PHYSICAL REVIEW VOLUME 72, NUMBER 1 JULY 1, 1947

Absolute Sensitivity of a Graphite Ionization Chamber

MELVIN LAX
*Massachusetts Institute of Technology, Cambridge, Massachusetts**
(Received February 10, 1947)

A thick-walled graphite ionization chamber is suggested as a simple instrument for measuring the total intensity in the x-ray beam emitted by the betraton. The chamber would consist of two layers of carbon, each about 1/5 of a radiation length, or 4.5 cm thick, separated by a thin gas layer whose width is not critical. The transverse dimensions of the carbon layers should be sufficiently large to cover completely the x-ray beam as well as to catch the electrons produced at an angle in the carbon. The absolute sensitivity of the ionization chamber is computed to an accuracy of about 5 percent by comparison with the ionization which would be produced in an infinitely thick carbon block.

1. INTRODUCTION

THE determination of cross sections for nuclear reactions induced by a high energy x-ray beam requires a knowledge of the total power in the beam and of the spectral distribution of this power.

Ionization chambers designed for measurements at several Mev have their sensitivities computed according to the assumption that the electrons produced by the x-ray beam have a maximum range less than the thickness of the ionization chamber walls.[1] If the walls are kept less than one radiation length[1a] in thickness, this assumption certainly fails for the x-ray beam emitted by the 100-Mev betatron.

A chamber consisting of two thick graphite layers separated by a thin gas layer was suggested by Bethe and Feynman, and preliminary calculations were made by Feynman for a chamber whose walls are $\frac{1}{5}$ of a radiation length, or 4.5 cm thick. Such walls will stop 25-Mev electrons. Below 25 Mev, the chamber will therefore behave as an ordinary ionization chamber. Above 25 Mev, there will be a reduction in sensitivity because the wall thickness will be less than the range of some of the electrons produced.

The purpose of this paper is to compute the sensitivity of such an ionization chamber taking into account: (1) Finite thickness of the graphite walls; (2) attenuation of the x-ray beam in the walls; (3) non-linearity in the range-energy relationship; (4) multiple scattering; (5) radiation.

The sensitivity of the chamber is determined predominantly by factors (1) and (2). Fortunately, the effects of these factors can be taken into account precisely. The range-energy nonlinearity is never more than 20 percent for electrons from 0 to 100 Mev, and the effect of this non-linearity on the final sensitivity is less than 10 percent. The non-linearity effect is corrected by using the range-energy relationship for electrons given by Rossi and Greisen.[2] Multiple scattering and radiation are hard to compute precisely, but their effects are kept to less than 5 percent by use of a light element, such as carbon, for the walls, and by an appropriate choice of wall thickness.

2. GENERAL DESIGN CONSIDERATIONS

The basic principle of the design is to reproduce as nearly as possible the ideal situation of an infinite carbon block in which electrons are produced uniformly throughout. In this case, the ionization energy produced per unit volume is equal to the energy given to electrons per unit volume by the x-ray beam. This will be true even if electron scattering takes place.

Unfortunately, uniform electron production is limited by two factors: (1) attenuation of the x-ray beam in the carbon, and (2), production of showers for thicknesses of a radiation length or

* This work was done while the author was employed at the Research Laboratory, General Electric Company, during the summer of 1946.

[1] G. C. Laurence, Can. J. Research **A15**, 16 (1937).

[1a] The term *radiation length* may be defined roughly as the distance in which the energy of a fast particle is reduced to $1/e$ of its original value by radiative processes. A more precise definition is given in B. Rossi and K. Greisen, Rev. Mod. Phys. **13**, 240 (1941).

[2] B. Rossi and K. Greisen, Rev. Mod. Phys. **13**, 240 (1941); see p. 247, Eq. (1.12).

more. Each of these factors requires that the thickness of the carbon block be small compared with a radiation length. It is this requirement which prevents the use of carbon blocks thick enough to stop the highest energy electrons produced by the x-ray beam.

The effect of using a carbon block of finite thickness T will now be considered. Electrons whose range R is less than T must come from a layer of thickness R adjacent to the gas in order to produce ionization in the gas. This is the same number of electrons as would be counted if the block were infinite in thickness.

On the other hand, if R is greater than T, the electrons come from a layer of thickness T instead of R, and the number of electrons crossing the gas layer is reduced in the ratio T/R. Assuming a linear range-energy relationship, $E=\epsilon R$, this ratio also represents the decrease in ionization in the gas compared with the case of the infinite block. Thus the efficiency for electrons, compared with an infinite medium can be represented by

$$\begin{aligned} \eta &= 1 && E<E_0, \\ \eta &= E_0/E=T/R && E>E_0, \end{aligned} \tag{1}$$

where $E_0=\epsilon T$ is the energy lost by an electron on traversing thickness T.

The electron efficiency η can be thought of as the fraction of the electron energy created per unit volume which is converted to ionization per unit volume. For energies less than E_0, this efficiency is unity because no electrons escape; those electrons which leave a given unit volume are replaced by others which enter it, so that all energy eventually goes into ionization. This result is true even if scattering takes place.

For energies greater than E_0, however, the amount of ionization produced in the gas layer depends on the angle at which the electrons cross it, and this is not quite compensated for by the change in thickness from which the electrons come. For these energies ($E>E_0$), therefore, it is necessary to keep the scattering small.

To keep the error less than 5 percent we should set[3]

$$\langle\theta^2\rangle_{\text{Av}}=(21/E_0)^2(T/2)<0.1, \tag{2}$$

where T is the thickness in radiation lengths, and $E_0=\epsilon T$ is the energy in Mev required to cross the block. The value $T/2$ is used, since electrons are produced, on the average, in the middle of the block. Using $E_0=\epsilon T$ we obtain

$$\epsilon^2 T>2000. \tag{3}$$

The largest collision losses per radiation length, ϵ, occur for the lightest elements (ϵ varies as $1/Z$). If carbon is used, $\epsilon=120$ Mev/(radiation length), and (3) gives $T>0.14$ radiation length.

We have already shown, however, that to keep attenuation and shower effects small, T must be small compared with a radiation length. This pretty well fixes the choice of T to be about $\frac{1}{5}$ of a radiation length, or about 4.5 cm of carbon.

The second carbon block serves the purpose of compensating for back scattering in the first block. It should be the same thickness as the first block, although this dimension is not critical.

3. SENSITIVITY CALCULATION

If one x-ray of energy W is absorbed, producing an electron of energy E, the fraction of the absorbed energy ending up as ionization is given by $(E/W)\eta(E)$, since E/W is the fraction of x-ray energy given to the electron and $\eta(E)$ is the fraction of electron energy ending up as ionization. In general, the x-ray of energy W will produce a distribution of electron energies $\sigma(W, E)dE$, so that the average fraction of absorbed x-ray energy which ends up as ionization is given by

$$S(W)=\frac{\int (E/W)\eta(E)\sigma(W,E)dE}{\int \sigma(W,E)dE}. \tag{4}$$

If $Wf(W)dW$ is the incident x-ray energy, and $1/G(W)$ is the absorption coefficient per radiation length in carbon, the absorbed energy in a thickness Δt radiation lengths is given by

$$(\Delta t/G(W))\,Wf(W)dW,$$

and the ionization energy produced in this layer is given by

$$I=\Delta t\int Wf(W)S(W)/G(W)dW, \tag{5}$$

$$I=\Delta t\langle S/G\rangle_{\text{Av}}\int Wf(W)dW, \tag{6}$$

[3] Reference 2, p. 263.

where a comparison of (6) and (5) defines the average sensitivity $\langle S/G\rangle_{\text{Av}}$ for a given incident spectrum.

Formulas (5) and (6) require a slight correction for the fact that the gas layer is not also made of carbon. This correction has been shown by L. H. Gray[4] to be simply the ratio of stopping powers in the gas to that in the carbon, this ratio being roughly independent of electron energy if the gas is also a light element. In our units, this correction factor is given by R_c/R_g, where R_c and R_g are the electron ranges in carbon, and in the gas, both measured in radiation lengths.

According to Eq. (4), $S(W)$ can also be written in the form

$$S(W)=P_cS_c(W)+P_pS_p(W), \tag{7}$$

where P_p is the relative probability for pair production in carbon (see Figs. 1a, and 1b), $P_c=1-P_p$, and S_c and S_p are the sensitivities which would prevail if only the Compton effect, or only pair production took place. Thus S_c is computed using Eq. (4) with σ replaced by the cross section for the Compton effect, and S_p is computed in a similar manner, except for a factor 2 because two electrons are produced in each pair.

The decomposition of $S(W)$ by means of Eq. (7) is useful because S_c and S_p are independent of the material chosen for the chamber walls, whereas P_c and P_p are not.

The sensitivities S_p and S_c were computed using $\eta(E)=1$ for $E<25$, $\eta(E)=25/E$ for $E>25$. Furthermore, the cross section for pair production was assumed to be independent of electron energy since it is very close to constant for electron energies from $E=0$ to $E=W-2\text{mc}^2=(W-1)$ Mev. Using Eq. (4) this gives

$$S_p(W)=(W-1)/W, \qquad W<26 \text{ Mev}, \tag{8a}$$

$$S_p(W)=50(W-13.5)/W(W-1), \qquad W>26 \text{ Mev}. \tag{8b}$$

It should be noted that Eq. (8a) is independent of the assumption of a constant cross section, and merely represents the fact that a gamma-ray of energy W gives energy $(W-1)$ to the electron and positron pair. All of this energy is converted to ionization if $W<26$ Mev, and only part of it if $W>26$ Mev.

[4] L. H. Gray, Proc. Roy. Soc. **A156**, 578 (1936).

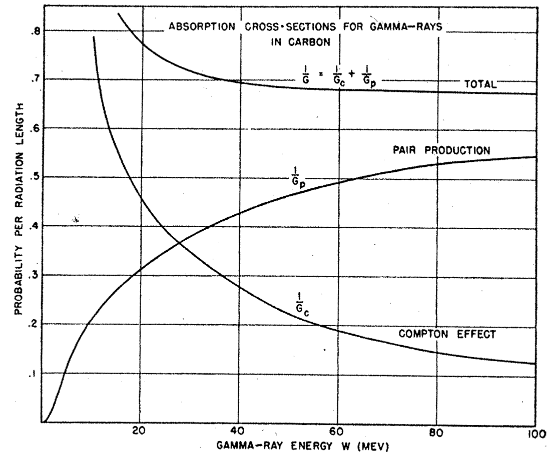

FIG. 1a. Absorption cross sections for gamma-rays in carbon.

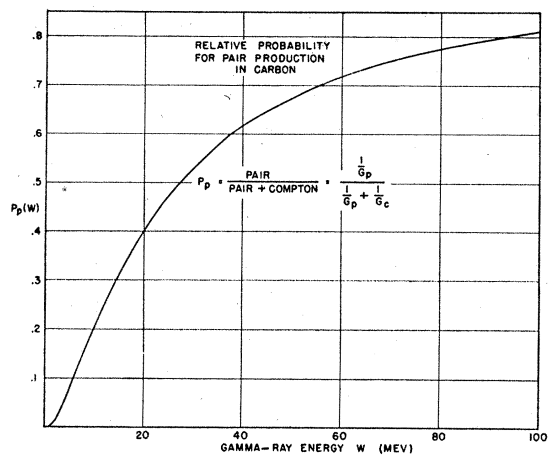

FIG. 1b. Relative probability for pair production in carbon.

For the Compton effect the cross section used was the usual one[5]

$$\sigma_c(W, E)dE=k(x)dx \\ =(c/\gamma)[1+x^2-x\sin^2\theta]dx/x, \tag{9}$$

where $x=W'/W$, $W'=W-E$ is the energy carried off by the recoil x-ray, and $\gamma=W/\text{mc}^2\simeq 2W$. The Compton recoil angle and energy are related by

$$\sin^2\theta=\frac{2(1-x)}{\gamma x}-\frac{(1-x)^2}{\gamma^2x^2}, \tag{10}$$

and x takes all values from $1/(2\gamma+1)$ to 1. The

[5] Reference 2, p. 251.

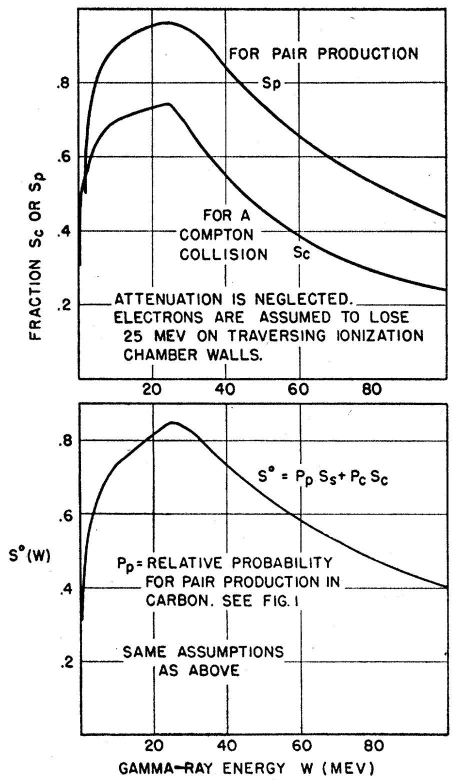

FIG. 2. Above: Fraction of gamma-ray energy which goes into ionization after one collision. Below: Average fraction of gamma-ray energy which goes into ionization after one collision in carbon. (The equation between the abscissae .6 and .8 should read: $S^0=P_pS_p+P_cS_c$.)

average fraction of the energy given to the recoils

$$\bar{x}(W)=\int xk(x)dx \Big/ \int k(x)dx \qquad (11)$$

varies from 0.20 at 100 Mev, to 0.30 at 10 Mev, to 0.55 at 1 Mev.

Using the Compton cross section (9) in (4) gives

$$S_c(W)=1-\bar{x}(W), \qquad W<25, \qquad (12a)$$

$$S_c(W)=\frac{25}{W}-\frac{8-9x_0+x_0^3+6x_0\ln x_0}{3+2\ln(4W+1)}, \qquad W>25, \qquad (12b)$$

where $x_0=1-25/W$.

$S_c(W)$ and $S_p(W)$ are shown in Fig. 2, together with the values of $S^0=P_cS_c+P_pS_p$ for carbon. The superscript ^{0} is used, because the sensitivity just obtained is uncorrected for attenuation, scattering, and radiation. In the next section, we shall find, that to a good approximation, including all corrections,

$$S(W)=e^{-T/G}S^0(W). \qquad (13)$$

Referring to Eq. (5), we see that the proper measure of the relative sensitivity of the chamber as a function of x-ray energy is $S(W)/G(W)$. This is plotted in Fig. 3. An examination of the curve shows that it can be fitted closely by

$$\begin{aligned} S/G&=0.84e^{-0.017W}, \quad W<25,\\ &=0.71e^{-0.011W}, \quad W>25. \end{aligned} \qquad (14)$$

Although this sensitivity varies by a factor of about 3 from the low energies to 100 Mev, the average sensitivity $\langle S/G\rangle_{Av}$ is not influenced much by the exact shape of the assumed spectrum $Wf(W)dW$. This is true for any smooth spectrum such as the betatron spectrum. For example, if we assume $Wf(W)=1$ from 0 Mev to 100 Mev, the resulting sensitivity $\langle S/G\rangle_{Av}$ is 0.45. On the other hand, the markedly different assumption $Wf(W)=1-0.01W$ gives $\langle S/G\rangle_{Av}=0.54$.

For simplicity, the sensitivity $\langle S/G\rangle_{Av}$ is computed for a flat spectrum up to any betatron energy U. The sensitivity thus obtained varies smoothly from 0.45 at 100 Mev, to 0.61 for betatron operation at 20 Mev (see Fig. 4). It should be noted that for betatron operation at energies less than 100 Mev, $\langle S/G\rangle_{Av}$ is even less sensitive to assumptions about the betatron spectrum since $S(W)/G(W)$ varies over a smaller range.

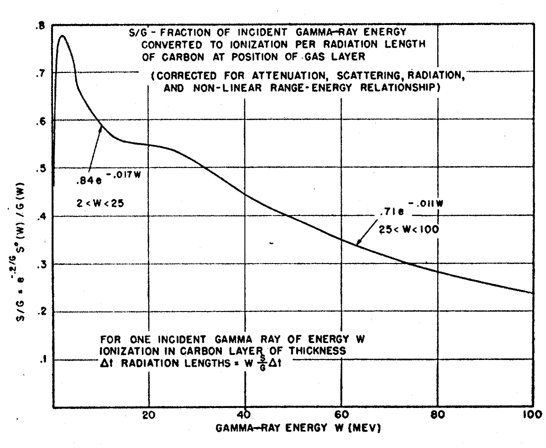

FIG. 3. S/G is fraction of incident gamma-ray energy converted to ionization per radiation length of carbon at position of gas layer (corrected for attenuation, scattering, radiation, and non-linear range-energy relationships).

4. FACTORS WHICH AFFECT THE ELECTRON EFFICIENCY

A. Attenuation

In place of the gamma-ray intensity at the surface of the ionization chamber, one should use an intensity averaged over the region in which those electrons are produced which cause the ionization in the gas layer. (If $R<T$ this layer is of thickness R, running from $T-R$ to T. If $R>T$, the layer is of thickness T.) The average decrease in sensitivity resulting from attenuation is thus given by

$$R<T,\quad \frac{1}{R}\int_{T-R}^{T} e^{-z/G}dz=\frac{G}{R}\,e^{-T/G}[e^{R/G}-1]$$

$$\simeq e^{-T/G}[1+R/2G], \qquad (15a)$$

$$R>T,\quad \frac{1}{T}\int_{0}^{T} e^{-z/G}dz=\frac{G}{T}[1-e^{-T/G}]$$

$$\simeq e^{-T/G}[1+T/2G]. \qquad (15b)$$

For purposes of computation, it is sufficiently accurate to replace $1/G$ by 0.7 in the brackets (see Figs. 1a and 1b) while retaining it in the exponential.

B. Non-Linearity in the Range-Energy Relationship

The efficiency for an electron of energy $E>25$ is not $25/E$, but is given by the actual fraction of energy lost by such an electron on traversing the block of thickness T.

$$\eta=[E(R)-E(R-T)]/E(R),\quad R>T, \qquad (16a)$$

$$=(25/E)\{[E(R)-E(R-T)]/25\}. \qquad (16b)$$

For a linear range-energy relationship, the number in braces { } in Eq. (16b) can be made exactly unity for all energies, simply by choosing T to be the thickness which will just stop a 25-Mev electron. In the non-linear case, it is better to choose T such that the bracketed member has an average value close to unity over the range from 25 Mev to 100 Mev. This can be done sufficiently accurately by choosing $T=0.2$ radiation lengths.

Below 25 Mev, the efficiency is $\eta=1$ regardless of a non-linear loss of energy to ionization along the path, since all of the energy is converted to ionization (except for a small amount of radiation).

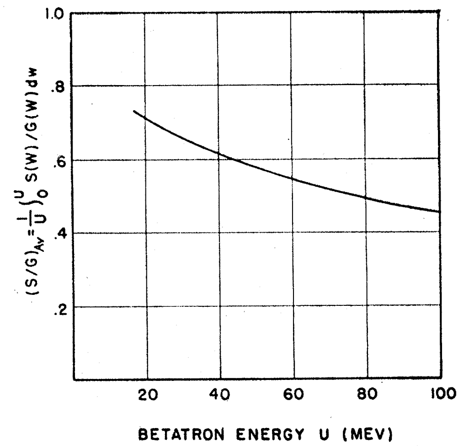

FIG. 4. Fraction of incident gamma-ray energy converted to ionization per radiation length if incident intensity is flat from 0 to U Mev. Ionization energy in gas layer = (incident energy) $\langle S/G\rangle_{Av}\Delta t(R_c/R_g)$, where Δt is the thickness of gas layer in radiation lengths, and R_c/R_g is the ratio of electron ranges in carbon and in the gas, both measured in radiation lengths.

C. Scattering

For electrons which have enough energy to traverse the block, scattering was kept small in the original design (cf. Eq. 2). The effect of this scattering is to increase the path in the block from T to T_s, where

$$T_s\simeq\int_0^T [1+\tfrac{1}{2}\langle\theta^2(t)\rangle_{Av}]dt, \qquad (17)$$

and $\langle\theta^2(t)\rangle_{Av}$, the mean-square deflection angle on traversing a finite thickness t, is given by[6]

$$\langle\theta^2(t)\rangle_{Av}=\int_0^t (21/E')^2dt. \qquad (18)$$

For a linear range-energy relationship, $E'=E-\epsilon t$,

$$\langle\theta^2(t)\rangle_{Av}=[(21)^2/E(E-\epsilon t)]t, \qquad (19)$$

and

$$T_s/T\cong 1-0.066[y+\ln(1-y)] \qquad (20)$$

where $y=25/E$, and $\epsilon=120$ Mev/(radiation length) in carbon.

The increase in collision loss due to this length-

[6] Reference 2, p. 264.

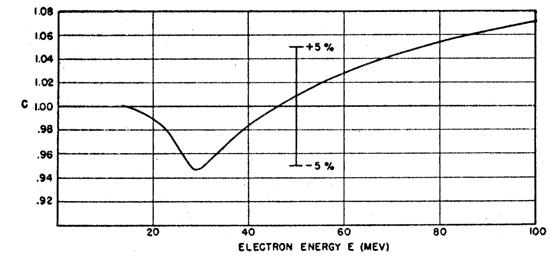

FIG. 5. C is the ratio of the actual fraction of electron energy converted to ionization, to the fraction used for computation in this paper. $C=\eta(\text{actual})/\eta(\text{used})$. The fraction used was: for $E<25$ Mev, $e^{-T/G}$; for $E>25$ Mev, $(25/E)e^{-T/G}$. The actual fraction takes into account attenuation, radiation, scattering and non-linear range-energy relationship.

ening of path is obtained by replacing T in (16) by T_s.

For electrons of energy less than 25 Mev, the scattering does not change the total path length, or the total ionization of an electron. However, it does reduce the thickness of the layer from which electrons can reach the gas from R to R', but this affects only the attenuation correction (15a) which is already a small correction.

An estimate of the straight line path R' is given by

$$R' \simeq \int_0^{t_0} [1-\tfrac{1}{2}\langle\theta^2(t)\rangle_{\text{Av}}]dt \tag{21}$$

$$\simeq R[0.6E-\ln(1+0.6E)]/0.6E, \tag{22}$$

when the upper limit t_0 is chosen so that

$$\langle\theta^2(t_0)\rangle_{\text{Av}}=2,$$

i.e., so that the integrand remains positive.

D. Radiation

For electrons of energy less than 25 Mev, the effect of radiation is to shorten the range from R to R_r, thus decreasing the ionization loss in the ratio $\mu=R_r/R$. If we represent the energy loss roughly by

$$dE/dt=\epsilon+fE, \tag{23}$$

where the second term is the contribution attributable to radiation, and $f\simeq 1-(1/E^{-\frac{1}{2}})$, we find an efficiency

$$\mu=R_r/R=1-(f^2/2\epsilon), \tag{24}$$

where $\epsilon\simeq 100$ Mev/(radiation length) for electrons of less than 25 Mev. The largest correction, at $E=25$ Mev is $\mu=0.92$.

For energies greater than 25 Mev, only high energy radiation produces a decrease in efficiency. For example, a 75-Mev electron will still lose about 25 Mev by collision, on traversing the block, unless it radiates more than 50 Mev, which is very unlikely. Thus above 25 Mev, the efficiency will be given by $\eta=(25/E)\mu$, where the factor μ as the result of radiation will increase rapidly from 0.92 to 1.

To estimate the effect of this radiation straggling, we use Bethe's formula[7] for the distribution of electron energies $w(U)dU$ for an electron of incident energy U_0, after traversing a plate of thickness t, radiation lengths

$$w(U)dU=\frac{dU}{U_0}\frac{[\ln(U_0/U)]^{(t-\ln 2)/\ln 2}}{\Gamma(t/\ln 2)}. \tag{25}$$

The effect of collision losses, which is neglected in (25), can be taken roughly into account by using for U_0 not the incident energy, but an average energy, say $(E-12.5)$ Mev, i.e., by regarding the radiation as taking place in the middle of the plate. This gives

$$\eta(E)=\int_0^{12.5} W(U)(12.5+U)/E\,dU + (25/E)\int_{12.5}^{E-12.5} W(U)dU, \tag{26}$$

$$\eta(E)=(25/E)\mu,$$

where μ increases from 0.92 at 25 Mev, to 0.985 at 40 Mev, to 0.997 at 100 Mev.

E. Summary

The efficiency, $S(W)$, with which x-ray energy is converted to ionization, was computed using an electron efficiency

$$\begin{aligned} \eta &= Ce^{-T/G} & E&<25, \\ \eta &= C(25/E)e^{-T/G} & E&>25, \end{aligned} \tag{27}$$

with $C=1$; whereas according to the preceding sections C is given by

$$\begin{aligned} C&=(1+0.35R')\mu & E&<25, \\ C&=(1+0.35T)\mu[E(R)-E(T_s)]/25 & E&>25. \end{aligned} \tag{28}$$

The correction factor C is plotted in Fig. 5.

[7] Reference 2, p. 256.

By a fortunate coincidence, the increase in efficiency for $E<25$ due to the term $(1+0.35R')$ is cancelled within 1 percent by the decrease caused by radiation μ. Also the thickness of the plate T and hence T_s, was adjusted to reduce the correction factor for $E>25$. Thus, on the average, C is close to unity. Since η is integrated once over electron energies, and then over x-ray energies, the effect of the fluctuations in C about the value unity can be neglected.

5. COMPTON RECOILS

In the preceding calculations, no account was taken of the possibility that some of the Compton recoil x-rays could have a second collision, thus producing additional ionization. For high energy x-rays pair production is the primary effect. The probability of a Compton collision is small, so that the number of recoil x-rays available for a second collision is small. Furthermore, the recoil x-rays, having lower energies, are more likely to produce a Compton collision, in which they transfer only a fraction of their energy to an electron.

This can be illustrated by a rough calculation for x-rays as low as 25 Mev in energy. If 100 x-rays of 25 Mev are incident, the total number of primary collisions is $100(0.2/G)\exp(-0.2/G) = 100(0.147)(0.863) = 12.7$. Since the relative probability of pair production is $P_p=0.47$, this corresponds to about 6 pairs and 6.7 Compton recoil electrons. The energy converted to ionization is $25[6S_p+6.7S_c] = 25[6(0.96)+6.7(0.74)] = 268$ Mev. The average energy carried off by each of the Compton recoils is $25(1-0.74) = 6.5$ Mev, or a total of 43.5 Mev.

The probability that these 6.5-Mev recoils will have a second collision is $0.1/G(6.5)\simeq 0.13$, since on the average the recoil traverses only 0.1 radiation lengths. The fraction of absorbed energy converted to ionization is $S^0(6.5)\simeq 0.67$. Thus the Compton recoil energy which eventually gets into ionization is $(43.5)(0.13)(0.67) = 3.8$ Mev, or 1.3 percent of the 268-Mev ionization obtained from the primary collisions. Note that by using $S^0(6.5)$, which neglects the attenuation factor $\exp[-0.1/G(6.5)]\simeq 0.9$, we have overestimated the secondary collisions in such a manner as to take into account roughly that there are tertiary and higher collisions.

A similar calculation at 10 Mev, shows that about 5 percent of the energy is recaptured by second collisions. Below 10 Mev, the error would be worse. However, this region is not too important since we are interested chiefly in the average sensitivity from 0 to 100 Mev.

ACKNOWLEDGMENT

The author wishes to express his appreciation to Dr. K. H. Kingdon and the other members of the staff of the Research Laboratory of the General Electric Company. The attitude of freedom and stimulation of research in the Laboratory was very gratifying and greatly facilitated this research.