available energy is about the same in the two cases, and in one case the alpha takes off from 10 percent to 15 percent of the energy.

For the fission tracks listed in Table I, a distribution in range can be plotted for individual fragments, since the alphas mark the point of origin. Such a distribution curve is shown in Fig. 7. With only 40 tracks, the accuracy of the curve is necessarily low, but it is felt that the two groups are within the accuracy of the data to resolve. Two groups might be expected from the two energy groups associated with the light and heavy fragments as shown by Jentsche[1] and Flammersfeld, Jensen, and Gentner.[2]

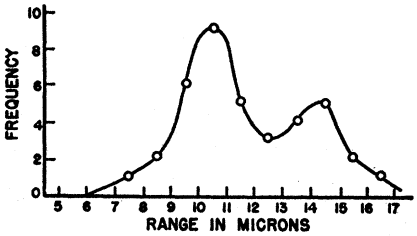

FIG. 7. Distribution in range of individual fragments in fission.

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[1] W. Jentsche, Zeits. f. Physik **120**, 165 (1943).

[2] A. Flammersfeld, P. Jensen, and W. Gentner, Zeits. f. Physik **120**, 450 (1943).

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Hyperfine Structure and Nuclear Moments of Columbium^{93}

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Measurements have been made on the hyperfine structure of 32 lines in the spectrum of CbI. Analysis of these structures supports the previously reported spin value of $4\frac{1}{2}$ units for the Cb^{93} nucleus and yields 32 hyperfine interval factors. These data, when used in conjunction with semi-empirical formulas for the coupling between the nucleus and an s electron in the configuration $4d^4\,5s$, lead to values for the nuclear g-factor and nuclear magnetic moment of 1.18 and 5.3 nuclear magnetons, respectively, for stable Cb^{93}. No nuclear electric quadrupole moment is detected.

INTRODUCTION

THE first published measurements and analysis of columbium hyperfine structure were by Ballard[1] who examined the intervals and intensities in ten visible lines. On a basis of this study he reported the nuclear spin as $4\frac{1}{2}$ units and the nuclear magnetic moment as 3.7 nuclear magnetons. Ballard's analysis was handicapped by a lack of information concerning the term structure of CbI, which was at that time very incompletely known. Subsequently, Meggers and Scribner[2] and Humphreys and Meggers[3] published very extensive classifications of the CbI lines and energy states, thus greatly facilitating a more detailed investigation of the hyperfine structure. We have measured the structure of 32 lines in the visible and by analysis of these data determined 32 hyperfine interval factors. Our analysis is in agreement with Ballard's value of $4\frac{1}{2}$ for the nuclear spin, but indicates a nuclear magnetic moment 43 percent larger than reported by him.

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[1] S. S. Ballard, Phys. Rev. **46**, 806 (1934).

[2] W. F. Meggers and B. F. Scribner, J. Research Nat. Bur. Stand. **14**, 629 (1935).

[3] C. J. Humphreys and W. F. Meggers, J. Research Nat. Bur. Stand. **34**, 477 (1945).

TABLE I. Hyperfine structure of CbI lines.

Wave-length (A.U.)	Intensity (estimated)	Component separations (cm^{-1})
6660.8*		0.000
		−0.305
		−0.567
		−0.800
	e	−1.416
5664.7	4	0.000
	2	0.122
	2	0.245
	2	0.365
	2	0.445
5350.7	5	0.000
	4	−0.109
	3	−0.199
	e	−0.344
5344.1*	7	0.000
	6	−0.152
	5	−0.282
	4	−0.397
	3	−0.490
	e	−0.607
5271.5	4	0.000
	2	0.126
	2	0.247
	2	0.371
	2	0.448
5180.3	5	0.000
	2	0.120
5160.3	3	0.000
	2	0.131
5134.7	2	−0.116
	10	0.000
	6	0.143
	1	0.298
5120.3	3	0.000
	2	−0.143
5100.2	5	0.000
	3	−0.174
5095.2	3	0.000
	2	−0.176
5079.0	5	0.000
	2	−0.154
5058.0	1	−0.323
	6	−0.148
	10	0.000
	2	0.132
5039.0	10	0.000
	7	−0.178
4989.0	3	0.000
	2	−0.215
4606.8	6	0.000
	5	−0.099
	e	−0.446
4546.8	5	0.000
	2	−0.119
4523	6	0.000
	5	0.133
	4	0.245
	3	0.326
4287.0	3	0.000
	2	0.158
4262.1	2	0.000
	1	0.168
4205	5	0.000
	4	0.108
	e	0.430
4195.1	5	0.000
	4	0.163
4192.1	6	0.000
	5	0.146
	4	0.283
	e	0.667
4190.9	6	0.000
	5	0.149
	4	0.292
	3	0.398
	e	0.733
4168.1	1	0.000
	1	0.258
	1	0.551
4163.7	5	0.000
	2	0.169
	3	0.273
4152.6	7	0.000
	6	0.141
	5	0.269
	4	0.382
	e	0.624
4139.7	8	0.000
	7	0.171
	6	0.321
	5	0.455
	4	0.573
	e	?
4137.1	6	0.000
	5	0.222
4123.8	5	0.000
	3	0.117
	2	0.216
4116.9	7	0.000
	6	0.287
4100.9	6	0.000
	5	0.125
	4	0.233
	e	0.433
4079.7	8	0.000
	7	0.145
	6	0.275
	5	0.384
	4	0.481
	e	0.618
4058.9	10	0.000
	9	0.191
	8	0.359
	7	0.510
	6	0.638
	5	0.747
	e	0.862

* As reported by Ballard.

EXPERIMENTAL DATA

In our observations the columbium spectrum was excited in helium in a water-cooled hollow cathode-discharge tube, thin sheets of metallic columbium being used to line the cathode cavity. Dispersion was provided by a Fabry-Perot interferometer with silvered mirrors, combined with a Littrow type prism spectrograph having glass optics.

Table I gives the results of our measurements on 32 line patterns and includes two additional lines of interest from Ballard's data. Component separations in each pattern are expressed in cm^{-1} with respect to the strongest line of the pattern. Since many of the structures are only partially resolved, the measured positions may refer only to points of maximum intensity in the pattern. The e's appearing in the intensity column indicate the estimated end of an incompletely resolved pattern. All lines here reported with the exception of λ4205 were measured on at least three photographic plates.

ANALYSIS

All evidence based on measured component intervals is in agreement with Ballard's assigned value of $I=4\frac{1}{2}$ for the nuclear spin of Cb^{93}. Because of the large indicated spin and the possibility of pattern distortion by a nuclear electric quadrupole, other spin values could not immediately be excluded on the ground of the observed intervals alone. In particular, careful consideration had to be given to the possibility of a spin of $3\frac{1}{2}$, which was also found to be consistent with observed interval ratios in resolved "flag" type patterns such as λ4059. Over-all consistency in the analysis of the numerous patterns favors overwhelmingly the spin of $4\frac{1}{2}$ units, however. No other assumed spin value permits a consistent interpretation of all the observed patterns. Accepting the spin as $4\frac{1}{2}$ units, it may be said that no nuclear electric quadrupole moment is detectible in Cb^{93} within the limit of accuracy of the present observations.

The classification and electron configurations

Table II. Classification of Cb lines.

		$4d^4(^5D)5s\ ^6D$					$4d^35s^2\ ^4F$			
	J	$\frac{1}{2}$	$1\frac{1}{2}$	$2\frac{1}{2}$	$3\frac{1}{2}$	$4\frac{1}{2}$	$1\frac{1}{2}$	$2\frac{1}{2}$	$3\frac{1}{2}$	$4\frac{1}{2}$
$4d^35s(^5F)5p\ ^6D^\circ$	$\frac{1}{2}$		5134.7							
	$1\frac{1}{2}$	5058.0		5160.3						
	$2\frac{1}{2}$		5039.0	5100.2	5180.3					
	$3\frac{1}{2}$				5095.3					
	$4\frac{1}{2}$				4989.0	5079.0				
$4d^4(^5D)5p\ ^6F^\circ$	$\frac{1}{2}$	4168.1	4195.1							
	$1\frac{1}{2}$	4137.1	4163.7	4205.3						
	$2\frac{1}{2}$		4123.8							
	$3\frac{1}{2}$			4100.9	4152.6					
	$4\frac{1}{2}$				4079.7	4139.7				
	$5\frac{1}{2}$					4058.9				
$4d^4(^5D)5p\ ^6P^\circ$	$1\frac{1}{2}$	4116.9								
	$2\frac{1}{2}$				4192.1					
	$3\frac{1}{2}$					4190.9				
$4d^35s(^5F)5p\ ^6F^\circ$	$\frac{1}{2}$						5664.7			
	$1\frac{1}{2}$									
	$2\frac{1}{2}$									
	$3\frac{1}{2}$			5120.5						
	$4\frac{1}{2}$									
	$5\frac{1}{2}$									
$4d^35s(^3F)5p\ ^4D^\circ$	$\frac{1}{2}$						5271.5			
	$1\frac{1}{2}$									
	$2\frac{1}{2}$								5350.7	
	$3\frac{1}{2}$									5344.2
$4d^35s(^3F)5p\ ^4F^\circ$	$1\frac{1}{2}$						4523.4			
	$2\frac{1}{2}$							4546.8		
	$3\frac{1}{2}$				4287.0					
	$4\frac{1}{2}$					4262.1				4606.8

reported by Meggers and Scribner[2] and Humphreys and Meggers[3] for the lines here of interest are given in Table II. The wave-lengths of the lines appear within the rectangles and the terms from which they originate along the margins, even terms above and odd terms to the left.

Graphical analysis procedures described elsewhere[4, 5] were applied to the observed line patterns and lead to the hyperfine interval factors for thirty-two terms given in Table III. Because some of the patterns are more completely resolved and hence permit more positive interpretation than others, the interval factors cannot be given equal weight regardless of the indicated numerical accuracy. They have, therefore, been rated *A*, *B*, and *C* to indicate the degree of confidence with which they may be regarded, *A* indicating that we consider both the interpretation and the measurement reliable, *C* that the interpretation is somewhat doubtful.

TABLE III. Hyperfine interval factors for CbI.

State	J	Interval factor (cm^{-1})		
$4d^4(^5D)5s\ ^6D$	$\frac{1}{2}$	0.060	±0.001	*A*
	$1\frac{1}{2}$	0.028		
	$2\frac{1}{2}$	0.024		
	$3\frac{1}{2}$	0.022		
	$4\frac{1}{2}$	0.021		
$4d^35s^2\ ^4F$	$1\frac{1}{2}$	0.021	±0.001	*A*
	$2\frac{1}{2}$	0.013	±0.001	*C*
	$3\frac{1}{2}$	0.004		
	$4\frac{1}{2}$	0.005		
$4d^35s(^5F)5p\ ^6D°$	$\frac{1}{2}$	0.057	±0.001	*A*
	$1\frac{1}{2}$	0.029		
	$2\frac{1}{2}$	0.028		
	$3\frac{1}{2}$	0.026		
	$4\frac{1}{2}$	0.024		
$4d^4(^5D)5p\ ^6F°$	$\frac{1}{2}$	0.049	±0.001	*A*
	$1\frac{1}{2}$	0.010		*C*
	$2\frac{1}{2}$	0.005		
	$3\frac{1}{2}$	0.004		
	$4\frac{1}{2}$	0.002		
	$5\frac{1}{2}$	0.000		
$4d^4(^5D)5p\ ^6P°$	$1\frac{1}{2}$	0.001		*C*
	$2\frac{1}{2}$	0.004		
	$3\frac{1}{2}$	0.004		
$4d^35s(^5F)5p\ ^6F°$	$\frac{1}{2}$	−0.026	±0.001	*A*
	$1\frac{1}{2}$	?		
	$2\frac{1}{2}$	?		
	$3\frac{1}{2}$	0.021	±0.001	*B*
	$4\frac{1}{2}$	?		
	$5\frac{1}{2}$	?		
$4d^35s(^3F)5p\ ^4D°$	$\frac{1}{2}$	−0.025	±0.001	*A*
	$1\frac{1}{2}$	?		
	$2\frac{1}{2}$	0.020	±0.003	*C*
	$3\frac{1}{2}$	0.025	±0.001	*B*
$4d^35s(^3F)5p\ ^4F°$	$1\frac{1}{2}$	0.000	±0.001	*B*
	$2\frac{1}{2}$	0.016	±0.001	*C*
	$3\frac{1}{2}$	0.018	±0.001	*B*
	$4\frac{1}{2}$	0.017	±0.001	*B*

NUCLEAR MAGNETIC MOMENT

An approximate calculation of the nuclear *g*-factor of Cb^{93} may be made by means of certain semi-empirical relations suggested by Goudsmit.[6] These relations connect the nuclear *g*-factor to the observed hyperfine coupling factors and other experimentally determinable parameters which depend upon the coupling between a single optical electron and the nucleus. Most favorable for this purpose are the five hyper-multiplets occurring in the ^6D states arising from the $4d^4(^5D)5s$ electron configuration. These five hyperfine interval factors are rather accurately known from our analysis and are theoretically favorable because the contribution of the $5s$ electron to the splittings is separable to a good approximation from that of the $4d^4$ group.

Goudsmit's relations expressing the nuclear *g*-factor in terms of the coupling between an *s* electron and the nucleus can be written in a single formula as follows:

$$g(I)=a(s)\frac{R^{\frac{1}{2}}Z_0 1840(3-4\alpha^2Z^2)(1-\alpha^2Z^2)^{\frac{1}{2}}}{8\alpha^2 Z W^{\frac{3}{2}}}.$$

Here W is the energy in cm^{-1} necessary to remove the *s* electron from the atom in that stage of ionization in which it is the only outer electron, and the other symbols have their usual conventional meaning. The coupling coefficient $a(s)$ for an *s* electron in a given configuration and state can be obtained from hyperfine structure data, provided there are available interaction relations expressing the observed interval factors in terms of the nuclear coupling factors for individual electrons of the configuration. A set of formulas directly applicable to the $4d^45s\ ^6D$ states has been derived by Fisher and Peck[5] for

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[4] R. A. Fisher and S. Goudsmit, Phys. Rev. **37**, 1057 (1931).

[5] R. A. Fisher and E. R. Peck, Phys. Rev. **55**, 270 (1939).

[6] S. Goudsmit, Phys. Rev. **43**, 636 (1933).

d^6s ^6D states.** These formulas may be written:

$$A(\tfrac{1}{2})=\frac{7}{15}a(s)-\frac{4}{3}a(d^4)-\frac{4}{3}b(d^4),$$

$$A(1\tfrac{1}{2})=\frac{13}{75}a(s)+\frac{2}{15}a(d^4)+\frac{74}{105}b(d^4),$$

$$A(2\tfrac{1}{2})=\frac{23}{175}a(s)+\frac{12}{35}a(d^4)+\frac{164}{245}b(d^4),$$

$$A(3\tfrac{1}{2})=\frac{37}{315}a(s)+\frac{26}{63}a(d^4)+\frac{222}{441}b(d^4),$$

$$A(4\tfrac{1}{2})=\frac{1}{9}a(s)+\frac{4}{9}a(d^4)-\frac{20}{63}b(d^4),$$

$$\Sigma A=a(s).$$

Here $A(\frac{1}{2})-A(4\frac{1}{2})$ are the hyperfine interval factors for the different states of the ^6D multiplet, the inner quantum number of the state being indicated in the parenthesis, $a(s)$ the coupling coefficient of the s electron, and $a(d^4)$ and $b(d^4)$ the orbital and spin parts, respectively, of the coupling coefficient for the d^4 electron group. Values of $a(s)$, $a(d^4)$, and $b(d^4)$ are determined by inserting in these formulas the experimental values of the A's from Table III. Since there are more equations than unknowns, consistency provides some check upon the accuracy of the formulas. Values of the coupling constants found to give best consistency are:

$$a(s)=0.155\text{ cm}^{-1},$$
$$a(d^4)=0.0089\text{ cm}^{-1},$$
$$b(d^4)=0.0004\text{ cm}^{-1}.$$

Substitution of these values into the formulas gives for the interval factors $A(\frac{1}{2})-A(4\frac{1}{2})$ respective values of 0.060, 0.028, 0.023, 0.022, and 0.021 cm^{-1}. These are to be compared with the values in Table III and seem to speak well for the validity of the formulas in this case.

An experimental value of $a(s)$ is now available for substitution into the formula for $g(I)$ given above. The value for W required by the formula is found by using the data of Humphreys and Meggers[3] to determine the energy change in going from the center of gravity of the ^6D and ^4D terms of the $3d^44s$ configuration in CbI to the center of gravity of the $3d^4$ ^5D in CbII. For this we use the value 51,200 cm^{-1}. Using for the other parameters the values $R=1.097\times10^5$ cm^{-1}, $\alpha=1/137$, $Z_0=1$, $Z=41$, we obtain for $g(I)$, the nuclear *g-factor* of the Cb^{93} nucleus, the numerical value 1.18. Multiplying this by the nuclear spin of $4\frac{1}{2}$ units gives a nuclear magnetic moment of 5.3 nuclear magnetons.

The value 1.18 here obtained for $g(I)$ is 43 percent larger than that reported by Ballard. While admittedly both results are approximations, we believe that somewhat more confidence may be placed in the larger value since it is based upon more complete hyperfine structure data, a knowledge of the ionization potential of the atom, and computations which take into account the coupling between the s electron and the nucleus in a more detailed way. The significance of the third figure in our g-value is, of course, doubtful.

** It is to be recognized that the assumption of a coupling factor like $a(s)$, which is constant for all states of a multiplet, is valid only within limits.