Another run was made in position B, where the energy was 88 Mev. This provides the points in Fig. 2 from 88 to 65 Mev. Finally, a run in position A was made, using $2'' \times 3'' \times \frac{1}{4}''$ carbon plates; here the neutron background was relatively large (about 40 percent) because the number of protons was small, but the accuracy was sufficient to show that the cross section does not vary appreciably in going from 88 up to 140 Mev. This last run was not included in the plot.

The striking feature of this result is that there is little if any variation in the $C^{12}(p, pn)C^{11}$ cross section between 60 and 140 Mev. This is not consistent with the picture of proton capture followed by pn evaporation, because the competition of other reactions at high excitation energies would reduce the cross section rapidly. One must assume a non-capture excitation or a (p, n) exchange with roughly constant energy transfer, followed by evaporation of a single neutron or proton. Dr. Serber has pointed out[4] that this is a reasonable assumption in this energy range.

The activity in the boric acid is itself of some interest. The (p, n) reaction in B^{11} gives a peak near the end of the range, as expected for that type of reaction; the second rise at higher energy is attributed to reactions forming C^{11} from oxygen, since its magnitude is consistent with the activity observed in BeO traversed by the same proton current.

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[1] A. C. Helmholz, E. M. McMillan, and D. Sewell, Phys. Rev., to be published.
[2] R. Serber, Phys. Rev., to be published.
[3] W. W. Chupp, E. Gardner, and T. B. Taylor, to be published.
[4] R. Serber, to be published.

On Quantized Space-Time

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RECENTLY Snyder[1] has developed a theory of quantized space-time that is invariant under Lorentz transformations. The theory is, however, not invariant under translations. Indeed, as he has pointed out, to make the theory invariant under translations one must have space-time coordinates forming a continuum.

It does seem desirable, however, to have an invariant theory under a wider group of transformations than the Lorentz transformations. (E.g., we may want to have homogeneity as well as isotropy in space-time.) This can be accomplished by proposing that space-time is curved.

As an example let us consider a de Sitter universe

$$-x_0^2+x_1^2+x_2^2+x_3^2+\xi^2=R^2, \tag{1}$$

which is a pseudosphere in a five-dimensional flat space: x_0, x_1, x_2, x_3, ξ. The group of linear transformations that leaves the quadratic form $-x_0^2+x_1^2+x_2^2+x_3^2+\xi^2$ unchanged could be approximated by the product of the ordinary Lorentz group and the group of translations, for points on the pseudosphere in the region $|x_0| \ll R$, $|x_i| \ll R$. If we define $L_i/\hbar$, $M_i/\hbar$, and $Rp_0/\hbar$ and $Rp_i/\hbar$ as the nuclei of these transformations, we would get the angular- and linear-momenta operators in the de Sitter universe. The p's are given by

$$\begin{aligned} p_0 &= i\hbar\frac{\xi}{R}\frac{\partial}{\partial x_0}+i\hbar\frac{x_0}{R}\frac{\partial}{\partial \xi}, \\ p_i &= -i\hbar\frac{\xi}{R}\frac{\partial}{\partial x_i}+i\hbar\frac{x_i}{R}\frac{\partial}{\partial \xi}. \end{aligned} \tag{2}$$

They satisfy

$$L_i=\xi(x_jp_k-x_kp_j)/R. \tag{3}$$

The commutators involving the p's are

$$[p_i, p_j]=i\hbar L_k/R^2,$$

and

$$[x_i, p_j]=i\hbar\xi\delta_{ij}/R. \tag{4}$$

Those involving the L_i, M_i, and x_μ's are as usual.

The above is, however, not the only theory that is invariant under the Lorentz transformations in five dimensions. We shall propose, in general, that x_0, x_1, x_2, x_3, ξ be Hermitian operators, and that after a Lorentz transformation in five dimensions they can be brought back to their original forms by a unitary transformation. This means that the nuclei of the unitary transformations, which we define to be $L_i/\hbar$, $M_i/\hbar$, and $Rp_0/\hbar$, and $Rp_i/\hbar$, still satisfy the same commutation relations (4). The commutation relations between the operators x_0, x_1, x_2, x_3, ξ are not fixed by the requirement of invariance. If we propose that they are given as follows,

$$\begin{aligned} [x_i, x_j]&=ia^2L_k/\hbar, \quad [x_0, x_i]=ia^2M_i/\hbar, \\ [\xi, x_i]&=ia^2Rp_i/\hbar, \quad [\xi, x_0]=ia^2Rp_0/\hbar, \end{aligned} \tag{5}$$

it is evident that the 15 operators $L_i/\hbar$, $M_i/\hbar$, $Rp_\mu/\hbar$, x_μ/a and ξ/a satisfy the same commutation relations as the nuclei of the group of Lorentz transformations in six dimensions with the basic quadratic form $-\eta_0^2+\eta_1^2+\eta_2^2+\eta_3^2+\eta^2+\zeta^2$. So a possible solution is

$$\begin{aligned} L_i &= i\hbar\left(\eta_k\frac{\partial}{\partial\eta_j}-\eta_j\frac{\partial}{\partial\eta_k}\right), \\ M_i &= i\hbar\left(\eta_0\frac{\partial}{\partial\eta_i}+\eta_i\frac{\partial}{\partial\eta_0}\right), \\ p_0 &= \frac{i\hbar}{R}\left(\eta\frac{\partial}{\partial\eta_0}+\eta_0\frac{\partial}{\partial\eta}\right), \quad p_i=\frac{i\hbar}{R}\left(\eta\frac{\partial}{\partial\eta_i}-\eta_i\frac{\partial}{\partial\eta}\right), \\ x_0 &= ia\left(\zeta\frac{\partial}{\partial\eta_0}+\eta_0\frac{\partial}{\partial\zeta}\right), \quad x_i=ia\left(\zeta\frac{\partial}{\partial\eta_i}-\eta_i\frac{\partial}{\partial\zeta}\right), \end{aligned} \tag{6}$$

and

$$\xi=ia\left(-\zeta\frac{\partial}{\partial\eta}+\eta\frac{\partial}{\partial\zeta}\right).$$

For this solution the eigenvalues of the space coordinates are discrete.

Returning to the general case, we should add Eq. (1) to the general Eqs. (4) and (5).

If we put $R=\infty$ we would get the special solution (2).

[1] H. Snyder, Phys. Rev. 71, 38 (1947).