On the Origin of the Soft Component of Cosmic Radiation

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The intensity of the soft component in the upper atmosphere is calculated on grounds of an improved version of an earlier theory, assuming that the soft component is produced by a short-lived meson which, along with ordinary mesons, is produced by primary protons. The position of the maximum is largely influenced by the mean free path of the primary protons. The results (which are largely independent of the details of the theory), including the latitude effect, are in good agreement with the experiments, thus showing that the bulk of the soft component can be understood by this mechanism. The absence of the east-west effect must be explained by a large angular straggling.

1. INTRODUCTION

RECENTLY a theory of the soft component of cosmic rays has been given[1] in which it is assumed that the incoming primary radiation consists solely of protons.

On interacting with matter at the top of the atmosphere, these protons produce mesons. The symmetrical theory of a mixture of pseudoscalar and transverse mesons was used, the former being responsible for the penetrating component, whereas the latter give rise to the major part of the soft component, through rapid β-decay.

This "mixed" meson theory was chosen as the one which leads to the best understanding of the nuclear forces. In fact, the β-decay of the heavier nuclei seems incomprehensible unless we assume the existence of a short-lived kind of meson. Since the mesons at sea level are probably pseudoscalar it seems likely that these short-lived mesons are the transverse mesons required by the theory. The existence of a short-lived kind of meson seems equally indispensable for an explanation of the soft component, for the following reasons:

It has been shown[2] that if the total primary radiation consisted solely of electrons, the soft component could be explained completely by the cascade theory. This, however, cannot be true. A large fraction of the primary radiation must consist of protons in order to account for the hard component. Recent rocket experiments[3] also seem to indicate that at least 70 percent of the incoming primaries are protons. Of the remaining 30 percent, some at least must be slow protons, which fail to penetrate a thickness of 15.2 cm of lead used in the experiments or produce only slow mesons which cannot penetrate the lead. Therefore, the fraction of primaries which may be electrons is likely to be something less than 30 percent.

This small fraction of the primaries alone cannot explain the intensity of the soft component. Some other origin is at least partly necessary. Thus, it is important to know whether the soft component can be explained, through secondary and tertiary effects, by a primary radiation of protons only. In the present theory, the major contribution is supplied by the transverse mesons which, decaying at once, produce primary electrons. These in turn increase by cascade multiplication, thus giving rise to the soft component. We shall see that the soft component can indeed be accounted for in this way. It may be added that the results would be practically the same if any other kind of short-lived meson exists (for instance neutrettos) which decays into electrons or photons.

We shall be concerned in this paper only with the bulk of the soft component in the higher part of the atmosphere, which will be seen to consist mainly of comparatively slow electrons. In particular the origin of the large extensive air showers will not be investigated here. It may well be that these necessitate a small energetic primary electron component. By assuming a primary proton component solely, we do not wish to exclude the possibility that a certain

[1] J. Hamilton, W. Heitler, and H. W. Peng, Phys. Rev. **64**, 78 (1943). (In the following quoted as HHP.)
[2] W. Heitler, Proc. Roy. Soc. **161**, 261 (1937); L. W. Nordheim, Phys. Rev. **51**, 1110 (1937).
[3] S. E. Golian, E. H. Krause, and G. J. Perlow, Phys. Rev. **70**, 776 (1946).

fraction of the primaries may be electrons. It is clear that this would not alter anything essential in our results: Since both primary electrons and primary protons give rise to about the same intensity (relative to the number of primary particles), any mixture of primary components would do the same.

Since the publication of the HHP paper,[1] certain modifications have become necessary in the theory, and this is the reason the work has been resumed. One of these modifications has been dealt with in detail in a recent paper by one of us.[4] It refers to the expressions for the cross sections for the production of a meson in a proton-nucleon collision. In order to calculate these cross sections, the method of Weizsäcker-Williams was used, according to which the field of a fast-moving particle, energy E, is considered as equivalent to a spectrum of virtual quanta, of various energies. One of these quanta, with energy ϵ, is then scattered by a second particle at rest. The process appears as an emission of a quantum of energy ϵ', while ϵ is lost by the fast particle, and the energy $\epsilon-\epsilon'$ is transferred to the particle at rest. There are two distinct contributions to the cross sections derived by this method: (I) where the virtual mesons ϵ are attributed to the actually moving nucleon, and (II) where the virtual mesons are those belonging to the nucleon actually at rest but considered from a Lorentz frame where the roles of the two nucleons are interchanged. It was the inclusion of contribution (II) which was not considered in HHP.[1]

These two contributions Φ^{I} and Φ^{II} must then be added to give the actual cross sections for meson production. The detailed discussion of this modification has been given in reference 4. The resulting cross sections will merely be quoted in Section 2.

There is a further important modification. It has been remarked by Janossy[5] that the cross section for meson production in a nucleon-nucleon collision, is of the same order of magnitude as the average area occupied by a nucleon in the nucleus. Thus, in its passage through a nucleus, a proton will, on the average, produce more than one meson. Thus the mesons are not produced uniformly, but in small groups. So we can no longer consider the nucleons of matter as being distributed at random, but as being concentrated in groups in the nuclei. An incoming proton will thus travel on the average a distance l before coming in contact with a nucleus, l being the mean free path of a proton in air. The mesons are then produced during the passage of the proton through the nucleus. This modification has the very important effect of shifting the position of the maximum intensity of the soft component (as function of depth) to greater depths. This is very satisfactory, as it will be remembered that previously (HHP) the maximum occurred too near the top of the atmosphere, when compared with the experimental results.

2. CROSS SECTIONS FOR MESON PRODUCTION

We quote the formulas of reference 4 for the two contributions Φ_t^{I} and Φ_t^{II} for the production of a transverse meson, of energy ϵ, by a nucleon with energy E:

$$\Phi_{tNR}{}^{I}d\epsilon=\frac{8}{3}f^2(D_t+D_p)\frac{d\epsilon}{\epsilon^3}\left(1-\frac{\epsilon}{M}\right),\qquad \frac{1}{f}<\epsilon<\frac{M}{2} \tag{1a}$$

$$\Phi_{tER}{}^{I}d\epsilon=\frac{16}{3}\frac{f^2}{M}D_t d\epsilon\left[\frac{1}{\epsilon^2}-\epsilon\left(\frac{2}{E}\right)^3\right],\qquad \frac{M}{2}<\epsilon<\frac{E}{2} \tag{1b}$$

[4] W. Heitler, Proc. Roy. Irish Acad. **50A**, 155 (1945); W. Heitler and P. Walsh, Rev. Mod. Phys. **17**, 252 (1945). A slightly different interpretation of the Weizsäcker-Williams method has recently been suggested by H. A. Bethe, Phys. Rev. **70**, 787 (1946). The fact that the number of virtual mesons accompanying a fast nucleon is greater than unity has led Bethe to assume that one has to deal here with a multiple emission of mesons. We think it rather doubtful that interpretation is really correct. The number of virtual quanta has no physical meaning other than the strength of force acting on the nucleon at rest, and there is no upper limit for it. In other words, in intermediate states "probabilities" larger than unity may occur, and do occur in theories of particles with integral spin. It is the nucleon at rest which emits the meson (but owing to the exchange character of the forces the fast nucleon loses the charge). We remark that, unless the cross section for meson production is smaller by an order of magnitude than is at present believed, or else the Weizsäcker-Williams method is completely wrong, it follows from the experiments directly that the number of virtual mesons must be larger than unity. A discussion of this fact on different lines has been given by one of us (H) in Proc. Roy. Irish Acad. **50A**, 1 (1944).

[5] L. Janossy, Phys. Rev. **64**, 345 (1943).

$$\Phi_{tNR}{}^{II}d\epsilon=\frac{4}{9}Mf^2(D_t+D_p)\frac{E-\epsilon}{E^2}d\epsilon \times\begin{cases}\left(f^3-\frac{8}{M^3}\right) & 1<\epsilon<\frac{2E}{Mf}\\ \frac{8}{M^3}\left(\frac{E^3}{\epsilon^3}-1\right), & \frac{2E}{Mf}<\epsilon<E\end{cases} \quad (1c)$$

$$\Phi_{tER}{}^{II}d\epsilon=\frac{32f^2}{M}D_t\frac{d\epsilon}{E^3}(E-\epsilon)\left[\frac{E-\epsilon}{M}-1\right], \quad 1<\epsilon<E-M. \quad (1d)$$

Each formula is valid only for the energy regions indicated: where they overlap, all contributions must be added. D_t and D_p are certain constants of which, however, our results will be practically independent, $f^2=0.13$ from the theory of nuclear forces. We also use the same natural meson units $c=\hbar=\mu=1$. Cross sections, therefore, are in units of $(\hbar/\mu c)^2=4.3\times10^{-26}$ cm² and energies in units of $\mu c^2=0.94\times10^8$ ev. In the absence of any conclusive information about the mass of the vector meson we have assumed it to be equal to that of the ordinary meson. The first part of our considerations will be independent of the unit for the thickness of matter traversed. In the section which depends on the cascade theory, we use cascade units. With this unit of length, cascade processes take place in the same way in all materials. In these units, the height of the atmosphere is 24, one cascade unit equaling 3.2 cm. Hg.

There are now two additional contributions to the cross sections (1b) and (1d) which were omitted previously (because of a numerical error in an earlier paper[6]) dealing with the cross section for scattering of a transverse meson by a nucleon in the extreme relativistic region. These contributions are:

$$\Phi_{tER}{}^{I}d\epsilon=2f^2D_td\epsilon\left[\frac{3K}{7\epsilon^{4/3}}-\frac{2}{M\epsilon^2}-\frac{3K}{7}\epsilon\frac{2^{7/3}}{E^{7/3}}+\frac{16\epsilon}{ME^3}\right], \quad \left(\frac{M}{2}<\epsilon<\frac{E}{2}\right) \quad (2b)$$

$$\Phi_{tER}{}^{II}d\epsilon=2f^2D_td\epsilon\left[-3K2^{1/3}\left\{\frac{E-\epsilon}{E^{7/3}}-\frac{1}{M^{1/3}}\cdot\frac{(E-\epsilon)^{4/3}}{E^{7/3}}\right\}+\frac{12}{M}\left\{\frac{E-\epsilon}{E^3}-\frac{(E-\epsilon)^2}{M\cdot E^3}\right\}\right], \quad (1<\epsilon<E-M) \quad (2d)$$

where

$$K=\frac{8\sqrt{3}\cdot2^{2/3}\cdot\pi\cdot f^{4/3}}{9M^{1/3}}.$$

In this earlier paper,[6] the total cross section Φ for the scattering of a transverse meson by a nucleon into a meson of any polarization, in the center of gravity system, was given as

$$\Phi=8\pi(\tfrac{1}{2}+2\bar{\lambda})/\epsilon^2$$

(total charge of system $=2, -1$, symmetrical theory), where $\bar{\lambda}$ was incorrectly given as

$$\bar{\lambda}=\pi f^4\tau^{2/3}/6\sqrt{3}$$

with

$$\tau=f^2\epsilon^2.$$

Although $\bar{\lambda}$ was seen to increase with ϵ, it was remarked that the smallness of $f^2=0.13$ made the term negligible, up to large values of ϵ (actually $\epsilon\sim60$). Thus in (HHP), $2\bar{\lambda}$ was neglected, and Φ was simply given the value $4\pi/\epsilon^2$.

The true value for $\bar{\lambda}$ is, however,

$$\lambda=\frac{\pi}{3\sqrt{3}}\cdot\frac{\tau^{2/3}}{2^{2/3}}-\frac{3}{16} \quad (3a)$$

for the scattering of a positive meson by a proton and

$$\bar{\lambda}=\frac{\pi}{3\sqrt{3}}\tau^{2/3}\cdot\frac{2^{2/3}}{3}\begin{pmatrix}1-\frac{1}{2^{4/3}} & \frac{1}{2^{1/3}}-\frac{1}{2}\\ \frac{1}{2^{1/3}}-\frac{1}{2} & \frac{3}{4}\end{pmatrix}-\frac{3}{16}\begin{pmatrix}1&0\\0&1\end{pmatrix} \quad (3b)$$

for the scattering of a negative meson by a proton. The first row and column refer to a scattered negative meson, the second to a neutretto. The quantity $2\bar{\lambda}$ is now comparable with $\frac{1}{2}$ for all energies and cannot be neglected.

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[6] W. Heitler and H. W. Peng, Proc. Roy. Irish Acad. **49A7**, 101 (1943).

The additional contributions to the cross sections (1) for meson production due to the inclusion of these $\bar{\lambda}$ terms are (2b) and (2d).

Now, it must be remarked that the formulas for the cross sections for meson scattering in the extreme relativistic region are already rather doubtful, as they have been derived under the assumption that the existence of multiple processes does not influence the scattering of a single meson. This fact has only been proved mathematically to be true for low energies;[7] for high energies it is not at all mathematically certain and may not be true, because at high energies a large number of possible multiplicities may occur, and even if each of them may have a small influence, the total effect may be large. Although Φ_{NR} arising from the non-relativistic energy region of meson scattering is unaffected (and the main contribution to the production of mesons of all energies comes from this region), there is an uncertainty of Φ_{ER} arising from the extreme relativistic region. In particular it may be doubtful whether the cross sections for scattering of a meson are really as large as is indicated by the $\bar{\lambda}$-terms, or whether the influence of multiple processes causes a large damping, making them much smaller. Furthermore, the expressions (3) are asymptotic, valid for $\epsilon\rightarrow\infty$. They are needed, however, mainly in the region $\epsilon=M-5M$, say. If (3) were valid down to the boundary between the non-relativistic and extreme relativistic regions ($\sim M$), the formulas for the scattering of a transverse meson valid in the two regions would differ by a factor almost 10 at the boundary. It is, therefore, highly probable that $\bar{\lambda}$ assumes its asymptotic value (3) only at extremely high energies and is much smaller than (3) in the energy region needed. For this reason, the calculations have been performed twice, with and without these extra terms. It will be seen that there is not much difference between the two results, for the following reason: Although the cross section for the production of mesons is considerably increased by including the $\bar{\lambda}$-terms, the energy loss, $\Phi_{\text{en. loss}}$, is also increased. The total number of mesons (and hence electrons) produced depends, however, on the ratio of the production cross section and the energy loss of the primary proton. This ratio does not change much. We shall need the modified expression for $\Phi_{\text{en. loss}}$ caused by the $\bar{\lambda}$-terms.

[7] W. Heitler and H. W. Peng, Proc. Camb. Phil. Soc. **38**, 296 (1942).

The original expression for the energy loss[4] was

$$E\Phi'_{\text{en. loss}}=[1.5E+22\log(E/M)-1.0]D_t\cdot 10^{-2} \quad (4)$$

and was approximated by the first term, so that[8]

$$E\Phi'_{\text{en. loss}}=D_t(1.5\times10^{-2}E). \quad (4')$$

The extra energy loss due to the inclusion of the $\bar{\lambda}$-terms is computed to be

$$E\Phi''_{\text{en. loss}}=D_t\times10^{-2}(6.7E-4.7E^{\frac{3}{4}}-12\log(E/M)-88). \quad (5)$$

This, in turn, is approximated by the formula

$$E\Phi''_{\text{en. loss}}=D_t(4.9\times10^{-2}E), \quad (5')$$

for the values of E which are most important. Equation (5′) has to be added to (4′). Then

$$(-dE/dx)=NE\Phi_{\text{en. loss}},$$

where N equals the number of nucleons contained in a cylinder of unit length, and of cross section $(\hbar/\mu c)^2$. Thus the distance traveled by a fast nucleon while losing energy from E_0 to E is

$$x_{E_0,\,E}=k^{-1}\log(E_0/E), \quad (6)$$

where $k=N\cdot\Phi_{\text{en. loss}}$ and has the value $N\cdot D_t 6.4\times10^{-2}$ and $N\cdot D_t 1.5\times10^{-2}$, respectively, according to whether the $\bar{\lambda}$-terms are included or not.

There is one further remark to be made about the numerical values of the constants D. These values depend on the lower limit of the impact parameter $b_{\min}$ used in deriving the number of virtual quanta. In particular, the number of transverse mesons, with energy ϵ, was found to be[6]

$$q_{tr}d\epsilon=(d\epsilon/\pi\epsilon)(f^2D_t+g^2C),$$

where both D_t and C are functions of ϵ and of the impact parameter $b_{\min}$. Now $b_{\min}$ is certain to be of the order $\hbar/Mc=0.1$ in our units. If $b_{\min}$ is identified with this value then D_t was found to be 165. C was found to be negligible. Now the

[8] Eqs. (4) and (4′) are in fact not directly proportional to D_t, but are proportional partly to the factor (D_t+D_p) and partly to (D_t+2D_p). However, since D_t and D_p depend on the impact parameter in such a way (see below) that the ratio D_p/D_t is virtually constant, and D_t is very much larger than D_p, $E\cdot\Phi_{\text{en. loss}}$ is roughly proportional to D_t.

actual value of D_t depends rather sensitively on $b_{\min}$ and decreases when $b_{\min}$ is taken larger. If $b_{\min}$ is taken to be $2\hbar/Mc$, the value of D_t would decrease by roughly a factor 3. However, it will be seen that this dependence of the number of virtual mesons, on the impact parameter does not affect the results. For D_t occurs in all formulas relating to the total number of mesons produced only in the ratio

$$D_t/\Phi_{\text{en. loss}}$$

where $\Phi_{\text{en. loss}}$ is the cross section for energy loss. Since $\Phi_{\text{en. loss}}$ itself is directly proportional to D_t,[2] this ratio is quite independent of $b_{\min}$, provided that the order of magnitude of $b_{\min}$ is correct.

On account of this fact, the absolute values of the cross section for meson production and the energy loss of the proton are very uncertain as long as one is forced to use the Weizsäker-Williams method, and both may be considerably smaller than the values given in references 1 and 4, if it should turn out that $b_{\min}$ is to be taken larger than $\hbar/Mc$. Nevertheless, the total number of mesons appearing at any depth lower than the meson producing layer is more reliably given by the theory as it is independent of the actual value of $b_{\min}$.

3. THE TOTAL NUMBER OF ELECTRONS PRODUCED

We assume a spectrum of primary incoming protons, whose energy distribution at the top of the atmosphere we denote by

$$F(E_0)dE_0.$$

The actual process of meson production is roughly as follows: a primary proton on colliding with a nucleus produces a number of pseudoscalar and vector mesons during its passage through the nucleus. If its energy is not very high, the proton is probably brought to rest in one collision. If its energy is high, however, the proton may travel through two or three or even more nuclei, on an average, before being stopped. Since, however, our knowledge of the mean free path of a proton in air is very uncertain, and the accuracy of the theory not sufficient to compute it reliably, we simplify this rather complicated process, as follows: We assume that the proton is brought to rest in its passage through one (say, rather big) nucleus, and the mean free path of the proton is readjusted. This we introduce at a later stage of the considerations. During the passage through this one nucleus all the mesons are then assumed to be produced. Clearly, the total number of mesons produced by a proton is quite independent of the density of the matter traversed (in our case nuclear matter), and is quite independent of any unit of length. The above simplification of the picture can hardly affect the results appreciably. In particular, the position of the maximum and its height should be given correctly if for the mean free path of the proton its actual experimental value is taken.

Thus a proton, in its passage through the nucleus, will by (3) lose energy uniformly with distance according to the law e^{-kx}. We assume further that the primary spectrum is of the form

$$F(E_0)=A/E_0^{\alpha+1}. \qquad (7)$$

A is a normalization factor. The constant α is determined from the meson intensities at great depths. The value $\alpha=1.5$ represents the facts quite well. The number of primary nucleons at a depth x, with energy E, is given by (6) and (7)

$$F(E,x)dE=Ae^{-k\alpha x}(dE/E^{\alpha+1}). \qquad (8)$$

Then the number of transverse mesons produced at a depth x of nuclear matter within an energy interval $d\epsilon$ is

$$N\cdot d\epsilon\int_\epsilon^\infty \Phi_t(E,\epsilon)F(E,x)dE, \qquad (9)$$

N is the number of nucleons per unit volume. Φ_t is the cross section (1) and (2) for production of a meson ϵ by a nucleon of energy E, and $F(E,x)$ is the number of nucleons of energy E, at depth x, given by Eq. (8). These transverse mesons, having an extremely short lifetime, decay at once, each producing an electron and a neutrino. The probability of an electron of energy $\bar{\epsilon}$, being produced by a meson of energy ϵ in this way is $d\bar{\epsilon}/\epsilon$. The total number of electrons, within the energy interval $d\bar{\epsilon}$, produced in a layer d of nuclear matter is

$$N\cdot Ad\bar{\epsilon}\int_{\bar{\epsilon}}^\infty d\epsilon/\epsilon\int_\epsilon^\infty \Phi_{tr}(E,\epsilon)\int_0^d e^{-k\alpha x}(dE/E^{\alpha+1}).$$

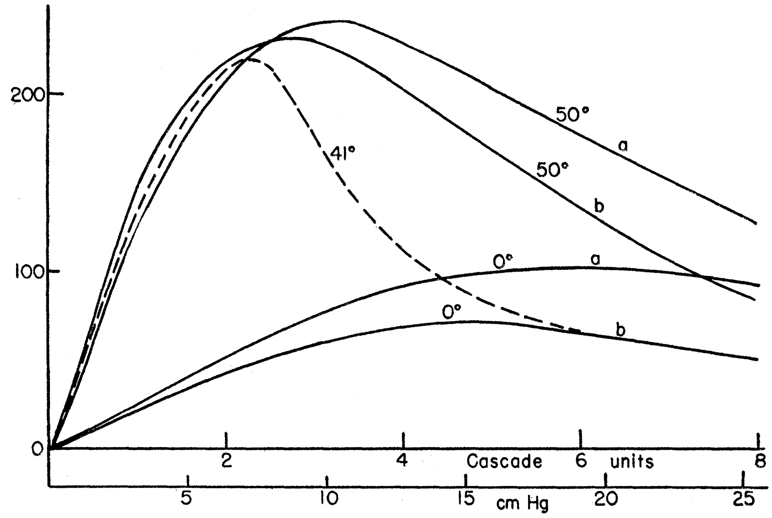

FIG. 1. Intensity of the soft component in the high atmosphere, at 50° and 0° latitude. Curve *a* with, and *b* without inclusion of the λ-terms. Dotted curve experimental at 41°. For the equator the mean free path of the primary protons was assumed to be 4.5 cascade units, at 50°, 2.5 cascade units. Normalization: 100 incoming primaries.

Obviously, only very small values of x contribute to this integral, so that one may extend the integration over x to infinity. There is a further point to be considered in integrating over x: the fact that the primary proton spectrum is cut off at a value E_ϑ of E_0 which depends on the geomagnetic latitude ϑ, imposes the following conditions

$$\begin{aligned} x &> (1/k)\log(E_\vartheta/E), & E &< E_\vartheta, \\ &> 0, & E &> E_\vartheta. \end{aligned}$$

Having performed the integration over x, we get the total number of electrons of energy $\bar{\epsilon}$

$$N(\bar{\epsilon})d\bar{\epsilon} = \begin{cases} \dfrac{N\cdot A}{k\alpha}\cdot\dfrac{d\bar{\epsilon}}{(E_\vartheta)^\alpha}\displaystyle\int_{\bar{\epsilon}}^{\infty}\frac{d\epsilon}{\epsilon}\int_{\epsilon}^{\infty}\Phi_{tr}(E,\,\epsilon)\frac{dE}{E}, & (E<E_\vartheta) \quad (10a) \\ \dfrac{N\cdot A}{k\cdot\alpha}d\bar{\epsilon}\displaystyle\int_{\bar{\epsilon}}^{\infty}\frac{d\epsilon}{\epsilon}\int_{\epsilon}^{\infty}\Phi_{tr}(E,\,\epsilon)\frac{dE}{E^{\alpha+1}}. & (E>E_\vartheta) \quad (10b) \end{cases}$$

For Φ we insert the cross sections (1) and (2). The limits $\epsilon\cdots\infty$, and $\bar{\epsilon}\cdots\infty$ have to be replaced by narrower limits, according to the regions of validity of the formulas (1) and (2). The integration can then be performed by simple analytical methods.

4. CASCADE MULTIPLICATION

The formulas (10) give the total number of electrons produced in the energy interval $d\bar{\epsilon}$ by the passage of a primary proton averaged over the primary energy spectrum through our "model nucleus," which we have assumed to be so large that the proton is stopped. If we now denote the mean free path of the proton in air by l, the chance of its colliding with a nucleus in a distance $d\xi$ is

$$e^{-\xi/l}d\xi/l.$$

Therefore, with the above assumptions, the number of electrons of energy $\bar{\epsilon}$ produced at a depth ξ from the top of the atmosphere because of the decay of transverse mesons is

$$N(\bar{\epsilon})\cdot e^{-\xi/l}\cdot d\xi\cdot d\bar{\epsilon}/l.$$

We measure ξ now in cascade units.

If $C(\bar{\epsilon}, x-\xi)$ be the cascade multiplication function (i.e., the number of electrons produced at a depth x, by a single primary electron, energy $\bar{\epsilon}$, at depth ξ) the total number of electrons of all energies, observed at depth x, because of the decay of transverse mesons, is

$$Z_t(x) = \int_0^x e^{-\xi/l}\frac{d\xi}{l}\cdot\int_0^\infty d\bar{\epsilon}\cdot N(\bar{\epsilon})C(\bar{\epsilon}, x-\xi). \quad (11)$$

This integral has been worked out by numerical integration for two different latitudes, $\vartheta=50°$ and $\vartheta=0°$ (equator). For C we have used the figures given by Bhabha.

As we do not know with sufficient accuracy, from theoretical considerations, the value of the mean free path l, we have taken, for a latitude of 50°, Schein's effective mean free path, derived from the absorption of the primaries.[9] This makes l equal to about 2.4–3 cascade units. We assume $l=2.5$. By taking for l the measured mean free path it is clear that we have also taken account of any secondary nucleons which further contribute to meson production.

Unfortunately, there is nothing known about l for the equator: all we know is that it is bound to be larger than in Europe, as the energies of the incoming primaries must be larger. Tentatively we take it as being 4.5 cascade units.

In addition to the electrons produced by the decay of transverse mesons, there are electrons produced by the decay of pseudoscalar mesons. Their number is smaller, but not negligible, compared with those caused by transverse mesons. As remarked in (HHP), their number is about $\frac{1}{3}$ of the total number of electrons produced in the upper layers of the atmosphere. They are therefore included by multiplying (11) by a factor $\frac{3}{2}$. (When the $\bar{\lambda}$-terms are included we have added half of the contribution without $\bar{\lambda}$.)

In Fig. 1, the intensity of the soft component for the top part of the atmosphere (normalized for 100 incoming protons) is plotted for $\vartheta=50°$ and the equator and calculated both with (curve *a*) and without (curve *b*) the $\bar{\lambda}$-terms. It is seen that there is very little difference between them. We have also plotted a recent experimental curve for the intensity measured by rocket experiments at White Sands, New Mexico, ($\vartheta=41°$N). The agreement between these curves is as good as can be expected, considering the inaccuracies of the theory. The position of the maximum has been shifted to lower heights than in (HHP), and appears to agree now with the experimental curve rather well. This is due mainly to the fact that a proper account was taken of the mean free path of the primaries, whereas in (HHP) the nucleons of air were considered as uniformly distributed. The electrons produced according to the theory are mainly of low energies, the vector mesons which produce them having themselves, on an average, comparatively low energies. If these alone are multiplied by the cascade process, they give a maximum too near the top of the atmosphere. The introduction of the mean free path causes the necessary shift towards lower heights.

It is clear from Fig. 1, that the latitude effect of the soft component is very large. The theoretical intensity at the equator is about 30 percent at 7.6 cm Hg of that at 50°. This agrees roughly with the measurements of the total ionization by Bowen, Millikan, and Neher.[10] Measurements of the soft intensity at the equator do not seem to exist.

There is, however, a difficulty in our theory which was already referred to in (HHP), and which still persists: the soft component does not seem to show an east-west effect. This seems to suggest that it must arise from an equal number of positive and negative primaries. It was suggested in (HHP) that since the soft component is produced by the protons in such an indirect way, and arises mainly from mesons and therefore electrons of low energy, the east-west effect may be blurred by a large angular spread of the electrons, as it would indeed follow from the theory.

The same view must be taken here—and in any theory that accounts for the soft component as arising indirectly from a primary proton component only. This point requires further clarification from the experimental side.

The results of this paper can be stated as follows: If the existence of a short-lived meson (transverse meson or neutretto) is assumed, the bulk of the soft component can be accounted for as due to a primary proton component only. The same will be the case if the primary radiation consists of any mixture of protons and primary electrons with similar energy spectra, but, of course, the meson component can only be accounted for if the primary proton component is not too weak. The absence of an east-west effect, if confirmed, must be explained by a large angular spread.

[9] M. Schein, M. Iona, and J. Tabin, Phys. Rev. **64**, 253 (1943). These experiments may not be sufficiently accurate yet. One may, *vice versa*, from the position of the maximum conclude that the mean free path of the protons in air must be between two and three cascade units.

[10] I. S. Bowen, R. A. Millikan, and H. V. Neher, Phys. Rev. **53**, 855 (1938).