THE

PHYSICAL REVIEW

A journal of experimental and theoretical physics established by E. L. Nichols in 1893

SECOND SERIES, VOL. 72, NO. 10 — NOVEMBER 15, 1947

Measurement of Transport and Inelastic Scattering Cross Sections for Fast Neutrons. I. Method*

H. H. BARSCHALL,** J. H. MANLEY, AND V. F. WEISSKOPF***
University of California, Los Alamos Scientific Laboratory, Los Alamos, New Mexico

(Received August 4, 1947)

An experimental method for measuring transport and inelastic scattering cross sections for fast neutrons is described. The cross sections are obtained from poor geometry and back scattering measurements by use of detector, the energy response of which rises above an adjustable neutron energy from zero to an almost constant value. A method of evaluation of the data is discussed.

1. INTRODUCTION

THE purpose of the work to be described in the present paper was to measure several of the quantities which would enable one to predict the behavior of fast neutrons in an extended medium, in particular, the reflection of fast neutrons from a thick layer of material. In the first part of the paper an elementary theory of the method used is given, a second part will describe the experimental results, in a third part the evaluation of the experiments will be discussed.

2. DEFINITIONS

The elementary theory to be given in the present first part of the paper will apply only to thin scatterers, while in the third part the finite thickness of the scatterers used will be taken into account. If the scatterer contains N atoms/cm^2 and if the total cross section of an atom for interaction with a fast neutron is σ, the following approximation holds for a thin scatterer: $\exp(-N\sigma)\approx 1-N\sigma$.

Since the scattering process may result in a change of energy as well as direction of the neutron, the notation $\sigma(\theta, E, E_0)$ will be used to denote the probability that a neutron of initial energy E_0 is scattered into a unit solid angle with a deviation θ from its original direction and into a unit energy range at an energy E. In terms of a current $I_0(E_0)$ incident on N scattering centers per square centimeter and a scattered current per unit solid angle and per unit energy range $I(\theta, E, E_0)$

$$\sigma(\theta, E, E_0)=I(\theta, E, E_0)/NI_0(E_0).$$

The total cross section for elastic scattering is defined by

$$\sigma_e(E_0)=\int_0^{\pi}\int_{E_0'}^{E_0}\sigma(\theta, E, E_0)dEd\omega,$$

where

$$d\omega=2\pi\sin\theta d\theta$$

and E_0' is the maximum energy loss permitted

* This paper is based on work performed at Los Alamos Scientific Laboratory of the University of California under Contract No. W-7405-eng-36 for the Manhattan Project and the information contained therein will appear in Division V of the Manhattan Project Technical Series as part of the contribution of the Los Alamos Laboratory.
** Now at the University of Wisconsin.
*** Now at the Massachusetts Institute of Technology.

without excitation of the scattering nucleus. It is to be noted that E and θ are not independent.

The cross section for inelastic scattering is defined for our purpose as

$$\sigma_i(B, E_0) = \int_0^\pi \int_0^B \sigma(\theta, E, E_0) dE d\omega$$

which states that the scattered neutron has an energy less than B. The expression "total inelastic cross section" will refer to all energies of the scattered neutrons as well as all angles:

$$\sigma_i(E_0) = \int_0^\pi \int_0^{E_0'} \sigma(\theta, E, E_0) dE d\omega.$$

The expression "total cross section" will refer to $\sigma(E_0) = \sigma_e(E_0) + \sigma_i(E_0)$.

In order to describe the behavior of neutrons in an extended medium, it is convenient to use the transport cross section. The transport cross section differs from the total cross section in that the contributions of neutrons scattered through various angles θ are weighted according to a factor $(1-\cos\theta)$. The transport cross section σ_t is defined as

$$\sigma_t(E_0) = \int_0^\pi [\sigma_e(\theta, E_0) + \sigma_i(\theta, E_0)](1-\cos\theta) d\omega.$$

In the present work measurements of the transport cross section and of the cross section for inelastic scattering were attempted. An exact determination of these quantities would require a knowledge of the distribution in energy and angle of the scattered neutrons as a function of primary neutron energy. Such detailed measurements were not made, but the use of four primary neutron energies and three angular intervals permits deduction of approximate values of transport and inelastic cross sections.

3. PREVIOUS WORK

Many investigations of the inelastic scattering of fast neutrons and of the angular distribution of scattered neutrons have been carried out in the past.

Measurements of the inelastic scattering of fast neutrons have been performed using radioactive threshold detectors.[1,2] This method has the disadvantage that not enough radioactive detectors with different thresholds particularly in the region up to 3 Mev are available. The observation of gamma-rays produced offers another way of obtaining information on inelastic scattering.[1,3] It is difficult to obtain in this manner the distribution in energy of the inelastically scattered neutrons or reliable absolute values of the scattering cross section. The most complete information on the distribution in energy of inelastically scattered neutrons can be obtained by observing hydrogen recoils in a cloud chamber[4,5] or in a photographic plate. The only drawback of this method is the large amount of work required in evaluating the results.

The most complete study of the angular distribution of scattered fast neutrons was carried out by Kikuchi, Aoki, and Wakatuki[6] who found a strong forward maximum for the scattering of 3-Mev neutrons by heavy elements. Their measurements, however, do not enable one to distinguish between elastically and inelastically scattered neutrons.

Qualitative information on inelastic scattering has been obtained by measuring the distribution in energy of recoiling alpha-particles in an ionization chamber.[7] The method described in the present paper uses a similar technique. Instead of helium, hydrogen or deuterium serves as the detector gas in order to avoid the effects of the resonance in helium around 1 Mev[8] which affects both the scattering cross section and the angular distribution of the recoils.[9] The method used in the present work enables one to obtain more quantitative information about the inelastic scattering than the previous work.

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4. DETECTOR

To determine the distribution in energy of the scattered neutrons a threshold detector was used. The ideal threshold detector would count all the neutrons above a given adjustable energy with the same efficiency. No such detector is available, but a gas recoil detector may be used to approximate this condition in the following way: an ionization chamber or proportional counter is filled with a light gas (hydrogen or deuterium) and the stopping power of the gas is chosen so that the range of the recoiling gas nuclei is small compared to the dimensions of the counter. The recoil pulses are amplified by means of a linear pulse amplifier and all the pulses above a given size are counted. The energy corresponding to a minimum pulse height which is counted will be referred to as the bias energy B. If the scattering of the neutrons by the detecting gas is isotropic in the center of mass system the sensitivity of the counter will be proportional to $\sigma(E)[1-(B/E)]$, where $\sigma(E)$ is the total scattering cross section of the detector gas.[10] For the case in which $\sigma(E)$ is proportional to $E^{-\frac{1}{2}}$, as is approximately true for hydrogen over a wide range of energies, the theoretical energy sensitivity of the detector is shown in Fig. 1. As may be seen from Fig. 1, the energy above which neutrons are counted is not well defined, a fact which will lead to considerable uncertainty in the meaning of $\sigma_i(B, E_0)$.

5. GEOMETRY

Two kinds of measurements were carried out, poor geometry and back scattering experiments. The experiments using poor geometry employed the arrangement shown in Fig. 2. A circular disk scatterer of radius a was placed half-way between the source and the detector. As was first pointed out by R. F. Christy, if the neutron source is isotropic, there is a compensation between scattering into the detector from some disk elements and scattering out of the neutron beam by others such that a decrease corresponding to the scattering of all neutrons from θ_m to π is observed, where θ_m represents the maximum angle through which a neutron can be scattered by the disk (edge) and reach the detector.

Let Q be the number of neutrons emitted by the source per unit solid angle and per second and consider single elastic collisions. The current scattered into a detector of unit area is

$$I_1=(QN/4D^2)\int_0^{\theta_m}\sigma(\theta)d\omega$$

with $\theta_m=2\tan^{-1}(a/D)$. The current received by a detector of unit area subtending a solid angle of $1/4D^2$ at the source is reduced by the interposition of the disk. This reduction arises from the scattering out of the beam by the portion of the disk within the solid angle, so that the current at the detector from the reduced direct beam is

$$I_2=(Q/4D^2)\left[1-N\int_0^{\pi}\sigma(\theta)d\omega\right]$$

and the net current

$$I=I_1+I_2=(Q/4D^2)\left[1-N\int_0^{\pi}\sigma(\theta)d\omega\right].$$

$Q/4D^2=I_0$ is the current at the detector without the scatterer. Hence the above relation may be written, for a disk thin compared to the scattering mean-free path, in the exponential form as follows

$$I/I_0=\exp\left(-N\int_{\theta_m}^{\pi}\sigma(\theta)d\omega\right).$$

This result will be called the Christy theorem.

It is possible to vary θ_m by varying D and in this way to measure the angular distribution of the scattered neutrons. Experiments were carried out for values of θ_m of 30°, 60°, and 90°. The accuracy of this method for measuring angular

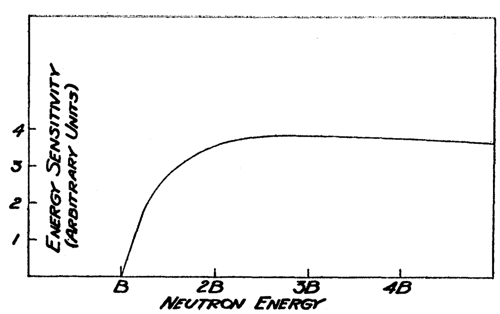

FIG. 1. Theoretical sensitivity of hydrogen detector as a function of neutron energy.

[10] H. H. Barschall and H. A. Bethe, Rev. Sci. Inst. 18, 147 (1947).

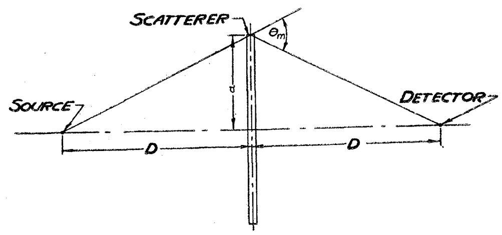

FIG. 2. Scattering geometry for poor geometry experiments.

distributions is affected by the need to take differences. In computing the transport cross section the errors will cancel, however, to a certain extent, since the differences are added again after they have been multiplied by the appropriate weighting factor.

For $\theta_m > 90°$ the method becomes very inaccurate because the transmission is very close to unity. Therefore the scattering into the back hemisphere was investigated by observing the reflected neutrons. As the number of reflected neutrons is small compared to the number of neutrons coming directly from the source, it is necessary either to shield the detector from the direct neutrons by means of a shadow cone or to use a directional detector. The shadow cone defines an annular ring on the disk scatterer from which neutrons may be reflected so that these measurements are equivalent to measurements using ring scatterers. That this is so was checked by ring-disk comparison and by moving the detector off the scattering axis to explore the extent of the shadow.

6. SOURCE

Experiments were carried out for neutrons of 0.2-, 0.6-, and 1.5-Mev energy using the Li(p, n) reaction and at 3 Mev using the d–d reaction. The assumption of an isotropic neutron source made in deriving the Christy theorem is valid neither for the Li(p, n) nor for the d–d reaction. Both sources deviate from spherical symmetry both in intensity and in energy. The effect of this asymmetry can be made to cancel by an appropriate choice of the direction of the scattering axis (line through target, center of disk and center of detector) with respect to the bombarding ions. In Fig. 3 the axis of symmetry S–D makes an angle α_0 with respect to the incident ions. The angle φ is the azimuth of an element of the scatterer with respect to the plane defined by the ion beam and the axis S–D, and α is the angle between the direction of the proton or deuteron beam and that of a neutron emitted from the target. The Christy theorem is valid if the neutron intensity averaged over φ is independent of θ.

For scatterers consisting of heavy nuclei the energy of the elastically scattered neutrons is, within the accuracy of the measurements, the same as that of the primaries. The sensitivity of the detector for elastically scattered neutrons as a function of φ and θ can therefore be measured directly by moving the detector around the source. Let $R(\alpha, B)$ be the response of the detector for a bias energy B at a constant distance from the source. The variation of $R(\alpha, B)$ is due to the dependence of both intensity and energy on α, and, therefore, depends on the bias B. The response of the detector as a function of θ and φ, $R(\theta, \varphi, B)$, may be obtained from $R(\alpha, B)$ by using the geometrical relationship between the angles:

$$\cos\alpha = \cos\alpha_0 \cos \tfrac{1}{2}\theta + \sin\alpha_0 \sin \tfrac{1}{2}\theta \cos\varphi.$$

The Christy theorem holds, provided

$$\bar{R}(\theta, B) \equiv (1/2\pi) \int_0^{2\pi} R(\theta, \varphi, B) d\varphi$$

is independent of θ. If $R(\alpha)$ can be expressed as $R(\alpha) = c_1 + c_2 \cos^2\alpha$, the integral is independent of θ for $\tan\alpha_0 = \sqrt{2}$, and if $R(\alpha)$ is a linear function of $\cos\alpha$, the integral is independent of θ for $\alpha_0 = 90°$.

The anisotropy of the Li(p, n) reaction for 0.2-Mev neutrons is so great that it was not possible to find any angle α_0 for which $\bar{R}(\theta)$ was not a rapidly varying function. Therefore no experiments for which the Christy theorem had to be used were carried out at that energy.

For 0.6-Mev neutrons from the Li(p, n) reaction $\bar{R}(\theta)$ was found from the measured functions $R(\alpha, B)$ to be constant within 5 percent at $\alpha_0 = 60°$, so that $\alpha_0 = 60°$ was chosen for the experiments.

For 1.5-Mev neutrons from the Li(p, n) reaction no angle α_0 was found for which $\bar{R}(\theta)$ was independent of θ. For $\alpha_0 = 40°$ the variation of $\bar{R}(\theta)$ with θ was smallest, but even for this angle corrections had to be applied.

To denote cross sections measured in various geometries the angle θ_m for the geometry used will be written as a subscript, i.e., σ_{60} denotes the cross section measured in the geometry for which $\theta_m=60°$, and is computed from the relationship

$$I_{60}/I_0=\exp(-N\sigma_{60})=\exp\left[-N\int_{60°}^{180°}\sigma(\theta)d\omega\right],$$

where I_{60} and I_0 denote the neutron intensities measured at the detector with and without the scatterer in the 60° geometry. For σ_{30} the correction for asymmetry of the source is negligible. In order to correct the measured values of σ_{60} and σ_{90}, these cross sections should be replaced by corrected values σ_{60}^* and σ_{90}^* according to

$$\sigma_{60}^*=\sigma_{30}+\beta(\sigma_{60}-\sigma_{30}),$$

$$\sigma_{90}^*=\sigma_{60}^*+\gamma(\sigma_{90}-\sigma_{60}),$$

where

$$\beta=\left[\int_{30°}^{60°}\bar{R}(\theta)\sin\theta d\theta\right]\Big/\left[\int_{30°}^{60°}R(\alpha_0)\sin\theta d\theta\right],$$

$$\gamma=\left[\int_{60°}^{90°}\bar{R}(\theta)\sin\theta d\theta\right]\Big/\left[\int_{60°}^{90°}R(\alpha_0)\sin\theta d\theta\right].$$

For the 3-Mev neutrons from the d–d reaction $\alpha_0=60°$ was chosen for most of the experiments. This choice was based on the fact that the angular distribution of the $D(d,p)H^3$ reaction may be expressed as $c_1+c_2\cos^2\alpha$. This would suggest $\alpha_0=55°$. Since the dependence of the detector on energy will probably introduce a term in $\cos\alpha$, the somewhat larger value of 60° was used.

The correction for the scattering by light elements is more involved. Even if the source were spherically symmetric, the response of the counter for elastic scattering would vary with the scattering angle because of the recoil energy loss. In order to compute this correction the energy and the intensity distribution of the source must be known. The energy of the emitted neutrons as a function of α, $E(\alpha)$, may be computed using the laws of conservation of energy and momentum. The distribution in angle of the neutrons from the source $Q(\alpha)$, may be determined from the measured function $R(\alpha)$ and the measured energy sensitivity of the detector using $E(\alpha)$.

The coefficients β and γ for light elements are

$$\beta=\left[\int_{30°}^{60°}\bar{Q}(\theta)\bar{S}(\theta)\sin\theta d\theta\right]\Big/\left[\int_{30°}^{60°}Q(\alpha_0)S(0)\sin\theta d\theta\right],$$

$$\gamma=\left[\int_{60°}^{90°}\bar{Q}(\theta)\bar{S}(\theta)\sin\theta d\theta\right]\Big/\left[\int_{60°}^{90°}Q(\alpha_0)S(0)\sin\theta d\theta\right],$$

where

$$\bar{Q}(\theta)=(1/2\pi)\int_0^{2\pi}Q(\alpha)d\varphi,$$

and $S(\theta,\varphi)$ is the response of the detector to neutrons emitted in the direction $(\theta/2,\varphi)$ and elastically scattered into the detector with an energy loss depending on θ, $S(0)=R(\alpha_0)$ is the response of the detector to neutrons which are emitted in the direction α_0 and have not lost energy by recoil.

7. EVALUATION

In the evaluation of the measurements the following assumptions or approximations were made:

(a) $\sigma_i(\theta)$ is independent of θ. This assumption appears reasonable from theoretical considerations. There is some experimental evidence that the inelastic scattering is isotropic (cf. reference 7). The assumption did not lead to any contradictions in the evaluation of the data.

(b) Neutrons are considered as scattered elastically if they are detected at the highest bias used in the experiments. Since some neutrons may lose little enough energy in inelastic collisions to be counted at the highest bias some inelastic scattering may be included in the cross section for elastic scattering.

The cross section $\sigma_e(\theta)$ is approximated by three average values in the following three

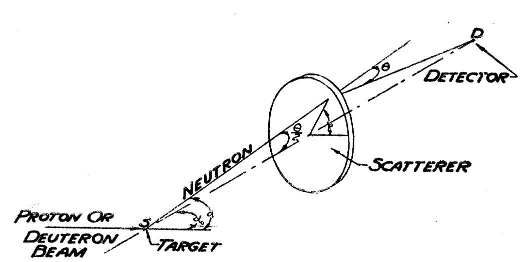

FIG. 3. Geometry used in scattering experiments.

angular intervals: 30° to 60°, 60° to 90°, 90° to 180°. The value for the interval 90° to 180° was taken as that measured in a back scattering geometry corresponding to an average angle of 135°. There is some experimental evidence, however, that the elastic back scattering may not be isotropic. The assumption that the scattering cross section for 135° is characteristic for the whole back hemisphere may, therefore, be in error. The contribution to the transport cross section of $\sigma_e(\theta)$ for $0° \leqslant \theta \leqslant 30°$ was neglected. This does not introduce any appreciable error, even if there is considerable forward scattering, since

$$\tfrac{1}{2}\int_{0°}^{30°}(1-\cos\theta)\,\sin\theta d\theta = 0.0045$$

or less than $\frac{1}{2}$ percent of the integral over all values of θ.

In the evaluation of the data the difference between the cross sections for two values of θ_m is first corrected for the asymmetry of the source and the energy loss in the case of light elements. From the corrected values of the differences, corrected values of σ_{60} and σ_{90} are determined which are used in the subsequent calculations.

The next step in the evaluation is the calculation of the cross section for inelastic scattering. This may be obtained in several ways: from the transmission data alone, from the back scattering, and from a comparison of transmission and back scattering.

From the measured cross sections for back scattering at various biases, σ_{BS}, the cross sections for inelastic scattering are obtained as first differences. Although the measurements of back scattering included only a specific angle in the back hemisphere, σ_{BS} is computed to refer to a 4π solid angle.

A comparison between the cross section for $\theta_m = 90°$ and the cross section for back scattering may be used to obtain values for the cross section for inelastic scattering for the following reasons. The value σ_{90} measured at a bias B is the sum of the cross section for scattering of all neutrons into the back hemisphere and the cross section for neutrons scattered into the front hemisphere with an energy such that they are not detected at the bias B. The cross section for the scattering of neutrons into the back hemisphere with an energy above the bias B is $\frac{1}{2}\sigma_{BS}$. Therefore $\sigma_{90}-\frac{1}{2}\sigma_{BS}$ is the cross section for the scattering of neutrons below the bias B, and allows one to obtain fairly reliable values for the inelastic scattering.

The average of the inelastic scattering obtained from σ_{BS} and from $\sigma_{90}-\frac{1}{2}\sigma_{BS}$ was taken as the most reliable determination of σ_i. The transmission data were then adjusted to give the bias effect corresponding to the previously calculated inelastic scattering. The adjustments necessary were in almost all cases within the experimental error of the measurements.

The value of $\sigma_{90}-\frac{1}{2}\sigma_{BS}$ for the lowest bias used yields the sum of the cross sections for inelastic scattering below the lowest bias and for absorption.

The transport cross section was obtained by adding for a given bias the contributions of the three angular intervals, weighted according to $(1-\cos\theta)$, and the cross section for inelastic scattering below the same bias. The transport cross section, according to its definition, is independent of the bias.

This method of evaluation assumes that a neutron is scattered only once in the scatterer. A method of evaluation taking into account multiple scattering will be given in the third part of this paper.