This normalization differs from the one in Whittaker. *Equations (A.6), (A.7), (A.8) define the quantities* E_r, $Q_i^{(r)}$, $P_i^{(r)}$ *uniquely*, except for (immaterial) factors of absolute value 1 in front of $Q_i^{(r)}$ and $P_i^{(r)}$.

When we substitute *these* $Q_i^{(r)}$ and $P_i^{(r)}$ into (A.3) and (A.4), we find that the Hamiltonian reduces to (A.2). To check on the commutation rules (A.5) we use the inversion formulae

$$a_r = i\sum_{i=1}^{N}(Q_i^{(r)*}p_i - P_i^{(r)*}q_i), \qquad (A.9)$$

$$a_r^{+} = -i\sum_{i=1}^{N}(Q_i^{(r)}p_i - P_i^{(r)}q_i). \qquad (A.10)$$

The identities in the $P_i^{(r)}$ and $Q_i^{(r)}$ given ahead of Eq. (A.8), together with Eq. (A.8) itself, then suffice to establish that (A.5) follows from the commutation rules for the p, q.

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On the Rise of the Wire-Potential in Counters

S. A. KORFF
New York University, University Heights, New York 53, New York

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The rate at which the potential of the central wire in a counter changes when a count takes place is considered. The electron collection-time is found to be between 1 and 3×10^{-8} second. The subsequent variations in potential are controlled by the motion of the positive ions. Some experiments are reported, in which rises of 100 volts per microsecond were observed. The factors controlling fast counting are briefly discussed.

INTRODUCTION

FOR some time it has been well known that the potential of the central wire of a counter, either operating as a proportional or as a Geiger counter, rises quite rapidly to its maximum value. Various estimates of the rise-time have been made, the usual values being between one-half and four microseconds, some estimates falling outside either limit. The quantity in question is of great importance in determining the maximum speed with which counters can be operated. However, it is not immediately obvious how or whether it is connected with the much longer "dead time" or "recovery time," studied by Stever and others, nor with the much shorter electron travel time. It was, therefore, felt that it might be worth while to explore the factors involved and to see whether the rise time could be related to other already understood properties of counters and gas discharges.

MOTION OF THE ELECTRONS

It is evident that we have to discuss the motion of the electrons from the time of the formation of the first electron in the counter (the initial ionizing event) to the appearance of the electrons on the sensitive portions of our measuring device. Let us view the phenomenon in two steps. First we shall discuss the motion of the electron in the gas, including the avalanche formation. The second step will take the electron from the point on the central wire where it is collected to the measuring instrument.

Consider first an electron formed in the sensitive volume of a counter. We shall not discuss the process whereby the electron came to be separated from an originally neutral atom or molecule but shall assume that the electron came into being as a result of the initial ionizing event. The electron finds itself in the field produced by the applied voltage. Since almost all counters have cylindrical geometry, we can write for the field E, in volts per centimeter, the expression:

$$E = 2q/r = V/(r\log(b/a)), \qquad (1)$$

where q is the charge per unit length on the wire, r is the radius measured from the center of the wire, V is the applied potential in volts, b is the radius of the cylinder, and a is the radius of the

TABLE I. Field as function of radius inside typical counter.

Radius from wire center (cm)	Field (volts/cm)
5×10^{-3} (wire surface)	37,400
0.01	18,700
0.05	3,740
0.1	1,870
0.5	374
1.0 (at cylinder)	187

wire in centimeters. Let us assume that we have a counter in which the cylinder has a radius of 1 cm and a wire of 5×10^{-3} cm (approximately 4-mil wire). If the potential applied to the counter is assumed to be 1000 volts, we can compute the field in volts per cm at any point inside the counter. The results of such a computation are presented in Table I.

The electron, as it travels toward the central wire, impelled by the field, will make collisions with atoms or molecules present in the counter. The classical mean free path l of an electron in a gas, between collisions with gas molecules, will be given nearly enough by the relation:

$$l=4/N\pi R^2, \qquad (2)$$

where l is the mean free path in cm, N is the number of atoms or molecules per cc, and R is the average collision radius of each. If we take N as 2.7×10^{18} per cc (corresponding to a gas pressure of 7.6-cm Hg in the counter) and R as 10^{-8} cm, then the mean free path in this counter is 4.8×10^{-3} cm. Although Eq. (2) will give only approximate values for l, we may consider the mean free path established nearly enough, since both R and N may be considerably varied in any practical case.

We shall now consider how much energy an electron will gain in one free path at any given point in the counter. Let us divide the counter volume into three sections with differing properties. The outermost region will be called the low field region. In this region, the electron will gain less than 2 volts per free path. The next region we shall call the medium field region. Here the electron gains between 2 and 15 volts per free path. The region next to the wire is the high field region, and in this region the electron will acquire 15 volts or more per free path. The reasons for the selection of these particular values will be discussed below. The energy gain dV, in electron-volts, will be related to the field through:

$$dV=-E(r)dr, \qquad (3)$$

where $E(r)$ is the field at radius r, and dr is the radial distance traversed by the electron. We neglect the third derivative term produced by variations in the field over so small a distance as a free path. If we take dr as equal to a mean free path, then we find that dV becomes 2 volts at $r=0.4$ cm, and 15 volts where r is 0.06 cm. Hence the high field region occupies a space of only half a millimeter about the central wire.

A 2 volt electron making a collision with an atom or molecule will be able to transfer energy by inelastic collisions to the target atom in many cases. For example, the lowest level in sodium is at 2.01 volts. In molecules with bands in the red or infra-red it is lower, while only for hydrogen, helium, and argon is it substantially higher. If a polyatomic constituent is present, electrons will start making inelastic collisions at quite low energies. As soon as an electron makes inelastic collisions we may think of it as coming essentially to rest after each collision and starting from rest to travel its next free path. The energy transferred in this manner to the atom or molecule will be lost either by radiation or by de-excitation collisions. However, such radiation cannot produce new electrons, because there are no photoelectric work functions as low as 2 volts characteristic of any substances ordinarily used in counters. The energy will therefore eventually be dissipated thermally. As the electron gains more and more energy per free path nearer the wire, there will be more and more available energy levels of the atom or molecule to which it can transfer energy, and consequently all further collisions will also be inelastic.

The electron now enters the high field region. Here the electron is a fraction of a millimeter from the wire and will gain, per free path, the 15.6 volts needed to ionize argon. Here the familiar Townsend avalanche process will start. This process has been fully described elsewhere[1] and will not be considered here. The result of ionizing collisions will be as before, that the electron will lose almost all of its energy per

[1] S. A. Korff, *Electron and Nuclear Counters* (D. Van Nostrand and Company, Inc., New York, 1946).

collision and may be regarded as starting from rest after each encounter.

The total time required for the electron to move from the place where it is produced to the wire will now be discussed. To know this we must know the average velocity. It is evident that the total time will equal the sum of the times dt required to traverse each free path. Hence we may write, for T, the total time,

$$T=\sum dt=\sum l/v(r), \tag{4}$$

where the sum is to be taken over all free paths. Again neglecting the third derivative over any one free path, the average velocity v in any free path is one half the final velocity, and this may be computed from the energy gained, since $\frac{1}{2}\ mv^2=eV$. Therefore, the average velocity v depends on the field, and since this varies inversely as the radius, $v(r)$ varies as the inverse square root of the radius. Since an electron starting at the cylinder gains about 0.2 volt between collisions, and since this energy rises only to 0.5 volt by the time the electron has traversed more than half the counter, it is obvious that the electron spends most of its time in the low field regions. Inserting actual values, we recall that a 1-volt electron has a speed of 5.93×10^7 cm/sec. Hence for an average energy of $\frac{1}{4}$ volt, the average velocity will be about 3×10^7 cm/sec., and it will require around 3×10^{-8} sec. for the electron to traverse the 1000 free paths between cylinder and wire.

The Townsend avalanche process, on this picture, will be characterized by an average electron velocity of perhaps 5 times the figure cited, and for a distance of 0.5 mm, and hence requires between 1 and 2×10^{-10} sec. to complete. It will be appreciated that the exact times involved will depend on the nature and amount of the gas used and on the voltage applied. However, if more gas is put in, the mean free path is shortened, but a higher voltage has to be applied. These two factors operate inversely to each other as far as the time is involved, so that 3×10^{-8} sec. will be the proper figure for many ordinary counters.

We are now ready to consider what happens after the electrons have arrived on the central wire. The central wire will generally be connected to the grid of the first tube in the detecting arrangement, and we shall assume that the wire, grid, and connecting leads have a distributed capacity of about 10^{-11} farad. We are interested in inquiring the rate at which the voltage of the grid will change.

When the Townsend avalanche is over, a sheath of positive ions will be left about the central wire. The role of this sheath in quenching the discharge and its influence on the rise-time has been thoroughly discussed by the Montgomerys.[2] The positive charges in the sheath will hold negative charges on the central wire. We may treat the problem by the familiar method of electrostatic images. It will be recalled that the image in a convex conductor of radius a, produced by a charge e at distance r from the center is $-ea/r$. This expression is exact for a sphere, and a good approximation for a cylinder. When the sheath is first formed and r is only a little greater than a, the image charge is roughly equal to the charge in the sheath, and almost all the electrons are held on the wire. However, as the sheath travels outward and r becomes large compared to a, the number of electrons held on the central wire lessens. The electrons are freed to travel to the grid of the tube and to cause its potential to vary.

We may estimate the speed of this process. The rise in potential of the grid will depend on the rate at which charge arrives, and this will depend on the rate at which charge is freed by the process we have just described. The freeing of charge depends on the time-rate of change of r, dr/dt. This in turn is in practice a function of the sheath position, or of the velocity of the positive ions. The velocity may be taken as varying with the field, and hence inversely as the radius.

Another way of viewing this problem is to consider the effect of the sheath as increasing the capacity of the central wire. As the sheath moves outward, the capacity decays down to the value determined by the counter geometry, and hence the potential of the system rises. The relation between charge q, potential V, and capacity C is the familiar equation $q=CV$. If we differentiate this with respect to time and

[2] C. G. and D. D. Montgomery, Phys. Rev. **57**, 1030 (1940).

recall that both C and V will vary, we obtain:

$$dq/dt = CdV/dt + VdC/dt. \quad (5)$$

We are interested in ascertaining dV/dt, and we can write down the factors on which dC/dt depends. Once the charge has been collected, dq becomes zero; the electrons are merely held and redistributed. The capacity of a cylindrical condenser is given by:

$$C = \tfrac{1}{2}\log(b/a) \quad (6)$$

per unit length, where b and a are the electrode diameters. In this case, we have to consider what happens when a, the wire diameter, remains constant, and b, the sheath, moves outward. The outward (radial) velocity of the sheath we shall assume to be proportional to the field. This assumption may be wrong in the high field region, but is nearly enough right in the region in which the sheath spends most of its time. The field varies inversely as the radius. The outward velocity, $d/dt(b)$, varies as $(1/r)$. Hence we have:

$$Kd/dt(\log b) = d/dt(1/C), \quad (7)$$

where K is a constant of proportionality involving $\log a$.

Inspection of this relation shows that the capacity will be large (and hence the potential of the grid will be small), while b is nearly equal to a and that after the sheath has moved out so that b is any substantial fraction of the size of the counter, the rate of change of the capacity will be small and the grid potential will approach a constant value. We may expect, therefore, that the rise of potential of the grid of the tube will give us a straight line on a semi-log plot and that the intercept on the time axis will give us a measure of the avalanche formation and electron collection time.

COMPARISON WITH EXPERIMENT

W. E. Ramsey[3] has made a careful series of measurements of the time-rate-of-change of the potential of the wire of a counter. Inspection of his curves will show at once that the predictions based on the above picture are admirably fulfilled. The predicted straight-line relationship suggested by Eq. (7) is found. The intercept noted by Ramsey is around 3×10^{-8} sec., which also agrees with the values computed above with the aid of Eq. (3) and the subsequent argument. He shows curves for argon-alcohol and for argon-oxygen counters. He finds that the argon-oxygen counter has a faster "breakdown" and shows an intercept of nearer 1.5×10^{-8} sec. This is also in accord with the point of view developed above, for in the polyatomic counters the electron will make inelastic collisions at a lower voltage because of the many available levels in the band-structure and so it will start to lose energy (radial velocity) at points further out in the counter.

In order to ascertain whether the above picture was also valid in the proportional region, a counter filled with BF_3 was used. It was connected to a P4 Synchroscope. A one megacycle timing wave gave a time calibration and a high resistance voltmeter gave the vertical pulse size scale. The counter wire was connected to ground through 100,000 ohms, and the distributed capacity of the wire and leads was estimated at about 30 mmf. Hence the pulses on the wire decayed with about a three-microsecond exponent. This quantity was made long compared to the rise of the pulse so that the exponential decay would not interfere with the rise-time measurements and so that the pulses would be large enough to be easily measured.

The wire was found to change in potential by about 25 volts in $\frac{1}{4}$ microsecond, i.e., at the rate of 100 volts per microsecond. It must be noted that the rate of rise is not uniform but is rapid at first and then becomes slower. Hence the rate is initially greater and afterwards less than 100 v/ms, and the figures refer to a particular magnitude and instant only. Further, in these tests it required the presence of about 2×10^8 electrons on the wire to cause a change in potential of 1 volt, so that even for large initial ionizing events a considerable gas amplification in the Townsend avalanche took place. Fluctuations were observed, the magnitude of which corresponded to about ten percent in the times required to rise 25 volts. The rates of change observed in these tests were similar to those found by Ramsey for his counters. The experi-

[3] W. E. Ramsey, Phys. Rev. **57**, 1022 (1940).

ments, therefore, bear out the picture developed above. We expect to make some further measurements at smaller pulse sizes.

One may also expect that the rise-time of the pulse will be slowed down if the capacity of the wire-system is increased. This was tested and found to be the case. Increasing the capacity also has the effect of decreasing the pulse size. Similar effects were noted by Ramsey[3] and are found in his curves.

DISCUSSION

In conclusion a few words may be said about high counting rates. In the Geiger region the normal recovery time is determined by the mobility of the positive ion sheath. This may be shortened by collecting the positives on the center wire, using the Simpson[4] circuit. The effective speed of operation of this circuit is limited by the spread-time of the sheath.[5] In the proportional region, the sheath does not spread the whole length of the wire,[1] and the recovery times can be made much more rapid.[6] The greater the gas amplification, however, the larger the number of positive ions that produce space charge which will limit counting speeds. At low amplifications, recovery times can be considerably shortened, but the recovery time still is the limiting factor. Only a small benefit can be achieved by operating the counter at a higher voltage. The field is increased, and hence the electron velocity is higher, but many more positives are formed at varying distances from the wire. The use of grids[7] is beneficial, since the positives have a smaller distance to travel. However, grids present added constructional difficulty and in the proportional region pose collection problems. A pulse sufficient in size to be easy to measure can be obtained in 10^{-7} sec., as we have shown. Therefore, an application of the Simpson circuit to a proportional counter will permit counting at rates up to several million per second. The matter is under study and further tests will be reported in due course.

[4] J. A. Simpson, Phys. Rev. **66**, 39 (1944).
[5] E. Wantuch, Phys. Rev. **71**, 646 (1947).
[6] C. O. Muelhause and H. Friedman, Rev. Sci. Inst. **17** 506 (1946).
[7] S. A. Korff, Phys. Rev. **68**, 53 (1945).

The Absorption Spectrum of InCl

HAROLD M. FROSLIE AND J. G. WINANS
University of Wisconsin, Madison, Wisconsin
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A study of the band spectrum of InCl has been carried out in absorption with a determination of vibrational, rotational, and electronic energy levels associated with the band system in the region 2650 to 3000 angstrom units. The B_v' and B_v'' values were found to be of the order of 0.11 wave numbers, from which can be calculated a nuclear separation of the order of 2.3×10^{-8} centimeter. Predissociation in the band system provided a value for the energy of dissociation of the ground state between 4.64 and 4.66 electron volts.

INTRODUCTION

WHILE studying the absorption of a mixture of Hg and In vapor, Winans, Davis, and Leitzke[1] observed a band system appearing in the region 2650 to 2800 angstrom units. The present study is the outgrowth of an attempt to verify the earlier observations and to study the bands under greater dispersion. By duplicating the conditions previously found necessary to produce the bands, it was possible to photograph them with sufficient dispersion to make an accurate measurement of the isotope shift. These measurements showed that the bands were due to InCl and not HgIn, as first supposed.

A survey of the literature showed that this same band system had been observed previously a number of times. Grotrian[2] obtained the

[1] J. G. Winans, Frances Davis, and Victor Leitzke, Phys. Rev. **57**, 1079 (1940).
[2] W. Grotrian, Zeits. f. Physik **12**, 229 (1923).