldsymbol{\pi}_2) + \beta_1 U_1 + \beta_2 U_2\\ - \lambda_1(\boldsymbol{\sigma}_1\cdot\mathbf{H})\beta_1 - \lambda_2(\boldsymbol{\sigma}_2\cdot\mathbf{H})\beta_2\end{aligned} \tag{8}$$

where $\boldsymbol{\pi}_1$ and $\boldsymbol{\pi}_2$ are the usual generalized momenta containing the vector potential caused by the external magnetic field (see Eq. (2′)), λ_1 is the anomaly in the magnetic moment of the proton and λ_2 that of the neutron. In this expression we will only carry terms to order v^2/c^2 in the U. With this understanding, the wave equation is

$$\mathbf{E}\varphi = E\varphi, \tag{9}$$

where φ is now a 16-component wave function. In analogy to Eq. (1), we write φ in terms of 4-component functions φ_1, φ_2, φ_3, φ_4. We choose φ_1 in such a way that it is the zero order function, φ_2 to be of order $v\varphi_1/c$ in the first particle, φ_3 to be of order $v\varphi_1/c$ in the second particle, and φ_4 to be of the order $v^2\varphi_1/c^2$. The equations for φ_1, φ_2, φ_3, and φ_4 are then

$$\begin{aligned}E\varphi_1 = c(\boldsymbol{\sigma}_1\cdot\boldsymbol{\pi}_1)\varphi_2 + c(\boldsymbol{\sigma}_2\cdot\boldsymbol{\pi}_2)\varphi_3 + V\varphi_1\\ - \lambda_1(\boldsymbol{\sigma}_1\cdot\mathbf{H})\varphi_1 - \lambda_2(\boldsymbol{\sigma}_2\cdot\mathbf{H})\varphi_1,\end{aligned} \tag{10a}$$

$$\begin{aligned}(E+Mc^2)\varphi_2 = c(\boldsymbol{\sigma}_1\cdot\boldsymbol{\pi}_1)\varphi_1 + c(\boldsymbol{\sigma}_2\cdot\boldsymbol{\pi}_2)\varphi_4\\ - V\varphi_2 + \lambda_1(\boldsymbol{\sigma}_1\cdot\mathbf{H})\varphi_2 - \lambda_2(\boldsymbol{\sigma}_2\cdot\mathbf{H})\varphi_2,\end{aligned} \tag{10b}$$

$$\begin{aligned}(E+Mc^2)\varphi_3 = c(\boldsymbol{\sigma}_1\cdot\boldsymbol{\pi}_1)\varphi_4 + c(\boldsymbol{\sigma}_2\cdot\boldsymbol{\pi}_2)\varphi_1\\ - V\varphi_3 - \lambda_1(\boldsymbol{\sigma}_1\cdot\mathbf{H})\varphi_3 + \lambda_2(\boldsymbol{\sigma}_1\cdot\mathbf{H})\varphi_3,\end{aligned} \tag{10c}$$

$$\begin{aligned}(E+2Mc^2)\varphi_4 = c(\boldsymbol{\sigma}_1\cdot\boldsymbol{\pi}_1)\varphi_3 + c(\boldsymbol{\sigma}_2\cdot\boldsymbol{\pi}_2)\varphi_2\\ + V\varphi_4 + \lambda_1(\boldsymbol{\sigma}_1\cdot\mathbf{H})\varphi_4 + \lambda_2(\boldsymbol{\sigma}_1\cdot\mathbf{H})\varphi_4,\end{aligned} \tag{10d}$$

where $M = m_1 + m_2 = 2m_1$ and $V = V_1 + V_2$.

These equations are to be treated by the method indicated in the previous section. Terms only as high as v^2/c^2 are to be carried, and then the magnetic moment is to be determined by treating the terms proportional to the magnetic field as perturbations. In the equations for the unperturbed system, there will be contributions due to retardation effects in U_1 and U_2; but these effects will only affect the magnetic moment through their influence on the wave function. Thus, for example, the amount of D state contained in the deuteron wave function may be caused partly by tensor forces which arise in zero order and partly by tensor forces which arise in order v^2/c^2. A distinction between these two interactions is not necessary for our purpose, since we are primarily interested in determining the amount of D state from a measurement of the magnetic moments and not in calculating directly what the amount of D state should be. Therefore, in the determination of the corrections to the formula relating the magnetic moment to the wave functions only terms of zero order in the potential such as that given by Eq. (7) will play a role.

It was mentioned in Section 2 that the corrections to the anomalous part of the magnetic moment are essentially independent of the nature of the force field. This follows directly from the proper manipulation of Eqs. (10). The final result which need not be repeated again in the following sections is

$$\epsilon_a = (2/(Mc)^2)[\lambda_1(\boldsymbol{\sigma}_1\cdot\mathbf{p}) + \lambda_2(\boldsymbol{\sigma}_2\cdot\mathbf{p})](\mathbf{H}\cdot\mathbf{p}), \tag{11}$$

where ϵ_a is the correction of order v^2/c^2 to the perturbation energy which arises from the anomalous moments, and $\mathbf{p}$ is the momentum of the proton which is equal to the negative of the momentum of the neutron in the center of gravity coordinate system. It is to be remembered that λ_1 and λ_2 are very nearly equal and opposite ($\lambda_1 = 1.790$, $\lambda_2 = -1.910$ in nuclear magnetons). Since the ground state of the deuteron is a triplet state, the spins $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ are parallel so the two terms in Eq. (11) very nearly cancel. The residual is small compared with the effects that are of interest, so in the following ϵ_a will be neglected.

The other terms may be obtained by the methods indicated in Section 2. The results are somewhat different because of the change in sign of the potential terms which occurs in Eqs. (10). This is a characteristic difference between the scalar field and the vector field. The final result for the perturbation energy due to the magnetic field, H, is

$$\epsilon_s=-\frac{e\hbar}{Mc}[(\mathbf{L}_1+\boldsymbol{\sigma}_1)\cdot\mathbf{H}]-\lambda_1(\boldsymbol{\sigma}_1\cdot\mathbf{H})$$

$$-\lambda_2(\boldsymbol{\sigma}_2\cdot\mathbf{H})+\frac{e\hbar}{Mc}[(\mathbf{L}_1+\boldsymbol{\sigma}_1)\cdot\mathbf{H}](2E-T)/Mc^2$$

$$+\frac{rV'}{4Mc^2}\frac{e\hbar}{Mc}\left[(\boldsymbol{\sigma}_1\cdot\mathbf{H})-\frac{(\boldsymbol{\sigma}_1\cdot\mathbf{r})(\mathbf{r}\cdot\mathbf{H})}{r^2}\right], \quad (12)$$

where E is the binding energy of the deuteron, $\mathbf{r}$ is the distance between the two particles, and V' is the derivative of V with respect to r.

Under the assumption that the only interaction between the particles is given by the central potential V, the average value of the last term in Eq. (12) is found to be

$$(e\hbar/Mc)\langle(\boldsymbol{\sigma}_1\cdot\mathbf{H})-\tfrac{1}{3}(\boldsymbol{\sigma}_1\cdot\mathbf{H})\rangle\langle T/2Mc^2\rangle, \quad (13)$$

as a consequence of the Schroedinger equation. The angular brackets are meant to indicate the average over the wave function. The ground state of the deuteron may be assumed to be a ^3S state since the effects of the tensor force are being neglected. Then the average value of this term is found to be

$$(e\hbar/Mc)(\boldsymbol{\sigma}_1\cdot\mathbf{H})\langle T/3Mc^2\rangle,$$

and the relativistic correction to the magnetic moment of the deuteron (measured in nuclear magnetons) becomes

$$\Delta\mu=-2\langle E-T/3\rangle/Mc^2. \quad (14)$$

For the deuteron $E/Mc^2=-1.2\times10^{-3}$ and $\langle T/Mc^2\rangle$ may be taken to be about[9] 5.4×10^{-3}. Then the correction to the moment is found to be

$$\Delta\mu=6.0\times10^{-3},$$

so about 0.88 percent of the deuteron moment would be ascribed to this relativistic effect. The correct relation between the moment of the deuteron and those of the neutron and proton is[1]

$$\mu_D=(\mu_n+\mu_p)(1-3D/2)+3D/4+\Delta\mu, \quad (15)$$

where D is the D state probability. The above estimate of $\Delta\mu$ leads to $D=5.1$ percent as compared to $D=4$ percent obtained by Arnold and Roberts.[3]

4. TWO-BODY PROBLEM WITH VECTOR FIELD

The treatment given in Section 3 can now be repeated using a 4-vector interaction in place of the scalar interaction. For this purpose we introduce a 4-vector field, $\mathcal{A}_1$, which arises from particle 2 and acts on particle 1. $\mathcal{A}_1$ may be taken to be the solution of the equation

$$\Box\mathcal{A}_1-\kappa^2\mathcal{A}_1=\mathcal{S}_2.$$

Again, only that approximation to the solution of this equation which neglects retardation effects is required. To this approximation one finds

$$A_1^4=iV_1, \quad (16a)$$

$$\mathbf{A}_1=(\mathbf{p}_2V_1+V_1\mathbf{p}_2)/2m_2c \quad (16b)$$

where $\mathbf{A}_1$ is the 3-vector representing the spatial part of $\mathcal{A}_1$. Since the spatial components $\mathbf{A}_1$ are of order v/c times the fourth component, it is to be expected that they will only contribute to the correction to the magnetic moment.

The Hamiltonian for the two interacting particles may now be written in the form

$$\mathbf{E}+Mc^2=m_1c^2\beta_1+m_2c^2\beta_2$$

$$+c\left[\boldsymbol{\alpha}_1\cdot\left(\boldsymbol{\pi}_1-\frac{1}{c}\mathbf{A}_1\right)\right]$$

$$+c\left[\boldsymbol{\alpha}_2\cdot\left(\boldsymbol{\pi}_2-\frac{1}{c}\mathbf{A}_2\right)\right]+V_1+V_2. \quad (17)$$

The contributions of the anomalous moment to the Hamiltonian have been ignored for the reasons given above. The perturbation treatment of the wave equations which follow from this

[9] This estimate of the kinetic energy is based on the work of Rarita and Schwinger (reference 1) so it is not strictly consistent with our assumption of no D state. In any case, this calculation serves only as an indication of the effect. Certain numerical constants which are not given in reference 1 were kindly provided the author by M. E. Rose who has recalculated them.

Hamiltonian leads to the result

$$\epsilon_v = -\frac{eh}{Mc}[(\mathbf{L}_1+\boldsymbol{\sigma}_1)\cdot\mathbf{H}] - \lambda_1(\boldsymbol{\sigma}_1\cdot\mathbf{H}) - \lambda_2(\boldsymbol{\sigma}_2\cdot\mathbf{H}) + \frac{eh}{Mc}[(\mathbf{L}_1+\boldsymbol{\sigma}_1)\cdot\mathbf{H}]T/Mc^2 - \frac{rV'}{4Mc^2}\frac{eh}{Mc}\left[(\boldsymbol{\sigma}_1\cdot\mathbf{H}) - \frac{(\boldsymbol{\sigma}_1\cdot\mathbf{r})(\mathbf{r}\cdot\mathbf{H})}{r^2}\right] - \frac{2V}{Mc^2}\frac{eh}{Mc}(\mathbf{L}_2\cdot\mathbf{H}). \quad (18)$$

The last term arises from the spatial part of the vector potential. It has been simplified by making use of the relation Eq. (16b) between the spatial part of the vector potential and the ordinary potential.

The V' term in Eq. (18) is again given by the expression (13) on the assumption that the ground state of the deuteron is a ^3S state, and the last term vanishes. Consequently, the correction to the magnetic moment is found to be

$$\Delta\mu = -2\langle T/3Mc^2\rangle = -3.6\times10^{-3},$$

in agreement with the results of Margenau and Caldirola. The D state probability which would be obtained from Eq. (15) using this correction is 3.4 percent. Thus there is a wide difference between the results obtained with the two different types of field.

5. THE EFFECT OF TENSOR FORCES

In the foregoing discussion it has been assumed that the fields are spin independent in zero approximation. Thus they do not include a tensor interaction of the type required to account for the quadrupole moment of the deuteron, except for the fact that some tensor interaction would arise in higher approximations in v/c. It seems desirable to treat the problem on the assumption that the tensor interaction arises in zero order, since this interaction appears to be quite strong.[1] Such an interaction can be included in the discussion by introducing additional fields, either scalar or vector.

Since the results of the calculation for a central field are already ambiguous, it does not seem worth while to go into very great detail with the tensor interaction. As a matter of fact, it is well known that the introduction of tensor forces in a field theory usually carries with it potentials proportional to r^{-3} for which there is no proper solution of the Schroedinger equation.[2] To avoid these difficulties the treatment of the fields will only be carried far enough to indicate the manner in which the tensor interaction is to be inserted in Eqs. (10a–d), or their equivalent. Then the usual tensor interaction will be inserted in the indicated manner.

The scalar field may be modified by the addition of a term

$$P_1 = k\eta_1(\boldsymbol{\gamma}_1\cdot\mathbf{grad}_1)W_1$$

where W_1 is a solution of the equation

$$\Box_1 W_1 - \kappa^2 W_1 = \eta_2(\boldsymbol{\gamma}_2\cdot\mathbf{grad}_2)(\boldsymbol{\gamma}_2\cdot\mathcal{S}_2),$$

and η_1 is the pseudoscalar obtained by taking the product of the four components of the 4-vector $\boldsymbol{\gamma}_1$, and η_2 is similarly defined for the second particle. The indicated gradients are to be interpreted in the four-dimensional sense. Again, only the zero-order contributions of W_1 are of interest and in this approximation we find

$$P_1^0 = k\beta_1(\boldsymbol{\sigma}_1\cdot\mathrm{grad}_1)(\boldsymbol{\sigma}_2\cdot\mathrm{grad}_2)V_1 \quad (19)$$

where the gradients are now to be interpreted in the three-dimensional sense, and the matrices $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ are the four-dimensional matrices obtained by taking the direct product of the ordinary two-dimensional spin matrix with a two dimensional unit matrix. This result is a consequence of the fact that the time derivations occurring in Eq. (19) are of the order of v/c times the spatial gradients, so they may be neglected.

The introduction of P_1 and of the analogous P_2 in the Hamiltonian given by Eq. (8) leads to an expression containing a tensor interaction. The feature which is of importance for our considerations is that P_1 is to be multiplied by β_1 on introduction into the Hamiltonian so that the tensor interaction term appears with the factor $\beta_1^2=1$. Therefore, the sign of the tensor interaction term is the same in all four of the equations which are the analog of Eqs. (10a–d). If we now introduce a simple tensor force of the type

$$K_1 = k[(\boldsymbol{\sigma}_1\cdot\mathbf{r}_1)(\boldsymbol{\sigma}_2\cdot\mathbf{r}_2) - \tfrac{1}{3}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)(\mathbf{r}_1\cdot\mathbf{r}_2)]V_1/2r_1r_2 \quad (20)$$

and make use of the fact that the signs of K_1 and the analogous K_2 are to be the same in all four of the Dirac equations, the additional correction to the magnetic moment due to the tensor forces can be calculated. The resulting expression is

$$\epsilon = \epsilon_s + \epsilon' \tag{21}$$

where ϵ_s is given by Eq. (12), and ϵ' is

$$\begin{aligned}\epsilon' = \frac{1}{2Mc^2}\frac{e}{Mc}\{ &(\boldsymbol{\sigma}_1\cdot\mathbf{r}\times\mathbf{H})\\ &\times[\hbar(\boldsymbol{\sigma}_1\cdot\mathrm{grad}_1K)/2i - kV(\boldsymbol{\sigma}_2\cdot\mathbf{r})(\mathbf{r}\cdot\mathbf{p})/r^2]\\ &+(kV/3)[(\boldsymbol{\sigma}_1\cdot\mathbf{r}\times\mathbf{H})(\boldsymbol{\sigma}_2\cdot\mathbf{p})\\ &+(\boldsymbol{\sigma}_2\cdot\mathbf{r}\times\mathbf{H})(\boldsymbol{\sigma}_1\cdot\mathbf{p})]+(k\hbar/3i)[(\boldsymbol{\sigma}_1\cdot\mathbf{r})\\ &\quad\times(\boldsymbol{\sigma}_2\cdot\mathbf{r}\times\mathbf{H})V'/r - 2(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2\cdot\mathbf{H})V]\},\end{aligned} \tag{22}$$

with $K = K_1 + K_2$.

In a similar manner, the behavior of a tensor term which is introduced in the vector field may be obtained. In this case one may add to the 4-vector potential an additional vector interaction which is given by

$$\mathbf{B}_1 = k\eta_1(\gamma_1\cdot\mathbf{grad}_1)Q_1$$

where Q_1 is a 4-vector potential satisfying the equation

$$\Box Q_1 - \kappa^2 Q_1 = \eta_2(\gamma_2\cdot\mathbf{grad}_2)\mathcal{S}_2. \tag{23}$$

Again, to the approximation which is of interest, we find

$$\mathbf{B}_1{}^0 = k\beta_1\beta_2(\boldsymbol{\sigma}_1\cdot\mathrm{grad}_1)(\boldsymbol{\sigma}_2\cdot\mathrm{grad}_2)\mathcal{A}_1{}^0, \tag{24}$$

and here, also, the important feature for our purpose is the fact that the potential appears with the factor $\beta_1\beta_2$. This factor leads to changes in sign which do not arise in the ordinary vector potential terms. The procedure has again been to introduce a tensor interaction of the form

$$\begin{aligned}\mathbf{B}_1{}^0 = k\beta_1\beta_2[&(\boldsymbol{\sigma}_1\cdot\mathbf{r}_1)(\boldsymbol{\sigma}_2\cdot\mathbf{r}_2)\\ &-\tfrac{1}{3}(\mathbf{r}_1\cdot\mathbf{r}_2)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)]\mathcal{A}_1{}^0/2r_1r_2\end{aligned} \tag{25}$$

into the Hamiltonian Eq. (17). The corrections to the magnetic moment may then be computed in a straightforward manner with the result

$$\epsilon = \epsilon_v - \epsilon' + \epsilon'' \tag{26}$$

where ϵ_v is given by Eq. (18), ϵ' by Eq. (22) (note the negative sign of this term), and ϵ'' by

$$\begin{aligned}\epsilon'' = \frac{1}{4Mc^2}\frac{e}{Mc}\{&(\hbar/2i)[(\boldsymbol{\sigma}_1\cdot\mathrm{grad}_2K)(\boldsymbol{\sigma}_1\cdot\mathbf{r}\times\mathbf{H})\\ &-(\boldsymbol{\sigma}_1\cdot\mathbf{r}\times\mathbf{H})(\boldsymbol{\sigma}_1\cdot\mathrm{grad}_2K)+(\boldsymbol{\sigma}_1\cdot\mathbf{r}\times\mathbf{H})\\ &\times(\boldsymbol{\sigma}_1\cdot K\mathbf{p})-(\boldsymbol{\sigma}_1\cdot K(\boldsymbol{\sigma}_1\cdot\mathbf{r}\times\mathbf{H})\mathbf{p})\}.\end{aligned} \tag{27}$$

The correction to the moment is affected in two ways by the action of the tensor force. First there are the new terms ϵ' and ϵ'' given by Eqs. (22) and (27) which are to be added to the moments. Then there is the effect of the appreciable amount of D state contained in the wave function of the ground state. The latter effect introduces corrections to the expression (13) which has been used in computing the last term of Eq. (12). Also, cross terms between the S and D states appear for this term and all other angle dependent terms such as ϵ' and ϵ'' and the next to last term in Eq. (18).

Although the tensor force is strong ($k\approx 9/4$), the D state amplitude is small enough[1] (20 percent of the S state amplitude) for a rough estimate of the correction to be made without taking into account the change in the wave function. This requires a determination of the average values of ϵ' and ϵ'' in a ${}^3S$ state. It is easily found that

$$\langle\epsilon'\rangle \approx (e\hbar/Mc)HT/4Mc^2, \tag{28}$$

and

$$\langle\epsilon''\rangle = 0. \tag{29}$$

The final results for the corrections to the moment are then found to be for the scalar field,

$$\Delta_s\mu = 0.46\times 10^{-2}, \tag{30}$$

and for the vector field,

$$\Delta_v\mu = -0.22\times 10^{-2}. \tag{31}$$

The corresponding D state probabilities which are to be obtained from Eq. (15) are found to be $D_s = 4.8$ percent and $D_v = 3.8$ percent, respectively. These results are, of course, very rough since the cross terms between the S and D states have been neglected. They serve, however, to demonstrate that the tensor interaction decreases the difference between the vector and scalar fields.

6. CONCLUSION

Only two special types of nuclear field have been considered here. The "true" field may differ from these or it may be composed of a combination of the various possibilities.[10,11] Therefore, the wide difference between the relativity effects on the magnetic moment obtained for the two special types of field considered indicates that the D state probability in the deuteron cannot at present be accurately determined from the measured moment. It also becomes clear that the measurements of Arnold and Roberts[3] cannot be considered as evidence that a small perturbation of the intrinsic magnetic moments of the neutron and proton does not occur when the particles are bound together in the deuteron. At best, one can conclude that such changes amount to no more than one or two percent of a nuclear magneton.

These results do indicate that an accurate determination of the D state probability from other data, such as the quadrupole moment, may be used to gain information concerning the transformation properties of the nuclear fields. For this purpose it would be necessary to assume that the changes in the intrinsic moments discussed in the preceding paragraph can be neglected.

To obtain the D state probability from the quadrupole moment, more detailed information concerning the neutron-proton interaction potential (but not the transformation properties of the fields) would be required. In particular, it would be most desirable to be able to distinguish between the very short range square well potential used by Rarita and Schwinger[1] in obtaining their estimate of $D=3.9$ percent and the longer range exponential type of potential used by Bethe[2] in obtaining the estimate $D=6.7$ percent.

Even if the required information were available, there would still be serious doubts concerning the additivity of the intrinsic neutron and proton moments. It appears that present knowledge of the ground state of the deuteron is in a most unsatisfactory state.

The author had the good fortune to see, through the kind offices of Professor A. Roberts, a paper[12] on the same subject by Professor Breit before completing this manuscript.

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On the Time Required for the Fission Process*

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An experiment has been done to determine if any fissions were delayed by as much as 10^{-8} sec. Less than 5×10^{-5} of the fissions were delayed by this time.

It is of interest to detect any measurable delay between the absorption of a neutron and the occurrence of fission. Feather[1] demonstrated by an ingenious experiment that *some* fissions occurred within 5×10^{-13} sec., the time required for the compound nuclei formed by the absorption of very fast neutrons by uranium to be stopped in solid matter. His observed effect, however, was about one-third of that calculated. The experiment described here was designed to find if *any* fissions were delayed.

The experimental arrangement is shown in Figs. 1 and 2. Nearly pure U^{235} was plated uniformly, 10 $\mu g/cm^2$, as oxide on platinum strips 20 cm$\times$0.9 cm which were fastened to the side

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** Now at Cornell University.
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