In conclusion we can say that the magnetic deflection of a beam of charged particles in magnetized iron should be quantitatively observable if high energy protons or deuterons are used instead of mesons.[1] A modification of the generally accepted result

$$\mathfrak{b} = \mathbf{B}$$

may be expected if there are short range forces modifying interpenetration of proton and electron. The Coulomb force alone will give such an effect, but it appears barely at the threshold of observation.

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Interpretation of the Triton Moment*

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In order to account for the measured magnetic moment of the triton it is necessary to assume that the wave function in the ground state is a linear combination of ^2S, ^2P, ^4P, and ^4D functions. An attempt is made to determine the amplitudes of these functions from the magnetic moment on the assumption that the intrinsic nucleon moments are additive and relativistic effects are negligible. With certain reasonable assumptions concerning the nature of the wave functions, it is found that the relative probabilities for finding the system in the ^2P, ^4P, and ^4D states satisfy the relation shown by the curves in Fig. 1. Wherever the results would otherwise be arbitrary, the wave functions have been chosen in such a way as to minimize the amount of P state, with the exception that only the lowest one-particle configurations have been considered. If the amplitude of the ^2S state is taken to be as large as possible, the wave function contains no ^4D state, 8 percent ^4P state, and 17 percent ^2P state. A wave function of this form would seem to indicate that there is a spin-orbit coupling other than the tensor interaction acting among nuclear particles. In the other extreme case that the wave function contains a maximum of the ^4D function, the ^2S state probability is zero, the ^4D probability is 22 percent, the ^4P is 30 percent, and the ^2P is 48 percent. If the wave function of He^3 has the same form as that of H^3, the He^3 moment would be expected to lie on one of the curves shown in Fig. 2.

1. INTRODUCTION

THE recent measurements[1,2] of the magnetic moment of the triton give a value about 6.7 percent greater than that of the proton. If the ground state of the triton were a pure $^2S_{\frac{1}{2}}$ state, it would be expected that the moment would be equal to the proton moment. It is believed, of course, that the ground state is not a pure ^2S state but contains an admixture of ^2P, ^4P, and ^4D states.[3] A theory based on simplifying assumptions leads[4] to the conclusion that the presence of these states should result in a reduction of the moment instead of the observed increase. However, it has been pointed out[5] that cross terms between the various states in the expression for the magnetic moment have been neglected in the simple theory. These may be positive and could, therefore, account for the large moment.

It is the purpose of this paper to obtain a general expression for the magnetic moment in terms of the amplitudes of the various wave functions and thereby to gain some information concerning the nature of the ground state wave

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function of the triton. The expression for the moment will be found to consist of a sum of terms of three different types. The first are the diagonal elements which are uniquely given in terms of the constant amplitudes of the wave functions. The second are cross terms which involve overlap integrals between the "radial" parts of the wave functions. These "radial" wave functions actually are not purely radial but are also functions of the cosine, q, of the angle between the vector connecting the two neutrons and the vector connecting the proton to the center of mass of the two neutrons.

The third set of terms consists of cross terms involving overlap integrals between one radial function and the derivative with respect to q of another such function. These may be very large if the wave functions contain very high configurations, that is, if the individual particles have very high orbital angular momenta. However, it seems likely that such high configurations do not occur in the ground state, since in the ground state the wave function adjusts itself in such a way as to minimize the kinetic energy of the system. For that reason, it will be assumed in the final analysis that these cross terms vanish, or, more specifically, that the radial functions do not depend on q. This assumption eliminates a great deal of the arbitrariness from the results.

Considering then terms of only the first two types, it is found that the observed moment can be accounted for only if the D state probability is less than that of either the ^2P or ^4P states. This conclusion appears to be at variance with current ideas concerning the nature of the triton wave function.[3] If it is accepted that the interaction term responsible for the mixing of states is the tensor interaction, then the ^4D state would be directly coupled to the ^2S state but the P states would not be. Therefore, it might be expected that the D state probability would be larger than the P probabilities.

This expectation is based on the premise that the wave function is predominately a ^2S state. There is the possibility that the wave function contains little or no ^2S state; that is, that the advantage gained through the large average value of the tensor interaction in the P and D states might be large enough to over-compensate the correspondingly large kinetic energy, in which case the energy would be a minimum for a small S state probability.

Further information concerning these questions may be obtained experimentally by means of a measurement of the moment of He^3. This paper includes a discussion of the relation between the moment of He^3 and the various possible mixtures of states which are consistent with the observed moment of H^3.

In this discussion, no consideration is given to the possibility that the intrinsic moments of the neutron and proton are not additive. Also, the relativistic correction to the triton moment is ignored.[6]

2. THE WAVE FUNCTIONS

The possible forms of the triton wave functions, with respect to their dependence on the spins of the particles, have been given by Gerjuoy and Schwinger.[3] We denote by $\boldsymbol{\rho}$, the unit vector in the direction of the distance between the two neutrons and $\mathbf{r}$, the unit vector in the direction of the distance from the center of gravity of the neutrons to the proton. If $\boldsymbol{\sigma}_3$ is the Pauli spin operator for the proton and $\boldsymbol{\sigma}_{12}=(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)/2$, where $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ are the Pauli operators for the two neutrons, then the wave functions have the following form:

$$^2S:\quad \psi_1=\psi f_1, \tag{1a}$$

$$\psi_2=(\boldsymbol{\sigma}_{12}\cdot\boldsymbol{\sigma}_3)(\mathbf{r}\cdot\boldsymbol{\rho})\psi f_2, \tag{1b}$$

$$^2P:\quad \psi_3=\boldsymbol{\sigma}_3\cdot(\mathbf{r}\times\boldsymbol{\rho})(\mathbf{r}\cdot\boldsymbol{\rho})\psi f_3, \tag{1c}$$

$$\psi_4=[\boldsymbol{\sigma}_{12}\cdot\mathbf{r}\times\boldsymbol{\rho}+i(\boldsymbol{\sigma}_3\cdot\mathbf{r})(\boldsymbol{\sigma}_{12}\cdot\boldsymbol{\rho}) -i(\boldsymbol{\sigma}_3\cdot\boldsymbol{\rho})(\boldsymbol{\sigma}_{12}\cdot\mathbf{r})]\psi f_4, \tag{1d}$$

$$^4P:\quad \psi_5=\left[\boldsymbol{\sigma}_{12}\cdot\mathbf{r}\times\boldsymbol{\rho}+\frac{i}{2}(\boldsymbol{\sigma}_{12}\cdot\mathbf{r})(\boldsymbol{\sigma}_3\cdot\boldsymbol{\rho}) -\frac{i}{2}(\boldsymbol{\sigma}_3\cdot\mathbf{r})(\boldsymbol{\sigma}_{12}\cdot\boldsymbol{\rho})\right]\psi f_5, \tag{1e}$$

$$^4D:\quad \psi_6=[(\boldsymbol{\sigma}_{12}\cdot\mathbf{r})(\boldsymbol{\sigma}_3\cdot\mathbf{r})-(\boldsymbol{\sigma}_{12}\cdot\boldsymbol{\rho})(\boldsymbol{\sigma}_3\cdot\boldsymbol{\rho})] \times(\mathbf{r}\cdot\boldsymbol{\rho})\psi f_6, \tag{1f}$$

$$\psi_7=[(\boldsymbol{\sigma}_{12}\cdot\mathbf{r})(\boldsymbol{\sigma}_3\cdot\mathbf{r})+(\boldsymbol{\sigma}_{12}\cdot\boldsymbol{\rho})(\boldsymbol{\sigma}_3\cdot\boldsymbol{\rho}) -\tfrac{2}{3}(\boldsymbol{\sigma}_{12}\cdot\boldsymbol{\sigma}_3)](\mathbf{r}\cdot\boldsymbol{\rho})\psi f_7, \tag{1g}$$

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$$\psi_8=[(\boldsymbol{\sigma}_{12}\cdot\mathbf{r})(\boldsymbol{\sigma}_3\cdot\boldsymbol{\rho})+(\boldsymbol{\sigma}_{12}\cdot\boldsymbol{\rho})(\boldsymbol{\sigma}_3\cdot\mathbf{r})-\tfrac{2}{3}(\mathbf{r}\cdot\boldsymbol{\rho})(\boldsymbol{\sigma}_{12}\cdot\boldsymbol{\sigma}_3)]\psi f_8, \quad (1\mathrm{h})$$

$$\psi_9=[(\boldsymbol{\sigma}_{12}\cdot\mathbf{r}\times\boldsymbol{\rho})(\boldsymbol{\sigma}_3\cdot\mathbf{r}\times\boldsymbol{\rho})-\tfrac{1}{3}(\mathbf{r}\times\boldsymbol{\rho})^2(\boldsymbol{\sigma}_{12}\cdot\boldsymbol{\sigma}_3)](\mathbf{r}\cdot\boldsymbol{\rho})\psi f_9. \quad (1\mathrm{i})$$

The function ψ is given by

$$\psi=(4\pi\sqrt{2})^{-1}(\chi_1^+\chi_2^- - \chi_1^-\chi_2^+\chi_3^m, \quad (2)$$

where the χ are the spin wave functions of the indicated particles. The functions f_j are functions of the distances corresponding to $\mathbf{r}$ and $\boldsymbol{\rho}$, and they are also even functions of the quantity $q=(\mathbf{r}\cdot\boldsymbol{\rho})$. For simplicity, they will be described as "radial" functions. The extra factors $(\mathbf{r}\cdot\boldsymbol{\rho})$ which are displayed in these equations, but not in those tabulated by Gerjuoy and Schwinger, are introduced in order to satisfy the Pauli principle for the two neutrons. The normalization conditions for the radial functions take the form

$$\tfrac{1}{2}\int |f_1|^2=1, \quad (3\mathrm{a})$$

$$\tfrac{3}{2}\int q^2|f_2|^2=1, \quad (3\mathrm{b})$$

$$\tfrac{1}{2}\int q^2(1-q^2)|f_3|^2=1, \quad (3\mathrm{c})$$

$$\tfrac{3}{2}\int (1-q^2)|f_4|^2=1, \quad (3\mathrm{d})$$

$$\tfrac{3}{4}\int (1-q^2)|f_5|^2=1, \quad (3\mathrm{e})$$

$$\int q^2(1-q^2)|f_6|^2=1, \quad (3\mathrm{f})$$

$$\tfrac{1}{3}\int (1+3q^2)q^2|f_7|^2=1, \quad (3\mathrm{g})$$

$$\tfrac{1}{3}\int (3+q^2)|f_8|^2=1, \quad (3\mathrm{h})$$

$$\tfrac{1}{3}\int (1-q^2)^2q^2|f_9|^2=1. \quad (3\mathrm{i})$$

The integrals indicated in these conditions are to be taken over the variable q (limits: -1 to $+1$) and over the magnitude of the distance between the neutrons and the magnitude of the distance from the center of the neutrons to the proton.

The wave function in the ground state of the triton is expected to be a linear combination of the nine functions given in Eq. (1); that is,

$$\Psi=\sum_{j=1}^{9}\alpha_j\psi_j. \quad (4)$$

The coefficients, α_j, and the form of the functions, f_j, could only be determined by solving the Schroedinger equation for the three-body problem. It is our purpose to express the magnetic moment of the triton in terms of the α_j and certain integrals over the f_j. Then we can hope to get some idea concerning the quantities α_j and f_j from the observed moment. The magnetic moment of the nucleus is given by

$$\mu=\mu_p(\Psi,\sigma_3^z\Psi)+\mu_n(\Psi,[\sigma_1^z+\sigma_2^z]\Psi)+(\Psi,L_3^z\Psi), \quad (5)$$

where L_3^z is the z-component of the orbital angular momentum of the proton, μ_p is the magnetic moment of the proton, and μ_n is the magnetic moment of the neutron. In this expression, the wave function, Ψ, is taken to be that function for which the magnetic quantum number of the total angular momentum is $+\frac{1}{2}$. By making use of Eq. (4), the magnetic moment can be expressed in terms of the matrix elements of the spin and orbital angular moment operators. Thus

$$\mu=\sum_{j,k}\alpha_j^*\alpha_k\{\mu_p(j|\sigma_3^z|k)+\mu_n(j|\sigma_1^z+\sigma_2^z|k)+(j|L_3^z|k)\}, \quad (6)$$

where the j, k refer to the wave functions ψ_j, ψ_k. In the next section it will be shown that a considerable fraction of these matrix elements vanish, so that this expression is not quite so formidable as it looks on first sight.

3. THE MATRIX ELEMENTS

In evaluating the matrix elements which appear in Eq. (6), it is convenient first to determine which of the elements vanish. The only diagonal elements that would be expected to vanish are those corresponding to the mixing of two S states by an orbital angular momentum operator. Thus:

$$({}^2S|L_3{}^z|{}^2S)=0, \tag{7}$$

where the indicated ${}^2S$ states refer to an arbitrary linear combination of the functions ψ_1 and ψ_2.

If the states to be mixed are orthogonal in the space coordinates, then the matrix element of the spin operators vanishes, since the spin operator will not remove the space orthogonality. Similarly, the matrix elements of the orbital angular momentum vanish if the functions are orthogonal in spin. It follows that:

$$\begin{gathered}({}^2S|\sigma_j{}^z|{}^2P)=0, \quad ({}^2S|\sigma_j{}^z|{}^4P)=0,\\ ({}^2S|\sigma_j{}^z|{}^4D)=0, \quad ({}^2P|\sigma_j{}^z|{}^4D)=0,\\ ({}^4P|\sigma_j{}^z|{}^4D)=0,\end{gathered} \tag{8}$$

where $j=1, 2, 3$. Also:

$$\begin{gathered}({}^2S|L_3{}^z|{}^4P)=0, \quad ({}^2S|L_3{}^z|{}^4D)=0,\\ ({}^2P|L_3{}^z|{}^4P)=0, \quad ({}^2P|L_3{}^z|{}^4D)=0.\end{gathered} \tag{9}$$

It is now possible to resort to symmetry arguments to show that other elements vanish. The operators, $\sigma_j{}^z$ and $L_3{}^z$, do not involve $\boldsymbol{\varrho}$ (see Eq. (12)), so they are unchanged by the transformation $\boldsymbol{\varrho}\rightarrow-\boldsymbol{\varrho}$. Therefore, if the functions to be mixed by the matrix element have opposite symmetry under this transformation, the element vanishes. A study of the functions given in Eq. (1) leads to the new results:

$$(1|\sigma_j{}^z|2)=0, \quad (3|\sigma_j{}^z|4)=0, \quad (3|\sigma_j{}^z|5)=0, \tag{10}$$

and

$$(1|L_3{}^z|4)=0, \ (2|L_3{}^z|3)=0, \ (3|L_3{}^z|4)=0. \tag{11}$$

It will be noted that, apart from the factors f_k, each of the functions $\psi_1\cdots\psi_9$ is either symmetric or antisymmetric under interchange of $\mathbf{r}$ and $\boldsymbol{\varrho}$. It has already been pointed out[4] that use can be made of this property, since the operator $L_3{}^z$ is given, in units of $\hbar$, by

$$L_3{}^z=2\left(x\frac{\partial}{\partial y}-y\frac{\partial}{\partial x}\right)\Big/3i, \tag{12}$$

while the z-component of the total angular momentum is given by

$$L^z=\left[\left(x\frac{\partial}{\partial y}-y\frac{\partial}{\partial x}\right)+\left(\xi\frac{\partial}{\partial \eta}-\eta\frac{\partial}{\partial \xi}\right)\right]\Big/i, \tag{13}$$

where x, y, z are the components of $\mathbf{r}$ and ξ, η, ζ, those of $\boldsymbol{\varrho}$. The two terms in Eq. (13) clearly have the same matrix element between two wave functions, both of which are either symmetric or antisymmetric under interchange of $\mathbf{r}$ and $\boldsymbol{\varrho}$. Thus the matrix element of either term is one-half the matrix element of L^z. It follows that for two such functions, ψ_l and ψ_m:

$$(l|L_3{}^z|m)=\tfrac{1}{3}(l|L^z|m). \tag{14}$$

The functions ψ_5 and ψ_6 are both antisymmetric under interchange of $\mathbf{r}$ and $\boldsymbol{\varrho}$, so they satisfy the condition for the validity of Eq. (14). In addition, ψ_5 and ψ_6 are orthogonal, so that the matrix element of the z-component of the orbital angular moment vanishes, since both functions are proper functions of this operator:

$$(5|L^z|6)=0. \tag{15}$$

Then, according to Eq. (14),

$$(5|L_3{}^z|6)=0. \tag{16}$$

Equation (14) may also be used to evaluate the diagonal elements of $L_3{}^z$. Considering first the ${}^2P$ functions, both are seen to be antisymmetric under interchange of $\mathbf{r}$ and $\boldsymbol{\varrho}$, so any linear combination has the same property. Thus

$$({}^2P|L_3{}^z|{}^2P)=\tfrac{1}{3}({}^2P|L^z|{}^2P)=2/9. \tag{17}$$

Similarly,

$$({}^4P|L_3{}^z|{}^4P)=\tfrac{1}{3}({}^4P|L^z|{}^4P)=-1/9. \tag{18}$$

The situation is not quite so simple for the ${}^4D$ functions. ψ_6 is antisymmetric and ψ_7, ψ_8, ψ_9 are symmetric under the operation being considered. Therefore, there are cross terms between ψ_6 and the other three functions which depend on the more detailed properties of the functions. The other terms can be evaluated by the above method. If ${}^4D'$ denotes an arbitrary linear combination of ψ_7, ψ_8, and ψ_9, then:

$$\begin{aligned}(6|L_3{}^z|6)&=({}^4D'|L_3{}^z|{}^4D')\\ &=({}^4D'|L^z|{}^4D')=1/3.\end{aligned} \tag{19}$$

It is also a simple matter to evaluate the diagonal elements of $\sigma_j{}^z$ in the quartet states, since these functions are symmetric in $\sigma_1{}^z$, $\sigma_2{}^z$, and σ_3^z. Therefore,[7]

$$({}^4P|\sigma_j^z|{}^4P)=\tfrac{2}{3}({}^4P|S^z|{}^4P)=5/9 \quad (20)$$

and

$$({}^4D|\sigma_j^z|{}^4D)=\tfrac{2}{3}({}^4D|S^z|{}^4D)=-1/3. \quad (21)$$

The values of the diagonal elements of the spin matrices in the states ψ_1 and ψ_3 may be obtained immediately by noting that both functions are antisymmetric for interchange of σ_1^z and σ_2^z. Therefore,

$$(1|\sigma_1^z+\sigma_2^z|1)=(3|\sigma_1^z+\sigma_2^z|3)=0. \quad (22)$$

On the other hand, the sum of the three σ_j^z must be equal to twice the average value of S^z, or

$$(1|\sigma_3^z|1)=2({}^2S|S^z|{}^2S)=1 \quad (23)$$

and

$$(3|\sigma_3^z|3)=2({}^2P|S^z|{}^2P)=-1/3. \quad (24)$$

All other matrix elements may be obtained by direct computation. Since the required calculation is tedious and not at all illuminating, it will not be presented here. The results are:

$$\begin{aligned}&(2|\sigma_3^z|2)=-\tfrac{1}{3}, \quad (2|\sigma_1^z+\sigma_2^z|2)=4/3,\\&(4|\sigma_3^z|4)=\tfrac{1}{9}, \quad (4|\sigma_1^z+\sigma_2^z|4)=-4/9,\\&(5|\sigma_3^z|4)=\tfrac{2}{3}\int(1-q^2)f_5{}^*f_4,\\&(5|\sigma_1^z+\sigma_2^z|4)=-\tfrac{2}{3}\int(1-q^2)f_5{}^*f_4\end{aligned} \quad (25)$$

for the spin elements. The elements of the orbital angular momentum operator are:

$$(3|L_3^z|1)=\frac{1}{9i}\int(1-q^2)qf_3{}^*f_1', \quad (26a)$$

$$(4|L_3^z|2)=\frac{1}{3i}\int(1-q^2)f_4{}^*(f_2+qf_2'), \quad (26b)$$

$$(7|L_3^z|5)=\frac{2}{9i}\int q^2f_7{}^*f_5-\frac{1}{9i}\int q(1-q^2)f_7{}^*f_5', \quad (26c)$$

$$(8|L_3^z|5)=\frac{1}{18i}\int(3+q^2)f_8{}^*f_5-\frac{1}{9i}\int q(1-q^2)f_8{}^*f_5', \quad (26d)$$

$$(9|L_3^z|5)=-\frac{1}{18i}\int q^2(1-q^2)f_9{}^*f_5+\frac{1}{9i}\int q(1-q^2)^2f_9{}^*f_5', \quad (26e)$$

$$(6|L_3^z|7)=\frac{5}{9}\int q^2(1-q^2)f_6{}^*f_7+\frac{2}{9}\int q^3(1-q^2)f_6{}^*f_7', \quad (26f)$$

$$(6|L_3^z|8)=\frac{2}{9}\int q(1-q^2)f_6{}^*f_8', \quad (26g)$$

$$(6|L_3^z|9)=-\frac{1}{9}\int q^2(1-q^2)f_6{}^*f_9, \quad (26h)$$

where f_j' is the derivative of f_j with respect to q.

[7] The factor 2 arises from the fact that the Pauli operator, σ^z, is twice the spin.

4. THE MAGNETIC MOMENT

In terms of these matrix elements, the magnetic moment is given by Eq. (6). In order to bring out explicitly the symmetry character of the wave functions, we set

$$\begin{aligned}&\alpha_1=a_1S, \quad \alpha_2=a_2S, \quad \alpha_3=a_3\,{}^2P, \quad \alpha_4=a_4\,{}^2P,\\&\alpha_5=a_5\,{}^4P, \quad \alpha_6=a_6D, \quad \alpha_7=a_7D, \quad \alpha_8=a_8D,\\&\alpha_9=a_9D,\end{aligned} \quad (27)$$

where S^2 is the probability of finding the system in the ${}^2S$ state, $({}^2P)^2$ that of finding it in the ${}^2P$ state, etc. The numbers S, ${}^2P$, ${}^4P$, and D are chosen to be real. They must satisfy the normalization condition

$$S^2+{}^2P^2+{}^4P^2+D^2=1. \quad (28)$$

The a_k also must be normalized as follows:

$$|a_1|^2+|a_2|^2=1, \quad (29)$$

$$|a_3|^2+|a_4|^2=1, \quad (30)$$

$$|a_5|^2=1, \quad (31)$$

$$|a_6|^2+|a_7|^2+|a_8|^2+|a_9|^2 + \tfrac{2}{3}\Re\Big\{4a_7^*a_8\int q^2f_7^*f_8 - a_7^*a_9\int(1-q^2)q^2f_7^*f_9 - a_8^*a_9\int(1-q^2)q^2f_8^*f_9\Big\}=1. \quad (32)$$

$\Re\{\ \}$ denotes the real part of the quantity contained in the bracket. Equation (32) has a relatively complicated form because the functions ψ_7, ψ_8, and ψ_9 are not orthogonal to one another.

In terms of the coefficients defined by Eq. (27), the magnetic moment is:

$$\mu=\mu_p-\frac{4}{3}|a_2|^2S^2(\mu_p-\mu_n) - \frac{4}{3}\,{}^2P^2[\mu_p-\tfrac{1}{3}|a_4|^2(\mu_p-\mu_n)] + \frac{2}{9}\,{}^4P^2(5\mu_n-2\mu_p)-\frac{2}{3}D^2(2\mu_p+\mu_n) + \frac{2}{9}\,{}^2P^2-\frac{1}{9}\,{}^4P^2+\frac{1}{3}D^2+\mu_x, \quad (33)$$

where μ_x contains the cross terms. This last quantity is given by

$$\mu_x=\frac{2}{9}D^2\Re\Big\{a_6^*a_7\Big[5\int q^2(1-q^2)f_6^*f_7 + 2\int q^3(1-q^2)f_6^*f_7'\Big] + 2a_6^*a_8\int q(1-q^2)f_6^*f_8' - a_6^*a_9\int q^2(1-q^2)f_6^*f_9\Big\}$$
$$+\frac{4}{3}(\mu_p-\mu_n)\,{}^4P\,{}^2P\Re\Big\{a_5^*a_4\int(1-q^2)f_5^*f_4\Big\}$$
$$+\frac{2}{3}\,{}^2PS\mathcal{I}\Big\{\frac{1}{3}a_3^*a_1\int(1-q^2)qf_3^*f_1' + a_4^*a_2\int(1-q^2)f_4^*(f_2+qf_2')\Big\}$$
$$+\frac{1}{9}D\,{}^4P\mathcal{I}\Big\{2a_7^*a_5\Big[2\int q^2f_7^*f_5 \int q(1-q^2)f_7^*f_5'\Big] + a_8^*a_5\Big[\int(3+q^2)f_8^*f_5 - 2\int q(1-q^2)f_8^*f_5'\Big] - a_9^*a_5\Big[q^2(1-q^2)f_9^*f_5 - 2\int q(1-q^2)^2f_9^*f_5'\Big]\Big\}. \quad (34)$$

Here, $\mathcal{I}\{\ \}$ is the imaginary part of the expression in the bracket.

The expression Eq. (34) is so complicated that some assumptions concerning the wave function must be made in order to simplify it. We assume that the functions f_k are independent of q or

$$f_k'=0. \quad (35)$$

This assumption appears to be reasonable, since the wave function of the ground state will have such a form that the kinetic energy of the system is as small as possible. Therefore, it should be a very smooth function, in which case Eq. (35) would be approximately valid.

The functions f_k also depend on the magnitudes of the distances between the particles. In accordance with our assumption that these functions are smooth, it seems reasonable to assume that they all have about the same shape. Therefore, we take

$$\int f_j^*f_k=\Big[\int|f_j|^2\int|f_k|^2\Big]^{\frac{1}{2}}. \quad (36)$$

According to the well-known Schwartz inequality, this equation gives an upper limit on the magnitudes of the integrals involved. Consequently, the magnitudes of the coefficients of the cross terms in the expression for the magnetic moment are no greater than the values given by Eq. (36), so the estimates obtained below of the amount of admixed ^2P, ^4P, and ^4D states are lower limits.

If we now set

$$a_k = x_k + iy_k, \tag{37}$$

the cross terms in the magnetic moment become

$$\mu_x = \frac{2}{3\sqrt{3}} D^2 \left[\frac{5}{\sqrt{7}}(x_6x_7 + y_6y_7) - \frac{\sqrt{7}}{2}(x_6x_9 + y_6y_9) \right] + \frac{8\sqrt{2}}{9}(\mu_p - \mu_n)\, {}^4P\, {}^2P(x_5x_4 + y_5y_4) + \frac{2\sqrt{5}}{9} {}^4PD \left[\frac{2}{\sqrt{7}}(x_7y_5 - x_5y_7) + (x_8y_5 - x_5y_8) - \frac{\sqrt{7}}{10}(x_9y_5 - x_5y_9) \right]. \tag{38}$$

Here, use has been made of the normalization conditions given by Eq. (3) as well as the assumption Eq. (36). The normalization condition expressed by Eq. (32) now has the form

$$\sum_{j=6}^{9} (x_j^2 + y_j^2) + \frac{4}{\sqrt{7}}(x_7x_8 + y_7y_8) - (x_7x_9 + y_7y_9) - \frac{\sqrt{7}}{5}(x_8x_9 + y_8y_9) = 1. \tag{39}$$

The magnetic moment is still given by Eq. (33), with $|a_2|^2 = x_2^2 + y_2^2$ and $|a_4|^2 = x_4^2 + y_4^2$. The constants, x_j, y_j, are to be chosen in such a manner that they satisfy the conditions of Eqs. (29) to (31) and Eq. (39), and that they give the correct value of the magnetic moment, i.e.,[1]

$$\mu = 1.067\mu_p. \tag{40}$$

The eighteen constants are clearly not determined by these five conditions, so some further assumptions may be made in order to make the final results somewhat more specific.

The fact that the coefficient in Eq. (40) is larger than unity does lead to a considerable limitation on the choice of the constants, since the diagonal terms in Eq. (33) tend to reduce the moment below the proton moment. Therefore, it is necessary to take the non-diagonal terms to be positive and rather large. In order to minimize the negative diagonal terms in Eq. (33) we are led to choose

$$a_2 = 0, \quad |a_4|^2 = 1. \tag{41}$$

Since it seems likely that the amount of S state will be as large as possible, we might require that the constants x_j, y_j be chosen in such a way as to lead to the largest possible value of S^2. There is also some reason to guess that the D state probability will be large compared to the ^2P and ^4P probabilities.[3] Although it will be found that this condition cannot be satisfied, we will choose the values of the constants in such a way as to make D^2 as large as possible just to see how closely we can approach the desired result. No simple analytical method was found for choosing the constants x_j, y_j in such a way that D^2 would turn out to be a maximum. For this purpose, it is desirable to make the coefficients of the terms containing D in Eq. (38) as large as is consistent with Eq. (39). It was found by examination that the maximum amount of D state resulted when

$$\begin{aligned} &x_6 = y_6 = x_7 = y_7 = x_9 = y_9 = 0, \\ &x_5 = x_4 = -y_8, \\ &y_5 = y_4 = x_8. \end{aligned} \tag{42}$$

With these values of the constants, the relation between the amplitudes of the P and D states which is given by Eq. (40) is

$$2.12\,{}^4P\,{}^2P + 0.178\,{}^4PD - [0.50\,{}^2P^2 + 1.25\,{}^4P^2 + 0.759D^2] = 0.067, \tag{43}$$

where we have taken $\mu_p = 2.79$ and $\mu_n/\mu_p = -0.685$. The values of $^4P^2$, $^2P^2$, and D^2 which are given by this equation are shown in Fig. 1. It should be emphasized that these are not the only possible combinations of these constants which will agree with the triton moment because the choice of the x_j, y_j which has been made is rather special.

It is to be noted that the S state probability will be a maximum for $D^2 = 0$, $^4P^2 = 8$ percent and $^2P^2 = 17$ percent. D^2 is never as large as $^2P^2$, and it is at most equal to $4P^2$ in spite of the fact that the coefficients x_j, y_j have been chosen in such a way as to lead to as large a value of D^2 as possible. The largest value of D^2 is 22 percent with $^4P^2 = 30$ percent and $^2P^2 = 48$ percent. In this case the wave function contains no S state.

It is generally believed that the properties of the wave function of He^3 are the same as those of the wave functions of the triton. Therefore, the conclusions drawn here concerning the admixture of states in the triton may be assumed to hold also for He^3. This makes it possible to make certain predictions concerning the magnetic moment of He^3. It has been shown that the moment of He^3 is given in terms of the moment of H^3 by the relation[4]

$$\mu(He^3)+\mu(H^3)=\mu_p+\mu_n-2(\mu_p+\mu_n-\tfrac{1}{2})\times(3D^2-{}^4P^2+2^2P^2)/3. \quad (44)$$

The consequences of this equation are demonstrated in Fig. 2 which shows the relation between the moment of He^3 and the amounts of ${}^2P$, ${}^4P$, and ${}^4D$ states which satisfy the relationship shown in Fig. 1. These are not the only possible values for the He^3 moment, since certain specific assumptions have been made concerning the wave function in order to obtain Fig. 1. The value of the He^3 moment to be expected on the basis of the 4 percent of ${}^4D$ state and 0 percent P state found by Gerjuoy and Schwinger[3] is $\mu(He^3)/\mu_p=-0.763$, a value which seems to be well out of the range of possibilities allowed by the considerations put forth here.[8] Therefore, a measurement of the moment of He^3 should prove to be a definitive experiment for distinguishing between the two cases. If the results are in agreement with expectations, it would then be possible to obtain another relation between the probabilities of the various states by taking the horizontal intercept of the observed moment with the various curves in Fig. 2.

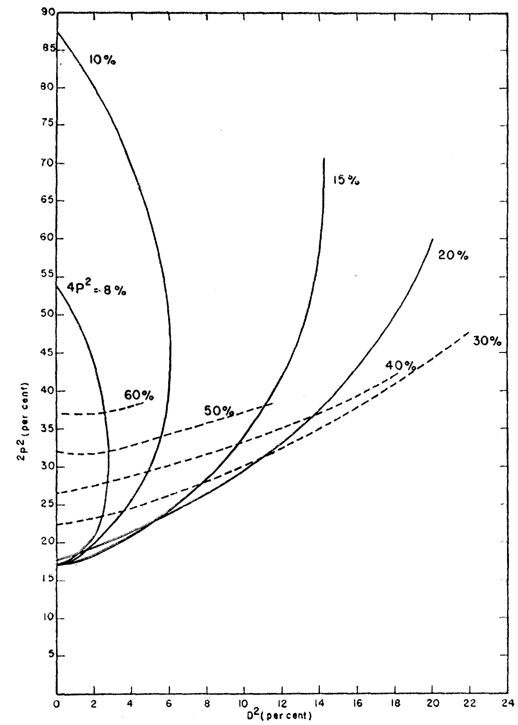

FIG. 1. Relation between ${}^2P$, ${}^4P$, and ${}^4D$ state probabilities required to account for experimental moment of the triton. The special assumptions made in obtaining these curves are expressed by Eqs. (35), (41), and (42).

5. CONCLUSION

The conclusion that the amount of D function is small compared to the amount of P function in the ground state of the triton is somewhat surprising if one believes that the tensor interaction is responsible for the admixture of states since, then, it would appear that the D state should play a predominant role. It is possible, of course, that this conclusion is a consequence of erroneous assumptions concerning the nature of the radial wave functions f_j. The derivatives of these functions have been neglected, and it can be seen that important terms could be introduced if the derivatives were not negligible. However, it has been found that the values of f_k' required to make these terms appreciable are quite large. To assume that it has such a large value would imply that the wave functions consist of products of one particle wave functions corresponding to high orbital angular momenta of the individual particles. This seems most unlikely. For the present, it seems reasonable to drop such terms.

There appear to be two essentially different wave functions of the triton which are consistent

[8] The Gerjuoy-Schwinger assumption of a small amount of ${}^4D$ function and even smaller amounts of the P functions is not consistent with the results obtained here because of the condition, Eq. (35). However, if the average value of f_1' happens to be large enough, in contrast to the requirement of Eq. (35), the term Eq. (26a) would give a contribution to the moment sufficient to account for the measured value even if the ${}^2P$, ${}^4P$, and ${}^4D$ probabilities are small. In this sense the measurement of the moment of He^3 might be considered as a test of the assumption expressed by Eq. (35). A large average value of f_1' would be rather surprising, since the usual assumption that f_1 be a function of $(r_{12}^2+r_{13}^2+r_{23}^2)$, where r_{mn} is the distance between particles m and n, would lead to $f_1'=0$.

with the measured magnetic moment. The amount of S state may be large and the amount of D state very small or zero. In this case, one might be forced to assume that a spin-orbit coupling plays an important role in determining nuclear structure. The other alternative is that there is little or no S state. This would imply that the tensor interaction has a sufficiently high average value to compensate the increase in kinetic energy which would appear for such a wave function. There would probably still be some difficulty in understanding the saturation of nuclear forces. A better understanding of this point could be obtained by carrying through a calculation of the binding energy of the triton on the assumption that there is little or no S state.

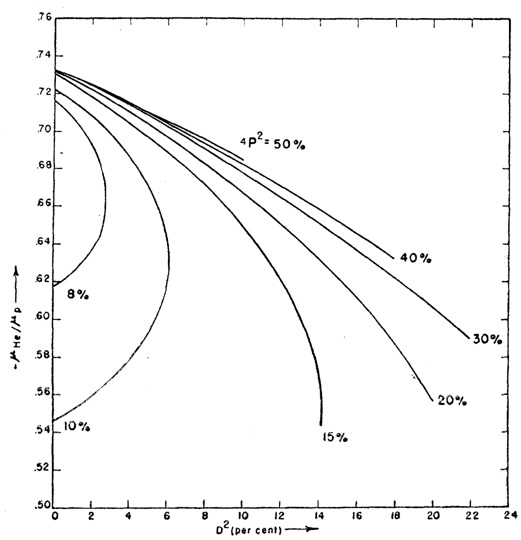

FIG. 2. The magnetic moment of He^3 in units of the proton moment. These curves have been obtained on the assumption that the relations shown in Fig. 1 apply also to He^3.

The possibility that the simple theory is entirely wrong should not be overlooked. The intrinsic moments of the neutrons and proton may be sufficiently perturbed by their mutual interaction to account for an appreciable fraction of the difference between the triton moment and the proton moment. Finally, relativistic corrections to the triton moment would be expected[6] and these may not be negligible compared to the effects under consideration.

The numerical work required for the construction of Figs. 1 and 2 was carried out by S. Moszkowski.