The γ-Radiation of Na^{24} and the Energy-Level Scheme of Mg^{24}

M. L. WIEDENBECK
Department of Physics, University of Michigan, Ann Arbor, Michigan
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SINCE Na^{24} can be produced easily and because it emits high energy γ-radiation it has long been of interest from the applied standpoint as well as from the standpoint of the structure of the levels giving rise to the radiation. Over the past several years the radiation has been investigated by many workers by various methods and in many cases the results have been inconsistent.

The sodium-β-spectrum has been most accurately studied by Siegbahn[1] and by Lawson[2] and has been found to be simple. Absorption experiments by Langer, Mitchell, and McDaniel[3] in 1939 have shown that the $\beta-\gamma$-coincidence rate is independent of the energy of the β-radiation, thus showing that a single β-transition takes place. The upper limit of the β-spectrum is 1.390 Mev.

The history of the γ-radiation following the β-decay of Na^{24} is somewhat more complex. This has, however, been described in considerable detail by Siegbahn.[1] It would appear that the most accurate results are those obtained by a beta-spectrograph study of the secondary electrons ejected from a body irradiated with the γ-rays in question. Such work has been carried out by Itoh,[4] Elliott, Deutsch and Roberts,[5] Mandeville,[6] and most recently by Siegbahn.[1] Their results all indicate that only two γ-rays are emitted by the Mg^{24} nucleus and these are of "nearly" equal intensity. The values given are 1.38 and 2.76 Mev.

The most simple arrangements of the levels of Mg^{24} would occur if the γ-rays were in cascade. However Guthrie and Sachs[7] have pointed out that such an arrangement would lead to a mass of Na^{24} which is $\sim$1.5 Mev higher than that calculated by Barkas[8] from a study of the fine structure of the mass defect curve. They further pointed out that this difficulty could be eliminated by assuming the level scheme shown in Fig. 1b with a branching ratio of 2:1.

Absorption measurements of the $\gamma-\gamma$-coincidences were carried out by Cook, Jurney, and Langer.[9] These indicate that the 2.8-Mev radiation is involved in the coincidence and hence supports level scheme of Fig. 1a.

Recently Barker[10] has measured the average

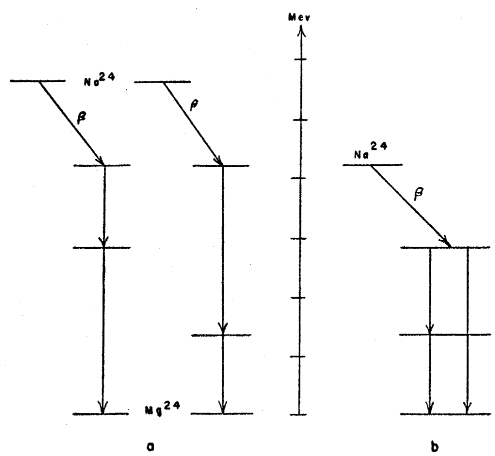

FIG. 1. "a" shows the two possible cascade energy-level diagrams of the decay of Na^{24}. "b" indicates the parallel decay proposed by Sachs.

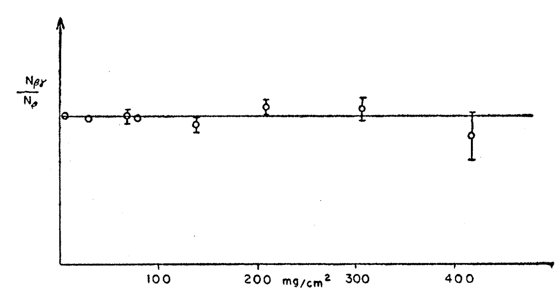

FIG. 2. $\beta-\gamma$-coincidence rate of sodium as a function of β-absorber thickness.

[1] K. Siegbahn, Phys. Rev. **70**, 127 (1946).
[2] J. L. Lawson, Phys. Rev. **56**, 131 (1939).
[3] L. M. Langer, A. C. G. Mitchell, and P. W. McDaniel, Phys. Rev. **56**, 962 (1939).
[4] J. Itoh, Proc. Phys. Math. Soc. Japan **23**, 605 (1941).
[5] L. G. Elliott, M. Deutsch, and A. Roberts, Phys. Rev. **63**, 386 (1943).
[6] C. E. Mandeville, Phys. Rev. **63**, 387 (1943).
[7] A. Guthrie and R. G. Sachs, Phys. Rev. **62**, 8 (1942).
[8] W. H. Barkas, Phys. Rev. **55**, 691 (1939).
[9] C. S. Cook, E. Jurney, and L. M. Langer, Phys. Rev. **70**, 985 (1946).
[10] E. C. Barker, Phys. Rev. **72**, 167(A) (1947), also private communication with E. C. Barker.

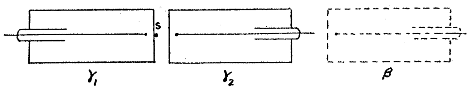

FIG. 3. Counting arrangement used in determining $N_{\gamma_1\gamma_2}$ and $N_{\gamma_1\beta}$. The source, S, is fixed rigidly to counter γ_1.

γ-energy per β-particle for Na^{24} by means of ionization methods. His results give 4.17 Mev (± 10 percent) per β-particle. This would also substantiate the level scheme indicated in Fig. 1a.

To check the simplicity of the Na^{24} β-spectrum, a determination of the $\beta-\gamma$-coincidence rate was made as a function of β-absorber thickness. The minimum thickness of absorber was equivalent to approximately 4 mg/cm^2 of aluminum. The curve, Fig. 2, is seen to be independent of the β-energy and again attests to the simplicity of the β-spectrum.

An additional method which can be used to distinguish between the level schemes of Figs. 1a and 1b lies in the $\gamma-\gamma$- and $\beta-\gamma$-coincidence rates. If we have two counters designated by γ_1 and γ_2 (Fig. 3), each having an efficiency $\epsilon_{1.4}$ for the 1.4-Mev γ-radiation and an efficiency $\epsilon_{2.8}$ for the 2.8-Mev γ-ray, then

$$N_{\gamma_1\gamma_2}=(2\sigma_{\gamma_1}\epsilon_{1.4}\epsilon_{2.8}/\epsilon_{1.4}+\epsilon_{2.8})N_{\gamma_2}, \qquad (1)$$

for level scheme of Fig. 1a. $N_{\gamma_1\gamma_2}$ being the $\gamma-\gamma$-coincidence rate, σ_{γ_1} the fraction of the total radiation intercepted by counter γ_1, and N_{γ_2} the counting rate of counter γ_2. Now if the counter γ_2 is moved farther from the source and converted to a β-counter, the $\beta-\gamma$-coincidence rate for scheme of Fig. 1a is:

$$N_{\beta\gamma_1}=\sigma_{\gamma_1}(\epsilon_{1.4}+\epsilon_{2.8})N_\beta. \qquad (2)$$

Dividing Eq. (1) by Eq. (2), and remembering that the source, S, is fixed rigidly to the counter γ_1, we have:

$$(N_{\gamma_1\gamma_2}/N_{\gamma_2})/(N_{\gamma_1\beta}/N_\beta)=2\epsilon_{1.4}\epsilon_{2.8}/(\epsilon_{1.4}+\epsilon_{2.8})^2. \qquad (3)$$

Similarly for level scheme of Fig. 1b we obtain the ratio

$$(N_{\gamma_1\gamma_2}/N_{\gamma_2})/(N_{\gamma_1\beta}/N_\beta)=3(\epsilon_{1.4})^2/2(\epsilon_{1.4}+\epsilon_{2.8})^2. \qquad (4)$$

TABLE I. Comparison of $\gamma-\gamma$ and $\beta-\gamma$ coincidence rates.

Radiator	$\dfrac{N\gamma_1\gamma_2/N\gamma_2}{N\gamma_1\beta/N\beta}$	Total relative efficiencies (brass = 1)
Brass	0.525	1
Cu	0.525	1
Al	0.51	1.2
Fe	0.502	1.06
Pb	0.517	0.95
Lucite	0.51	1.3

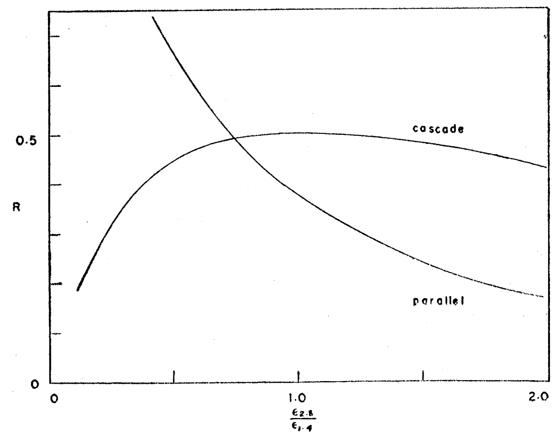

FIG. 4. Ratio, R, of $N_{\gamma_1\gamma_2}/N_{\gamma_2}/N_{\gamma_1\beta}/N_\beta$ as a function of the ratio of the counter efficiencies for the 2.8- and 1.4-Mev γ-rays.

It should be noted that these equations are derived on the assumption that the efficiency of the counters is low, actually they are $\sim$one percent efficient.

Equations (3) and (4) are plotted in Fig. 4. It is clear that scheme 1a gives rise to a ratio $(N_{\gamma_1\gamma_2}/N_{\gamma_2})/(N_{\gamma_1\beta}/N_\beta)$ nearly equal to 0.5 over a very wide range of efficiencies while scheme 1b gives a rapid variation in the region of interest.

The counters were of brass construction with a $\frac{1}{2}$-mil Cellophane end window. To convert the counter for γ-counting, a small cylindrical plug was inserted in front of the window.

Table I gives the results of six determinations made with various radiators in front of the thin-windowed counter ($\frac{1}{2}$-mil Cellophane). The total relative γ-efficiencies of the counters are also included in the table. It is seen that over this wide range of radiators the ratio remains essentially constant near the value 0.5. The experiment with the lead radiator was repeated with a greater thickness of lead to shift artificially the efficiency ratio ($\epsilon_{2.8}/\epsilon_{1.4}$) to a higher value. The counting rate with such an arrangement is low but within the statistical error the ratio $(N_{\gamma_1\gamma_2}/N_{\gamma_2})/(N_{\gamma_1\beta}/N_\beta)$ remains in the region of 0.5. It must, therefore, be concluded that the cascade level diagram of Fig. 1a is the correct one.

We wish to express our thanks to Professor Uhlenbeck for his discussions of this problem.