by use of an indium filter. Thus the measurements were limited in more than one way to neutrons absorbed strongly by indium.

BF3 gas in a steel cylinder was interposed in the collimated beam. The BF<sub>3</sub> was highly purified (the same gas as used in the thermal neutron transmission experiments described above). The transmission of the steel container filled with  $BF_4$  at 44 and 68 lb/in.<sup>2</sup> was compared with the transmission of the empty container. The density of gas used was determined by weighing the cylinder. The pressures used and the length of the cylinder (30 cm) were such that the transmissions were in an accurately determinable range (approximately a  $\frac{2}{3}$  transmission for the 68-lb sample).

The total cross section of BF<sub>3</sub> for indium resonance neutrons was measured as 107.1  $\times 10^{-24}$  cm<sup>2</sup>/atom. Assuming

> $\sigma_{\rm scattering}(F) = 3.7 \times 10^{-24} \, {\rm cm}^2$ ,  $\sigma_{\text{scattering}}(B) = 2 \times 10^{-24} \text{ cm}^2$ , indium resonance energy = 1.44 ev,

the boron absorption cross section for neutrons at velocity 2200 m/sec. is  $710 \times 10^{-24}$  cm<sup>2</sup>/atom.

The results of the three measurements are given in Table II.

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# Scattering of Fast Neutrons by Helium\*

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Measurements have been made of the angular distribution of helium recoils for incident neutrons of energies from 0.6 to 1.6 Mev. The distribution curves also permit estimates of the total cross section in this energy range. The results confirm the existence of a cross-section peak of about  $6.8 \times 10^{-24}$  cm<sup>2</sup> around 1 Mev and indicate, under the assumption of s- and p-wave scattering only, that the peak is double. But preliminary attempts to fit the data to Bloch's detailed theory of s- and p-scattering with a split p-level have not been successful, and the sign of the postulated splitting is not established.

# INTRODUCTION

SCATTERING theory for helium must satisfy quantitatively two previous sets of data as well as that reported here. Barschall and Kanner<sup>1</sup> obtained recoil distribution curves at neutron energies of 2.5 and 3.1 Mev, and Staub and Tatel<sup>2</sup> obtained the backward scattering cross section as a function of energy around 1 Mev, finding a peak (possibly double). The Barschall-Kanner curves are strongly anisotropic with a preponderance of forward scattering. Wheeler and Barschall<sup>3</sup> have shown that the data at 2.5 Mev can be fitted by the assumption of strong spin-orbit coupling, a p-wave resonance around 2.5 Mev, and a weak addition of d-wave of a size not unreasonably large for 2.5 Mev. But this data at 2.5 and 3.1 Mev cannot be matched with a simple theory involving only resonances around 1 Mev: an s-wave resonance would be isotropic and higher resonances at 1 Mev must fall off 'to insignificance at 2.5 and 3.1 Mev. Thus the two sets of data provide essentially two different problems, at any rate for analysis in terms of resonances. The Bloch formula for

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<sup>&</sup>lt;sup>2</sup> H. Staub and H. Tatel, Phys. Rev. 58, 820 (1940).

<sup>&</sup>lt;sup>3</sup> J. A. Wheeler and H. H. Barschall, Phys. Rev. 58, 682 (1940).

resonant p-wave scattering<sup>4</sup> used below can satisfy only one set, and we only consider the fitting of this formula to the data near 1 Mev of Staub and Tatel and of this paper. The main contribution from this paper is a group of recoil energy distribution curves for neutron energies from 0.6 to 1.6 Mev. While the authors have had opportunity to make only a preliminary theoretical analysis, it has seemed worth while to present these new data.

Earlier observations of scattering near 1 Mev were made by Gaerttner, Pardue, and Streib,5 who found a total cross-section peak around 0.8 Mev, and by Staub and Stephens,<sup>6</sup> who found a total cross-section peak near 1 Mev. Taken together, these studies indicated a double peak, but they were not sufficiently detailed to permit a quantitative check with theory on this score. The question of the double peak has been investigated in its theoretical aspects by Dancoff,<sup>7</sup> who concluded that a Thomas relativistic spin-orbit coupling should produce an inverted doublet (with size of splitting essentially unknown) while the tensor spin-orbit interaction of mesotron theory could be expected to produce a normal doublet with splitting of the order of 100 kev. None of the experimental data has determined whether the splitting is normal or inverted, nor is the size of the splitting established beyond the fact that it cannot be more than about 600 kev.

#### EXPERIMENTAL PROCEDURE

The measurements were made at the larger Van de Graaff generator loaned to the Los Alamos Laboratory by the University of Wisconsin. The lithium p-n reaction provided the neutrons. A cylindrical proportional counter,8 with guard rings at the end to define the counting volume, was filled with a helium-argon mixture and placed in the neutron flux; the helium recoil pulses were fed through a preamplifier and amplifier of rise-time  $\frac{1}{2}$  microsecond and RC decay time 20 microseconds into a discriminator



FIG. 1. Experimental arrangement for curves of energy distribution of recoils.

which counted simultaneously all pulses of a size above a set level ("integral count") and all pulses of a size within a set channel ("differential count"). Both settings were adjustable, and by moving the channel along the recoil energy scale while keeping its width constant, one obtained the energy distribution of the helium recoils. (The recoil energy was simply proportional to the pulse height.) Two kinds of data were taken: first, detailed curves of the energy distribution of the helium recoils for a given neutron energy (Figs. 2-6), and secondly, brief studies of the high energy end of these curves at successive neutron energies (Fig. 7). In each case, total counts above a certain recoil energy (determined by the need to eliminate  $\gamma$ -background) were recorded, and neutron flux was measured. Also proton current in the Van de Graaff generator was recorded during the runs.

In the case of the detailed curves the experimental arrangement was as follows: The scattering chamber was placed with its end about 9 inches from the lithium target, and its axis pointing to the target. (This puts the beginning of the counting volume about 11 in. from the



<sup>&</sup>lt;sup>4</sup> F. Bloch, Phys. Rev. 58, 829 (1940).

<sup>&</sup>lt;sup>5</sup> E. R. Gaerttner, L. A. Pardue, and J. F. Streib, Phys.

Rev. 56, 856 (1939). <sup>6</sup> H. Staub and W. E. Stephens, Phys. Rev. 55, 131 (1939).

<sup>&</sup>lt;sup>7</sup> S. M. Dancoff, Phys. Rev. **58**, 326 (1940). <sup>8</sup> P. G. Koontz and T. A. Hall, Rev. Sci. Inst. (to be published).



target.) The chamber axis made an angle of  $30^{\circ}$  with the target tube axis. Also off at an angle of  $30^{\circ}$ , and about 9 cm from the lithium target, was the flux monitor which was a U<sup>235</sup> fission foil placed in an ionization chamber. This arrangement is sketched in Fig. 1. The Van de Graaff generator was then set to give the desired neutron energy at  $30^{\circ}$ . The distance of the chambers from the target remained about as given throughout the runs, but was measured after each run to secure accurate flux measurement.

Data were taken with two different gas fillings, and under operation both as an ion chamber and as a proportional counter with gas multiplication of about 14. The two fillings were  $(\frac{1}{2}$  atmosphere of helium  $+2\frac{1}{2}$  atmospheres of argon), and  $(\frac{1}{2})$ atmosphere of helium+1 atmosphere of argon), the lower pressure being used to permit recording of slower recoils without interference from  $\gamma$ -background. The collecting potential was 1450 volts for use as an ion chamber and 2500 for use as a proportional counter with the higher pressure, and 1500 volts for use as a proportional counter with the lower pressure. At the higher pressure an  $\alpha$ -particle of 0.5 Mev has a range of 0.13 cm, and an  $\alpha$ -particle of 1 Mev a range of 0.22 cm; the corresponding figures for the lower pressure are 0.28 and 0.48 cm.

In the case of the curves covering the high energy recoils (Fig. 7), the arrangement of the helium chamber was the same, but the flux monitor used was a fission detector almost insensitive to energy, placed about 6 feet from the target.

# EXPERIMENTAL RESULTS

The results are summarized in Figs. 2–6. In clarifying their meaning, the following facts should be noted:

The curves obtained are almost the same for ion chamber and proportional counter operation. This was also the case in previous testing of this counter using the nitrogen n-p reaction. In the ion chamber case, because of the small rise and decay time of the amplifier and the low mobility of positive ions, the pulse is due mostly to the collection of the electrons, which travel over different distances from their point of origin to the central wire. Nevertheless, since ion chamber and proportional counter curves are quite similar, we may assume that the pulse is proportional to the energy of the recoil in each case<sup>9</sup> (except for wall effect, irregularity of multiplication, etc.). Hence the abscissas in Figs. 2-6 may also be considered to represent the recoil energy. Furthermore, writing

$$E = neutron energy$$

$$E_m = maximum energy of recoil$$

$$E_r = energy of recoil$$

$$\phi = angle of recoil$$

$$\theta = scattering angle of neutron, center of gravity
system$$

 $d\omega = 2\pi \sin\theta d\theta$ 

 $d\sigma/d\omega$  = cross section per unit solid angle in the center of gravity system,

we have

$$E_r = E_m \cos^2 \phi, \quad \theta + 2\phi = \pi$$



<sup>9</sup> The effect of "electron collection" is described briefly in reference 8. It is discussed in greater detail in the forth coming "Los Alamos Encyclopedia" but the necessary analyses were developed before the Manhattan Project began.

and

$$E_m = 4M_\alpha M_n / (M_\alpha + M_n)^2,$$

from which it follows that the curve of recoil distribution vs. recoil energy has the same shape as the curve of  $d\sigma/d\omega$  vs.  $\cos\theta$ , with  $\theta=0$ ,  $\cos\theta=1$  at zero recoil energy and  $\theta=\pi$ ,  $\cos\theta=-1$  at maximum recoil energy. Thus Figs. 2–6 give directly the differential cross section in the center of gravity system.

For each of these curves a check was made on  $\gamma$ -background, runs were made at different bias settings with a quartz shield blocking the lithium target from the proton beam. This procedure showed the effect of  $\gamma$ -rays from the bulk of the Van de Graaff generator, but eliminated  $\gamma$ -rays normally produced beyond the quartz shield in the target tube and lithium target. However, other measurements<sup>10</sup> have shown that at a distance of about 2 inches from the target,  $\gamma$ -rays from a target and  $\gamma$ -rays from the generator bulk are of about equal strength, and those from the target fall off as  $1/r^2$  with distance from the target; so that the  $\gamma$ -effect cannot be significantly larger than that observed in the procedure above. All data plotted were taken at points where the background is negligibly small.

The theory indicates (see below) that the differential curves should be parabolas. Parabolas have been fitted and are drawn in Figs. 2–6. At the high energy end of the curves, the experimental curve is broadened and decreased in height by the width of the channel of the discriminator; the parabolic curve has been ex-



<sup>&</sup>lt;sup>10</sup> These measurements were made by R. Taschek in connection with another experiment.



tended to a point where the area under it is about the same as that under the experimental curve. A further requirement was that the end point of the parabolic curve, corrected for channel width, should agree in height with the experimental peak. This requirement was met in each case to within 15 percent or better of the experimental peak height. In this way the abscissa of the high energy end can be determined to about 1 percent of the pulse height. The abscissa scale in Figs. 2–6 is arbitrary. The end of the parabola should be taken to correspond to the recoil of maximum energy, which is  $[4M_nM_{\alpha}/(M_n+M_{\alpha})^2]=16/25$  of the energy of the incident neutron.

The ordinate scale in Figs. 2–6 and Fig. 7 is also "arbitrary," being the actual number of counts observed. Most points represent the average of two points, so that twice as many counts are involved. When two or three runs were made on one point, repetition was on the whole well within statistical error. The "curve peaks" of Fig. 8 are also plotted on an arbitrary ordinate scale. These peaks are the same as those recorded in Fig. 7.

Figure 9 is a curve of the total scattering cross section vs. energy. The numbers plotted were obtained in two steps: (1) calculation of the cross section down to the integral bias, which depends solely on measured quantities, and (2) multiplication by a factor obtained by extrapolating the parabolic fit to zero recoil energy. (This factor is simply the ratio of areas under the full curve and under the curve down to the integral cut-off.)

The first step involved three quantities:



FIG. 7. Backward scattering differential curves.

neutron flux, number of helium atoms present, and integral count. The flux measurements have a probable error of about 5 percent, owing to the fact that only about 3000 fission monitor counts were obtained per run, and to uncertainties in relative distances of chamber and monitor from the target. The number of helium atoms present should be quite well known, the pressure having been measured by a mercury manometer, the temperature being known to a few degrees, and the active volume having been measured to about 3 percent. The error in the number of integral counts is negligible, as one obtains about 15,000 integral counts with each point on the curve. An associated error arises in the determination of the energy corresponding to the integral bias. This error is caused by the uncertainty in locating the exact position of the maximum recoil energy on the graph. This uncertainty amounts to about  $1\frac{1}{2}$  percent in the total cross section. Thus the probable error with perfect extrapolation would be 6 percent.



FIG. 8. Evidence of increased anisotropy at peak.

E(Mev)	$\sigma(10^{-24}~{ m cm^2})$
0.6	2.48
0.8	4.91, 4.63, 4.28
1.0	6.42, 6.75, 6.16
1.1	6.35
1.25	6.96, 6,70
1.35	5.66, 5.22
1.6	4.60

TABLE I. Total cross section vs. neutron energy.

The extrapolation itself is uncertain. It is poorest at lower neutron energies, where the area under the extrapolated curve is a larger fraction of the total, and where the curve cannot be fitted so well because its shape is not so well determined. The integral bias is always set as low as possible without  $\gamma$ -background interference; at the higher gas pressures this permits settings at about 0.22-Mev recoil energy and at the lower gas pressure settings of about 0.13 Mev. Thus the integral bias energy ranges from about  $\frac{1}{3}$  to  $\frac{1}{5}$  of the maximum recoil energy in the curves presented. While it is not certain that the curves should be simply parabolic, it is very unlikely that they will deviate much from a parabolic shape. (This would require strong *d*-wave or higher order scattering, which is very unlikely for energies as low as 2 Mev where the neutron wave-length in the center of gravity system is more than  $3 \times 10^{-12}$  cm.) The extrapolated area is probably correct to 25 percent or better (as a guess), so that the errors arising from extrapolation might be expected to range from 4 to 8 percent. This leaves the final probable error on the cross sections around 10 percent. This figure seems reasonable considering the spread in Fig. 9 at those energies for which more than one run was taken. Fig. 9 then gives the energy dependence of the cross section. The same points are tabulated in Table I.

### INTERPRETATION AND COMPARISON WITH THEORY

The Bloch theory is general, introducing few assumptions. The scattering near 1 Mev is built from s- and resonant p-waves. A spin-orbit interaction is allowed for; it is assumed that there are two resonant compound p-levels, of energies  $E_{1/2}$  and  $E_{3/2}$ ,  $\frac{1}{2}$  and  $\frac{3}{2}$  being the total angular momenta of the two levels. The only assumptions introduced are that all other levels of the compound nucleus do not affect the scattering of the neutrons at the energies used, that the spin-orbit forces are small enough so that the phases of the scattered waves  $\delta_l$  and the half-width  $\Gamma$  are about the same for the two levels, and that the phases  $\delta_l$ ,  $l \neq 0$ , for nonresonant components, are negligibly small.

The Bloch formula reads

$$\begin{aligned} \frac{d\sigma}{d\omega} &= \lambda^2 \left\{ \left[ \sin \delta_0 + e^{-i\delta_0} \left( \frac{\Gamma}{E_{3/2} - E - i\frac{1}{2}\Gamma} + \frac{1}{2} \frac{\Gamma}{E_{1/2} - E - i\frac{1}{2}\Gamma} \right) \cos \theta \right]^2 \\ &+ \frac{\sin^2 \theta}{4} \left[ \frac{\Gamma}{E_{3/2} - E - i\frac{1}{2}\Gamma} - \frac{\Gamma}{E_{1/2} - E - i\frac{1}{2}\Gamma} \right]^2 \right\} \end{aligned}$$

where  $d\sigma/d\omega$  is the cross section per unit solid angle in the center of gravity system,  $\lambda = \lambda/2\pi$ ,  $\lambda$  is the neutron wave-length in the center of gravity system,  $\delta_0$  is the phase shift of the *s*-scattered wave,  $\Gamma$  is the half-width of the He<sup>5</sup> levels, E is the energy of the incident neutron, and  $\theta$  is the neutron scattering angle in the center of gravity system. The quantities  $\Gamma$  and  $\delta_0$  are dependent on the incident neutron energy, the dependence of  $\delta_0$  not being well-known apart from the fact that it must be slowly varying. The quantities  $\Gamma$ ,  $\delta_0$ ,  $E_{3/2}$ , and  $E_{1/2}$  are the parameters left adjustable in fitting the data, for we have used the energy dependence<sup>11</sup>

$$\Gamma = 3\Gamma_0 E^{\frac{1}{2}}/(1+3.1/E),$$

with E in Mev and  $\Gamma_0$  a constant.

According to the Bloch formula (or to any superposition of *s*- and *p*-waves alone), the differential cross section as a function of  $\cos\theta$  is a parabola. Figures 2–6 are actually plots of  $d\sigma/d\omega$  vs.  $\cos\theta$ , with  $\theta=0$ ,  $\cos\theta=1$  at the origin and  $\theta=\pi$ ,  $\cos\theta=-1$  at the maximum recoil energy. The parabolic fit is good from 0.6 to 1.1 Mev, and is not too bad from 1.1 to 1.6 Mev. Furthermore, one can fit *any one* of the curves by using parameters near those suggested by Staub and Tatel from their data:  $\Gamma=0.4$  at 1 Mev,  $E_{3/2}=1.45$  Mev,  $E_{1/2}=1.05$  Mev,  $\sigma_0=1.5$  barns, for example, yield a good fit at 1 Mev

$$(\sigma_0 = 4\pi\lambda^2 \sin^2\delta_0).$$

Sets of parameters not very different from Staub and Tatel's may be found to fit any of the curves. Also, fits may be obtained both with the assumption of an inverted doublet  $(E_{3/2} < E_{1/2})$  and with the assumption of a normal doublet  $(E_{3/2} > E_{1/2})$ . But as yet no set of parameters has been found which, with the energy dependence of  $\Gamma$  and  $\delta_0$ mentioned, has provided a reasonably good fit for all the curves from 0.8 to 1.6 Mev; those parameters which suit the lower energy curves fit badly at the higher, and vice versa. As long as a good fit is not found, nothing can be said from this data as to whether the doublet is normal or inverted, or even as to the fundamental nature of the scattering. In particular, in addition to resonant *p*-scattering which should have the form of the Bloch formula, there still may be a significant contribution from a fairly constant *p*-background of non-resonant nature, a possibility which is not studied in this paper.

The data of Staub and Tatel indicate that the *p*-splitting may be significant, giving a double peak. (It is possible to draw a smooth single-peak curve through their data going scarcely beyond experimental error, but the points themselves indicate a double peak.) Figure 9 also presents the appearance of a double peak, although the accuracy is not good enough to assure that it is real. But if one assumes the correctness of the Bloch formula, Figs. 2–6 furnish indirect evidence



FIG. 9. Helium total scattering cross section.

<sup>&</sup>lt;sup>11</sup> We are grateful to Professor V. F. Weisskopf for his unpublished derivation of this relation, based on the analyses of V. F. Weisskopf and D. H. Ewing, Phys. Rev. 57, 472 (1940).

that the peak is double. In the case of zero splitting and neutron energy equal to the resonant energy, the Bloch formula reduces to

$$d\sigma/d\omega = \lambda^2(\sin^2\delta_0 + 9\cos^2\theta + 6\cos\theta\sin^2\delta_0).$$

Then if we take the ratio of differential cross section at  $\theta = \pi$  and at the minimum of the curve, we get

$$\frac{\sigma_{\pi}}{\sigma_{\min}} = \frac{9 - 5 \sin^2 \delta_0}{\sin^2 \delta_0 - \sin^4 \delta_0}.$$

The value of  $\sin^2 \delta_0$  is probably about 0.3. (We have  $\sigma_{\pi} = 4\pi\lambda^2 \sin^2 \delta_0$ ;  $\sigma_0$  has been measured to be<sup>12</sup>  $1.25 \times 10^{-24}$  cm<sup>2</sup> at thermal energies and probably remains the same or decreases slowly as energy increases;  $\lambda^2 \sim 1/E$ .) But in any event the ratio  $(9-5\sin^2 \delta_0)/(\sin^2 \delta_0 - \sin^4 \delta_0)$  cannot be less than 25, the minimum value reached at  $\sin^2 \delta_0 = 0.6$ . The largest ratio  $\sigma_{\pi}/\sigma_{\min}$  observed in the curves is only about 3, and the resonance cannot be so sharp that all the curves miss it badly. Therefore, on the assumption of the Bloch theory, it is necessary to have a splitting of the *p*-level in order to bring the theoretical ratio down.

There are two more points of interest in the data. The kinks in Figs. 4 and 5 were repeated too well to be caused by statistical fluctuation. They could be caused by *d*-waves except for the fact that the energy is still too low for strong *d*-scattering. There is no possibility of explaining them by simple *s*- and *p*-scattering. It is possible that such deviations from a smooth curve could be caused by the presence of a group of neutrons of lower energy, but other experiments have given no evidence for the existence of such a group.

A second point of interest is indicated in Fig. 8.

The total cross sections are those of Fig. 9, averaged. The peak heights, taken from Fig. 7 but plotted on an arbitrary scale, indicate the variation in the backward scattering cross sections. The Bloch formula predicts that forward and backward scattering cross sections will rise faster than the total cross section as resonances are approached. (This will be so unless the p-s interference term is too large, in which case either the forward or the backward cross section will rise slower than the total, while the other will rise still faster. It is unlikely that the interference term is this large.) In confirmation of the prediction, the peak heights of Fig. 8 rise faster than the integral counts.

## CONCLUSION

The interpretation given above must be considered as preliminary comment on the theory; it remains to be seen whether or not a more systematic approach will yield a detailed check between theory and the experimental distribution curves. At present, the data have not determined the adequacy of *s*- and *p*-scattering with spin-orbit interaction in treating the 1-Mev helium resonances, but it does indicate, assuming this adequacy, that there is a splitting of the *p*-level.

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<sup>&</sup>lt;sup>12</sup> J. Schwinger, Phys. Rev. 58, 1004 (1940).