conclude that the ratios of capture probabilities are proportional to Z.

8. CONCLUSION

The over-all conclusions can be summarized as follows. In condensed substances, both conductors and insulators, a negative mesotron is captured in its orbit nearest to the nucleus in about 10^{-13} second. In a gas the corresponding time is a little longer than is indicated by the ratio of densities. In particular, in normal air it is of the order of 10^{-9} sec. In both cases this time is very short compared with the mesotron natural lifetime of 2×10^{-6} sec. so that the mode of ultimate disappearance of the negative mesotron is governed by the balance between natural decay and typically nuclear phenomena leading to mesotron disappearance.

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Spin Dependence of Scattering of Slow Neutrons by Be, Al, and Bi

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Some information has been obtained on the spin dependence of scattering of slow neutrons by Be, Al, and Bi by measuring the scattering cross section for filtered neutrons. The result is that in none of these three cases does the sign of the scattering length change when the spin orientation is changed. But in the case of Be and Bi the magnitude of the scattering length for one spin orientation may be up to twice as great as that for the other spin orientation, and in the case of Al the variation may be by a factor of three.

SOME information on the spin dependence of the scattering of slow neutrons can be obtained by measuring the cross sections of some microcrystalline substances for filtered neutrons.[1,2]

When a slow neutron is scattered by an atom having nuclear spin I, two values for the scattering length[2] can be expected according to whether the spin of the neutron is parallel or antiparallel to I; these will be indicated by a_+ and a_-. If these two values are equal, there is no spin dependence of the scattering. In this case interference phenomena are not influenced by the spin, and the neutron waves scattered by the atoms behave as fully coherent. When a_+ and a_- are different, the coherent scattering of the atom is determined by an average scattering length: (see reference 2, formula (6))

$$a=\frac{I}{2I+1}a_-+\frac{I+1}{2I+1}a_+.$$

The remaining scattering behaves as incoherent for interference phenomena.

In order to discuss the significance of coherent and incoherent scattering it is necessary to distinguish between collisions in which the spin orientation is not changed and those in which it changes. The first type of collision is responsible for coherent scattering, the second for incoherent. The reason is that interference takes place only when the scattering is due to the cooperative action of all atoms. This is the case when there is no spin change to indicate which atom has been responsible for the scattering. If there is a spin change, however, the scattering is attributed to the individual action of that atom whose spin has changed.

One can prove by elementary quantum mechanics that the coherent scattering cross section is

$$\sigma_{\text{coherent}}=4\pi\left\{\frac{I}{2I+1}a_-+\frac{I+1}{2I+1}a_+\right\}^2, \qquad (1)$$

[1] H. L. Anderson, E. Fermi, and L. Marshall, Phys. Rev. **70**, 815 (1946).
[2] E. Fermi and L. Marshall, Phys. Rev. **71**, 666 (1947).

TABLE I. Scattering cross sections for slow neutrons.

Element	Spin	$\sigma\times10^{24}$ cm² for unfiltered neutrons $\lambda\sim1.8$A	$\sigma\times10^{24}$ cm² for BeO filtered neutrons $\lambda>4.4$A	$\sigma\times10^{24}$ cm² for graphite filtered neutrons $\lambda>6.7$A	Theoretical edge in A	Residual scattering cross section $\times10^{24}$ cm²	$\sigma_{\text{scattering}}$ $\times10^{24}$ cm²	Limits for value of a_+/a_-
Be	3/2	6.0	0.49	0.64	3.95	0.47	7.5	0.6 to 1.8
Al	3/2	1.7	0.93	1.23	4.67	0.21	1.5	0.25 to 3.3
Bi	9/2	7.2	1.67 to 1.4	0.93	8.00	0.85	8.9	0.5 to 2.1
Pb	—	9.8	4.6 to 1.9	1.61	5.7	0.85	11	—

and the incoherent scattering cross section is

$$\sigma_{\text{incoherent}}=4\pi\frac{I(I+1)}{(2I+1)^2}(a_+-a_-)^2. \qquad (2)$$

This last vanishes when a_+ is equal to a_-.

The scattering cross section σ is the sum of (1) and (2).

$$\sigma=4\pi\left(\frac{I+1}{2I+1}a_+^2+\frac{I}{2I+1}a_-^2\right). \qquad (3)$$

The scattering of neutrons by micro-crystalline substance is largely due to the Bragg reflections on micro crystals which happen to be oriented in the proper way. Such Bragg reflections are possible only if the wave-length λ is less than $2d$, where d is the maximum lattice spacing of the crystal. Correspondingly, a large drop in the total scattering cross section of micro-crystalline materials is usually observed at this critical wave-length. The cross section usually does not drop to zero, however. The residual scattering is partly caused by the effect of irregularities of the crystal, both permanent and those due to thermal agitation, and partly by the incoherent scattering of Eq. (2), and to the effect of isotopes if they exist.

If the residual cross section could be measured for a substance containing only one isotope and made of perfect crystals at absolute zero, a measurement of the incoherent cross section of Eq. (2) could be obtained. From it one could calculate $|(a_+-a_-)|$. Practically only an upper limit to this quantity can be obtained on account of the unavoidable irregularities of the sample.

Measurements have been performed on the residual cross section of micro-crystalline Be, Al, Bi, and Pb. A beam of neutrons from the thermal column of the Argonne heavy water pile was passed through a BeO filter (100 g/cm²) or a graphite filter (57 g/cm²). In this beam of filtered neutrons the transmissions of the various elements were measured for samples of increasing thickness. In some cases the cross section decreased appreciably with increasing thickness of the sample. This is due to a further filtering of the neutron beam by the sample, as will be discussed later.

The results of the measurement are summarized in Table I. The cross sections observed for neutrons filtered through BeO and through graphite are given in columns 4 and 5, respectively. For comparison the cross section observed for unfiltered neutrons is given in column 3. These last values, in particular those of Bi and Pb, are appreciably decreased by the fact that the samples had a coarse crystalline structure. Consequently there is extinction of the Bragg reflection within the individual single crystals causing an apparent lowering of the cross section.

The cross sections of columns 4 and 5 are much lower than those of column 3 because the filters have removed a large fraction of the neutrons capable of being Bragg-scattered.

For Be, the theoretical edge ($2d$) is 3.95A. Consequently both filters pass only wave-lengths longer than $2d$ and which cannot be Bragg-scattered. The increase in cross section from column 4 to column 5 is probably due to the Doppler effect resulting from thermal agitation of the Be nuclei. The residual cross section 0.47 given in column 7 has been chosen as the lower of these two values corrected for the small absorption of Be.

In the case of Al, the BeO filter does not completely eliminate all neutrons that may have Bragg reflections. Most of the rise in cross section from column 4 to column 5 is due to increase in the $1/v$ absorption Actually the minimum scattering cross section is observed with the graphite filter. This value of 1.23 is decreased by 1.02

because of absorption of the Al. The residual scattering cross section is therefore taken as 0.21.

For Bi, the theoretical edge $2d$ is 8.0A. Therefore neither filter is able to remove completely neutrons that can be Bragg reflected. For BeO filtered neutrons there is a decrease in cross section with increasing thickness of Bi presumably due to a partial filtering of the neutrons by the sample itself. The graphite filter, because of its larger critical wave-length ($2d$), gives a lower cross section. This last has been decreased by 0.08 because of absorption, and therefore 0.85 has been taken as the upper limit to the residual scattering.

The theoretical edge for Pb is at 5.7A. The BeO filter is therefore inadequate as is apparent from the fact that the cross section varies from 4.6 to 1.9 with increasing thickness of the absorber. As residual scattering cross section we have taken the one measured with the graphite filter corrected by 0.76 for absorption.

The values of the residual scattering cross section so obtained are collected in column 7 of the table. In column 8 are listed the scattering cross sections of the bound atoms obtained by correcting the cross sections of the free atoms by the factor $(A+1/A)^2$ for the reduced mass.

For the elements Be, Al, and Bi, which have only one isotope, the residual scattering cross section is due only to spin dependence of the scattering length and to crystalline irregularities. The assumption that the residual cross section is due entirely to spin dependence leads to an overestimate of the difference between a_+ and a_-. The limits for the ratio a_+/a_- given in column 9 are calculated with this assumption and using the data of columns 7 and 8 together with formulae (2) and (3).

In the three cases investigated it is found that a_+ and a_- have the same sign. This is in accordance with the fact[2] that almost all elements have a positive scattering length. If one is allowed to generalize from these few cases, it would appear that the same is true for the scattering lengths for individual spin orientations. This is quite plausible also on theoretical grounds. On the other hand, differences of as much as a factor of 2 in the two scattering lengths produce a relatively small amount of incoherent scattering. The observation of the residual scattering becomes a sensitive method for investigation of spin dependence only when the difference between the two scattering lengths is quite large. It is quite possible that this difference is much less than the maximum values given, and that the observed residual scattering is due largely to crystal irregularities. An analysis of the data for Pb has not been given because this case is complicated by the presence of several isotopes.

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