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Specular Reflection from Mercury Vapor*

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A quantitative study was made of the variation with vapor pressure of the diffusely scattered (absorbed and re-emitted) radiation from mercury vapor and of the regular reflection from a quartz-mercury vapor surface (specular reflection) which had been observed by Wood. The mercury vapor was contained in a quartz cell, and the intensity measurements were made with a photo-cell, with a stabilized low pressure discharge tube used as a source.

Both scattering and specular reflection were present over the range of vapor densities corresponding approximately to the range of condensed mercury temperatures between 100°C and 300°C. The scattered radiation increased up to 50°C and then fell off rapidly and was less than 0.5 percent at 300°C. Specular reflection was less than 0.5 percent below 100°C, and was measurable at 125°C, at which temperature the product of the cube root of the number of atoms per cubic centimeter and the wave-length was approximately seven. Over the range of densities corresponding to the range 150°C to 300°C, most of the energy of the 2537A radiation incident upon the mercury vapor was dissipated in the vapor.

The a.c. method, previously used in this laboratory for measurement of mean lifetimes, was used to study the persistence of the radiation in the quartz cell. For vapor pressures above 75°C there was no apparent change in the form of the curve giving the relation between the emission by the cell of scattered radiation and the elapsed time. Within the resolution of the timing method, about 1/40 of the lifetime of the excited state, the specular reflection showed no delay.

The optical constants, index of refraction and coefficient of absorption, of dense mercury vapor for the 2537A line were calculated from the reflectivity at two different angles of incidence. Their ratio remained nearly constant as the vapor pressure increased.

IT was shown by Wood[1] that at low vapor pressures mercury vapor irradiated with the resonance line 2537A showed only diffuse scattering, while at high pressures this was replaced by specular reflection. He illuminated with the sharp, unreversed line of a water-cooled mercury arc a small quartz bulb containing mercury vapor at various pressures. At low pressures the vapor throughout the entire bulb emitted radiation, but with increasing pressure the reradiating volume contracted toward the surface of entrance of the resonance line. He found that at a pressure corresponding to a condensed mercury temperature of 270°C the diffuse scattering had completely disappeared. However, at this temperature specular reflection by the quartz-mercury vapor surface was observable, and increased to considerable values with increase of pressure. This specular or metallic reflection occurred only at the resonance wave-length.

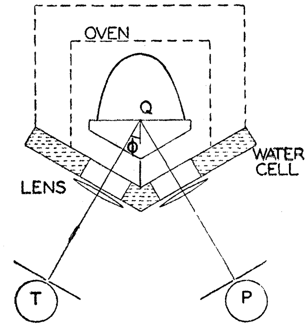

FIG. 1. Apparatus used for measurement of reflectivity. Slit width—1 mm; distance from slit to lens—30 cm. T—source; Q—quartz cell containing mercury vapor; P—photo-cell.

Rump[2] investigated the changes of line form caused by scattering and by reflection as a function of the pressure by means of absorption measurements, and concluded that specular reflection began at a lower temperature than that given by Wood. Schnettler[3] introduced the foreign gases H_2 and O_2, which quench the resonance radiation, and found that they affected the specular reflection only slightly. He interpreted this result as indicating that the lifetime of the excited atoms is not involved in specular reflection.

This paper describes a more detailed quantitative study of the diffuse scattering and of the

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[1] R. W. Wood, *Researches in Physical Optics*, II (Columbia University Press, New York, 1919).

[2] W. Rump, Zeits. f. Physik **29**, 196 (1924).

[3] O. Schnettler, Zeits. f. Physik **65**, 55 (1930).

specular reflection and their dependence upon pressure. A study was also made of the accompanying persistence or delay of the radiation. The use of the term "specular reflection" was found to be somewhat confusing, since in the experiments to be described the reflection from the inner boundary of the quartz container changed gradually from a "quartz-vacuum" reflection to a "quartz-mercury vapor" reflection and experimentally no distinction was made between them. Consequently in this paper the regular reflection will be referred to simply as "reflection." Diffuse scattering due to absorption and re-emission will be referred to simply as "scattering."

METHOD

The mercury vapor under study was contained in a quartz cell Q (see Fig. 1) with a plane front face, with provision made for varying the vapor pressure. Radiation of wave-length 2537A excited in a low pressure tube was allowed to fall on this surface and the scattered and reflected radiation studied. The investigation was divided into two parts: (1) A series of measurements to determine the variation in the reflectivity of the quartz-mercury vapor surface and the variation of the intensity of the scattered radiation with vapor pressure and temperature. (2) A study of the time elapsing between the incidence of the radiation on the mercury vapor and its re-emission.

In the first set of tests, radiation from a source T (see Fig. 1) collimated by a slit and lens, was incident upon the plane quartz-mercury vapor surface. When measuring only the reflection a second lens and slit were used before the photo-cell P, the scattered radiation being practically eliminated because of the small solid angle subtended at the quartz tube by the slit. By removing the second collimator and putting the photo-cell close to the quartz cell Q, intensity measurements were made which included all of the reflected and a considerable fraction of the scattered radiation. Using these two types of measurement the behavior of both the scattered and the reflected radiation could be separately studied.

The measurements of the persistence of the radiation from the mercury vapor in the quartz tube Q were made by a modification of the method previously used in this laboratory for measuring the mean lifetimes of excited atoms.[4] Radiation was produced in a thin layer of mercury vapor in the excitation cell E (see Fig. 2) by electron impact. Some of this radiation was incident upon the quartz tube Q, and the reradiated energy entered the photo-cell P. Alternating voltages were applied by an oscillator O to both the excitation cell E and to the photo-cell P in the same phase. Excitation in E occurred only during the positive half-cycle of the oscillator voltage. The direction of the current in the photo-cell reversed with reversal of the polarity of the oscillator voltage. At low frequencies, for which persistence of radiation had a negligible effect, radiation reached the photo-cell during only the positive half-cycles and there was no reverse photoelectric current. As the frequency was increased, the persistence of the radiation became appreciable and some radiation fell into the negative half-cycles and, since the average output of E was constant, the measured photoelectric current decreased. The result of this measurement is expressed by a curve giving the ratio R of the current at the frequency f to the current at the low frequency for which the effect of persistence was negligible. Such a curve will be referred to as an "R-f" curve. For the purpose of this investigation we can use the formula $R=1-a(1+S)$. This formula is based on the assumption that the photo-cell current changes sharply from positive saturation to negative saturation at zero voltage, which was approximately true. Here a is the fraction of the total radiation falling on the photo-cell in the

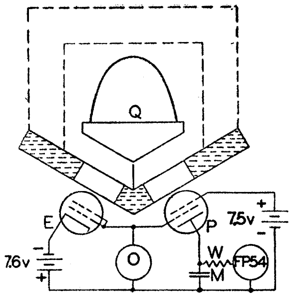

Fig. 2. Apparatus used for measurement of persistence. E—source; O—oscillator; P—photo-cell.

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[4] P. H. Garrett, Phys. Rev. **40**, 779 (1932).

TABLE I.

Temperature (°C)	N (atoms/cm^3)	$N^{1/3}\lambda$	n	κ	L/λ
100	6.9×10^{16}	4.8	1.00	0.00	—
215	5.4×10^{17}	21	0.98	0.045	1.8
300	4.1×10^{18}	43	0.94	0.165	0.48
350	1.0×10^{19}	62	0.89	0.33	0.24
400	2.2×10^{19}	85	0.84	0.53	0.15
450	4.5×10^{19}	112	0.81	0.69	0.12

TABLE II.

Temperature (°C)	N (atoms/cm^3)
0	6.0×10^{12}
10	1.6×10^{13}
26	6.5×10^{13}
45	2.6×10^{14}
50	3.7×10^{14}
75	1.8×10^{15}
115	1.4×10^{16}
150	6.3×10^{16}

negative half-cycles and S is the ratio of the current in the photo-cell for the same intensity of radiation when negative and positive voltages are applied, respectively. For very high frequencies a approaches 0.5 and R approaches $(1-S)/2$.

Since the radiation from the source E had some persistence, it was necessary to calculate the added persistence due to the quartz cell from the shift of the observed R-f curve from the curve obtained from the source E directly.

APPARATUS

The quartz cell[5] Q had an inside diameter at the front face of 3.2 cm. This face was a fused quartz prism of angle 109° so arranged that while only the quartz-mercury vapor boundary contributed to the reflection into the photo-cell, it was still possible to use a large aperture. A small side tube contained a small quantity of liquid mercury the temperature of which was altered to vary the vapor pressure. The upper part of the cell was maintained about ten degrees hotter than the side tube to prevent condensation of mercury on the surface. The temperatures and corresponding vapor pressures given in this paper are those of the liquid mercury. The corresponding density of the vapor expressed as the number of atoms per cubic centimeter (N) is given in Tables I and II.

The quartz cell was placed in a special oven. To protect the other apparatus from heat from the oven, the front wall was a water cell insulated on the inside. The radiation passed into and out of the oven through quartz windows. Each window consisted of a pair of polished, fused quartz plates between which distilled water was circulated. A third polished, fused quartz plate was used inside the furnace and was separated from the inner cooled plate by a thin layer of dead air to eliminate excessive heat losses. At high temperatures it was difficult to prevent the windows from clouding rapidly and quantitative measurements were troublesome. The upper part of the furnace was heated electrically and its temperature regulated manually. The air was stirred by a jet of compressed air. The non-uniformity of temperature throughout this part of the furnace did not exceed twenty degrees at the highest temperature used (450°C). The temperature of the liquid mercury in the side tube was controlled with a thermostat which was designed to allow rapid changes of the operating temperature. The lower part of the furnace was a cylindrical copper block with a hole bored along the axis to receive the side tube containing the liquid mercury. This block was heated by coils wound around it. Its temperature was controlled to about 0.1°C by a thermocouple in circuit with a potentiometer and galvanometer. The galvanometer spot illuminated a photo-cell in such a way as to control an amplifier the output of which was the heater current. The values of the temperature were obtained from the potentiometer settings.

The source T was a mercury vapor arc with a quartz capillary about 1.8 mm in diameter, and was observed transversely to avoid self-reversa of the 2537A line. It was operated at a pressure of 0.009 mm (45°C) and a direct current of about 40 milliamperes. Its output was stabilized by monitoring with a photo-cell which controlled through an amplifier the circuit which furnished the power supply for the arc. The intensity of the arc had no measurable random fluctuation, but showed a steady decrease of about one percent per hour owing to deposits on the surfaces of the capillary. This source was used with a

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[5] Made by the Amersil Company, Inc., Hillside, New Jersey.

Corning filter number 791, which absorbed wavelengths shorter than 2200A. The measurements were corrected for current produced in the photocell *P* by radiation other than the resonance line 2537A, amounting to about four percent of the total current. This arc was used when a high intrinsic intensity without self-reversal was needed for the measurements of reflection.

The photo-cell *P* was fabricated of nickel, and its active surfaces were coated with a film of magnesium. The envelope was of glass which transmitted about forty percent at 2537A. The photoelectric current approximately saturated at plus or minus one volt, and the ratio S of the sensitivity when the grid was negative to that with the grid positive was 0.28. The photoelectric currents were measured by an FP54 tube (see Fig. 2) used as an electrometer. The high frequency voltage was kept off the grid of the electrometer tube by a resistance W of 10^{10} ohms in series with the capacity to ground of the FP54 grid circuit, the impedance of which at the frequencies used was small compared to W. A condenser M of about sixty micro-micro farads completed the alternating current circuit through the photo-cell.

The excitation cell E employed a thermionic cathode, and an anode which consisted of a grid placed close to the cathode and a surrounding nickel mesh. The small distance between the cathode and the grid provided a large space current at low positive anode voltage. The direct anode voltage was 7.6 volts (see Fig. 2), the peak alternating voltage about 8.0 volts, the peak current about 100 milliamperes, and the average current 47 milliamperes. Under these operating conditions the curve of excitation as a function of time approximated half-sine waves. The envelope was of the same high transmission glass as was used for the photo-cell. The vapor pressure in the excitation cell was 1.8×10^{-4} mm, corresponding to 0°C. At this pressure the radiation from the cell had a greater persistence than that which would have resulted from only the single lifetime 1.08×10^{-7} second,[4] owing to the fact that about fifty percent of the radiation from the atoms excited by electron impact was absorbed and re-emitted in the cell.

A comparison of the line widths of the radiation from the sources E and T was made by measurements of absorption with a mercury vapor absorption cell. The absorption coefficient was within four percent of the same value for both sources.

REFLECTION AND SCATTERING

For the measurement of reflection alone, the apparatus of Fig. 1, described above, was used with both collimators. The ratio of the photoelectric current with the mercury vapor in the quartz cell to the current with the vapor frozen out with liquid nitrogen was measured at different temperatures. This gave the ratio of the reflectivity of the quartz-mercury vapor surface to that of the quartz-vacuum surface. Curve A of Fig. 3 shows this ratio as a function of temperature for an angle of incidence, ϕ, on the quartz-mercury vapor surface of 28.3°.

To find the variation with temperature of the scattered radiation, similar measurements were made with the photo-cell P placed as close as possible to the furnace window so that it would receive a maximum of scattered radiation in addition to that reflected. The same angle of incidence was used. The result is shown as curve B in Fig. 3. Curve B coincides with A within the precision of measurement (0.5 percent) for temperatures above 300°C, has a minimum at about 200°C, a maximum at 50°C, and falls to zero at low pressures. The variation with temperature of the scattered radiation is given by the difference between curves A and B, plotted as curve C, the scale of ordinates being shown at the right of the figure. This curve coincides with B below about 100°C. It should be noted, however, that only a fraction of the total scattered radia-

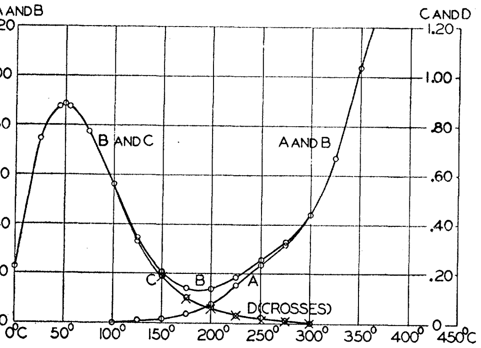

FIG. 3. Curve A—reflected radiation. Curve B—scattered plus reflected radiation. Curve C—difference between A and B. Curve D—delayed radiation.

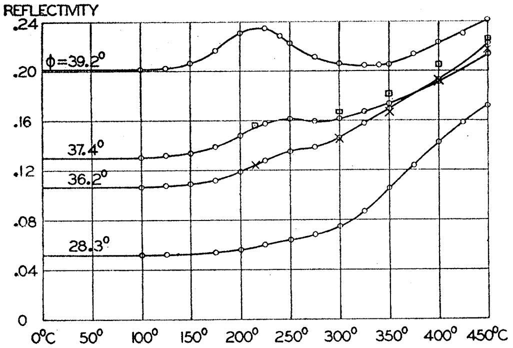

FIG. 4. Reflectivity *vs.* condensed mercury temperature for four values of the angle of incidence ϕ.

tion is involved in this measurement. This fraction could not be accurately computed since it depended upon the geometry of the system, varying somewhat with the vapor density in the cell.

Curves C and A show that the effect of the mercury vapor on the reflection was inappreciable (less than 0.5 percent) below 100°C and scattering disappeared (less than 0.5 percent) at 300°C. In the region 150°C to 300°C both were present, though weak, so that most of the energy of the 2537A radiation entering the quartz cell was dissipated in the vapor.

The absolute reflectivity of the quartz-mercury vapor surface was found by multiplying the ordinates of curve A by the reflectivity of the quartz-vacuum surface calculated from the index of refraction of the fused quartz. The results are shown in Fig. 4, the reflectivity being plotted as a function of the temperature. Curves for three other values of the angle of incidence, $\phi=36.2°$, 37.4°, and 39.2°, are also given. These were determined in the same manner as the values for 28.3°. It is interesting to note that the reflectivity for angles of incidence (39.2° and 37.4°) near the critical angle (41.6°) have each a maximum and a minimum of reflectivity and that the curves for smaller angles show reversals of the curvature at approximately the same values of the pressure.

The values for the mercury vapor of the constants n (index of refraction) and κ (coefficient of absorption) in the Fresnel's reflection formulas[6] were calculated for several values of the vapor density from the values of total reflectivity obtained for $\phi=39.2°$ and 28.3°. These values are given in columns four and five of Table I. In the first, second, and third columns are given the temperature, the number of atoms per cubic centimeter, N, and $N^{\frac{1}{3}}\lambda$. Here λ is the wave-length in the vapor of the 2537A line. In the sixth column is given the distance, L, in wave-lengths, in which the intensity of the resonance line was attenuated in the vapor by the factor e, as calculated from the values of κ.

The estimated error in the values of the optical constants was two percent or less for n, and for κ seven percent at 215°C, and fifteen percent at 400°C. These errors were primarily due to the probable error of 0.3° in determining ϕ, the effect of which on the optical constants increased with ϕ and the reflectivity.

Using the tabulated values of the optical constants, the reflectivities for the angles of incidence 37.4° and 36.2° were calculated. The calculated points for 36.2° are shown in the figure by crosses, which lie close to the experimental curve. The similar points for 37.4° are shown by squares, which lie above the corresponding experimental curve. This discrepancy was attributed to an error of determining the angle ϕ, and would largely disappear were the angle of incidence increased by 0.4°, which is a little greater than the estimated uncertainty in the measurement of the angle.

The values of $1-n$, κ, and the reflectivity for $\phi=28.3°$ are plotted in Fig. 5 as a function of $N^{\frac{1}{3}}\lambda$. From this figure it is seen that the increase of the reflectivity of the surface, caused by the mercury vapor and the attenuation, became measurable at approximately the same density ($N^{\frac{1}{3}}\lambda$ about 5). It shows also that the ratio of

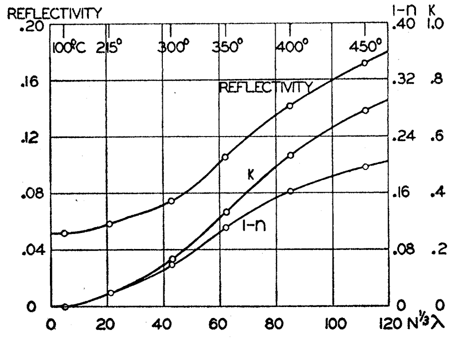

FIG. 5. Reflectivity, $1-n$ and κ as a function of $N^{\frac{1}{3}}\lambda$.

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[6] M. Born, *Optik* (Edwards Brothers, Inc., Ann Arbor, Michigan, 1943), pp. 29, 260.

$1-n$ to κ is almost independent of the increase of vapor pressure. The maxima and minima in the reflectivity curves of Fig. 4 are not dependent upon peculiarities in the variation of n and κ, but are largely due to the fact that the angles of incidence approach the critical angle.

PERSISTENCE

The persistence or delay of the radiation, caused by persistence in the quartz cell, was investigated by obtaining ratio-frequency (*R-f*) curves at temperatures ranging from 0°C to 450°C. The results are shown in Fig. 6. The curve marked "aluminum" was taken with an aluminum reflector substituted for the quartz cell. For this curve the only persistence involved was that in the source E. When the quartz cell was used, the added persistence, caused by the absorption and re-emission in the cell of some of the radiation from E, resulted in a further change in the division between the half-cycles of the illumination of the photo-cell and consequently gave *R-f* curves differing from the aluminum curve. Curves for nine values of the vapor pressure, indicated by the corresponding temperatures, are given in the figure. It is to be noted that as the temperature is increased the curves shift toward lower frequencies until a temperature of approximately 45°C is reached, after which the shift is toward higher frequencies. The 300°C and 450°C curves coincide with the aluminum curve within the precision of measurement of the curves, which was about one-half percent. It will be further noted that the curves differ markedly in shape, especially at the lower frequencies.

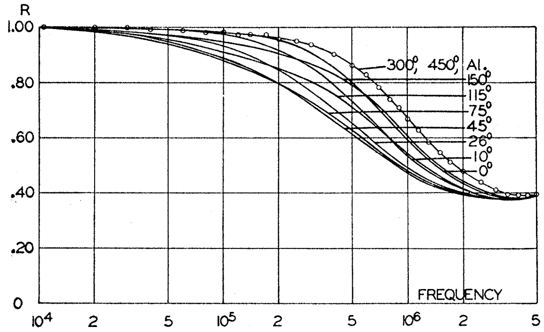

FIG. 6. Observed *R-f* curves for different values of the condensed mercury temperature. Frequency in cycles per second.

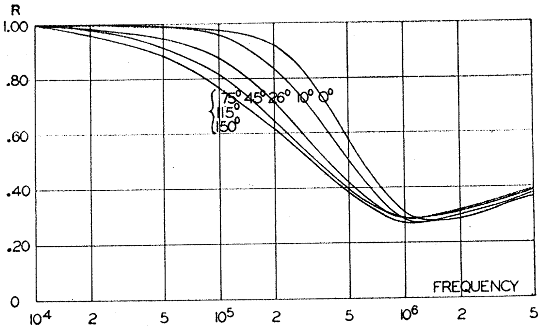

FIG. 7. Observed *R-f* curves for scattered radiation only.

Each of the measured curves, except the aluminum curve, can be considered as the composite of the two curves which would have been obtained if the radiation reflected from the quartz-mercury vapor surface and the radiation absorbed and re-emitted by the vapor in the cell could have been measured separately. The ordinates of the curves are equal to the sum of the corresponding ordinates of these two component curves multiplied by b and $1-b$, respectively, where b is the ratio of the intensity of the reflected energy to the total intensity received by the photo-cell. The values of b for the range 0°C to 150°C were found by measuring the ratio of the photoelectric current with the mercury vapor in the tube to the current when the vapor was frozen out, since below 150°C the reflection was practically the same as from the quartz-vacuum surface. With this data the *R-f* curves, which would have been obtained with the scattered radiation alone, were computed from the curves in Fig. 6, and are shown in Fig. 7. No curves are given for temperatures above 150°C since for these temperatures the scattered radiation was so small a fraction of the total that it was not possible to carry out the calculations with significant results. The 75°C, 115°C, and 150°C curves were indistinguishable, and presumably the curves for the scattered radiation for all higher temperatures would be identical with this limiting curve. Again it will be noted that as the temperature increases the difference between the shape of the curves and of that due to the direct radiation from E becomes more marked. This is especially evident in the higher temperature curves in which the ordinates fall off from unity at low frequencies. The precision

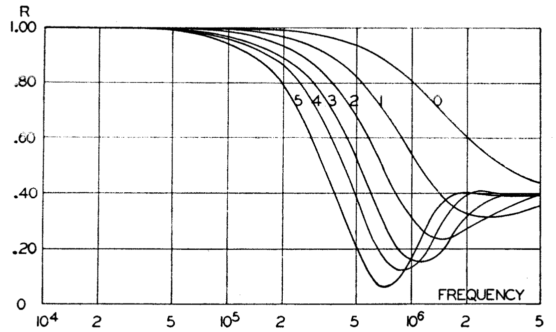

FIG. 8. Calculated *R-f* curves for 0 to 5 absorption re-emission steps.

of the points on the curves of Fig. 7 was estimated to be three percent for the 0°C and 150°C curves, and better for the others.

The maximum of the intensity of the scattered radiation received by the photo-cell occurred at about 50°C (see Fig. 3), which is very nearly the same as the temperature above which all the *R-f* curves have the limiting form and position. For higher temperatures the radiation is absorbed and dissipated in the quartz cell to a large degree, but the relation between the intensity of the radiation which escapes and the elapsed time is apparently unchanged.

It should be noted in discussing Fig. 7 that the forms of these curves for the scattered radiation depend upon the persistence in the source *E* as well as on the added delays in the quartz cell. The most probable process causing these delays is the simple absorption re-emission process with the lifetime 1.08×10^{-7} second. The explanation of the long persistence as caused by the formation of metastable atoms, which because of collisions of the first kind eventually radiate the resonance line, seems untenable in view of the small probability of this process.[7] Each observed curve is therefore considered as the composite of, first, *R-f* curves for the radiation which, emitted without experiencing absorption and re-emission in *E*, was absorbed and re-emitted once in the quartz cell, second, *R-f* curves for radiation which experienced two absorption re-emission processes in the source and in the quartz cell considered as a single system, third, curves for radiation which experienced three such delays, etc. Since we cannot determine the relative intensity of radiation in each of these groups as is necessary for an exact analysis of the experimental curves, we can only estimate these. In Fig. 8 are shown some *R-f* curves which were calculated from the characteristics of *E* and the photo-cell. The curve marked *O* is for the radiation emitted by excited atoms which reaches the photo-cell without experiencing absorption. Curve *1* is for radiation which has experienced one absorption re-emission process after its original emission, curve *2* for radiation which has experienced two absorption re-emission steps, etc. From the shape and position of these curves and of those in Fig. 7 it is obvious that for temperatures of 26° and higher a very large number of steps must have been involved if the form of the curves was due solely to these absorption re-emission processes. Computation of such curves as those of Fig. 8 for a large number of steps was impracticable. However, it can be shown that the intensity of radiation from a group of atoms excited at the time $t=0$ and experiencing m absorption re-emission steps will follow the law $t^m e^{-t/\tau}$, where τ is the lifetime and m is the number of absorption re-emission steps. This expression has a maximum at $t=m\tau$. Since the radiation was excited only during the positive half-cycles, the *R-f* curves will therefore have their minima approximately at the frequencies for which the maxima of $t^m e^{-t/\tau}$ occur one-half period after the excitation, or for $f=1/(2m\tau)$. From this relation it can be estimated that in the case of the limiting curve at least several percent of the radiation must have experienced several hundred absorption re-emission steps. A rough estimate gives the average number of such steps as about 40.

As already seen, the *R-f* curves for aluminum, and for the quartz cell at 450°C and at 300°C in Fig. 6 coincide within 0.5 percent. From the calculated *R-f* curves *0*, *1*, and *2* of Fig. 8 it can be seen that at a frequency of one magacycle the increased persistence resulting from the delay of one lifetime in the quartz cell decreases the ordinate about twenty percent. Since the shift in the curves is approximately proportional to the added persistence, it was estimated from the precision with which the 450°C and 300°C curves were determined, that if any persistence were introduced by the reflection it was less

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[7] C. Kenty, Phys. Rev. **42**, 823 (1932).

than one-fortieth of a lifetime. This, together with the fact that the scattered radiation became immeasurable at approximately the same temperature as did persistence of radiation in the quartz cell, leads us to assume that reflection introduces no delay at any temperature.

As an additional check on the behavior of the scattered radiation from the quartz cell in the region 150°C to 300°C, the alternating current method was employed in a somewhat different manner. The ratio, S, of the photo-current for negative and positive voltages on the grid of the photo-cell was approximately 0.28, and the persistence of the radiation reflected from the quartz-mercury vapor surface (persistence due only to E) resulted in more energy arriving at the photo-cell in the positive half-cycles than in the negative. Hence the reflected radiation $a=a_s\leq\frac{1}{2}$. In this test the phase of the voltage applied to the photo-cell was reversed with respect to that on the excitation cell, so that in the formula for R, S was replaced by $1/S$, i.e., $R=1-a_s(1+1/S)=1-4.57a_s$. Hence $R=0$, for the frequency for which $a_s=0.219$. The frequency was then determined for which the photoelectric current was zero with the quartz cell at 300°C, at which temperature there was no measurable persistence in the quartz cell. Using this frequency in the modified circuit, measurements of the photocurrent for different vapor pressures were made. The results are shown in Fig. 3 by curve D, the observed points being given by crosses. The scale of ordinates of D, shown at the right of the figure, is of course arbitrary and was adjusted to coincide with C at 150°C. The two curves are coincident within the precision of measurement. This result agrees with the conclusions that reflection was instantaneous for all pressures, and that the relation between the intensity of the absorbed and re-emitted (scattered) energy and the time which gave the limiting R-f curve of Fig. 7 remained unchanged at temperatures above 150°C.

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The Electromagnetic Shift of Energy Levels

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By very beautiful experiments, Lamb and Retherford[1] have shown that the fine structure of the second quantum state of hydrogen does not agree with the prediction of the Dirac theory. The $2s$ level, which according to Dirac's theory should coincide with the $2p_{\frac{1}{2}}$ level, is actually higher than the latter by an amount of about 0.033 cm^{-1} or 1000 megacycles. This discrepancy had long been suspected from spectroscopic measurements.[2,3] However, so far no satisfactory theoretical explanation has been given. Kemble and Present, and Pasternack[4] have shown that the shift of the $2s$ level cannot be explained by a nuclear interaction of reasonable magnitude, and Uehling[5] has investigated the effect of the "polarization of the vacuum" in the Dirac hole theory, and has found that this effect also is much too small and has, in addition, the wrong sign.

Schwinger and Weisskopf, and Oppenheimer have suggested that a possible explanation might be the shift of energy levels by the interaction of the electron with the radiation field. This shift comes out infinite in all existing theories, and has therefore always been ignored. However, it is possible to identify the most strongly (linearly) divergent term in the level shift with an electromagnetic *mass* effect which must exist for a bound as well as for a free electron. This effect should

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[1] Phys. Rev. **72**, 241 (1947).

[2] W. V. Houston, Phys. Rev. **51**, 446 (1937).

[3] R. C. Williams, Phys. Rev. **54**, 558 (1938).

[4] E. C. Kemble and R. D. Present, Phys. Rev. **44**, 1031 (1932); S. Pasternack, Phys. Rev. **54**, 1113 (1938).

[5] E. A. Uehling, Phys. Rev. **48**, 55 (1935).