Superconductivity and the Debye Characteristic Temperature

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A plot of the superconducting threshold temperatures as a function of the Debye characteristic temperatures of the known superconducting elements discloses a number of interesting relationships. These relationships, in conjunction with previously known empirical rules, are used to discuss the occurrence of superconduction among other elements. It is concluded that Sc, Y, and Sm are not likely to be found superconducting, but that Ce, Pr, Nd, and Ac (should its atomic volume prove to be of the order of 22 cm^3/mole) are expected to be superconducting. Data on Pa is very sketchy. The best that can be inferred is that Pa should be superconducting with a threshold temperature in excess of 2°K.

THUS far little connection has been found between the occurrence of superconductivity among the elements and the other physical properties of the elements. One of the two connections which have been pointed out is that the superconducting elements occupy groups II to V in the periodic table except for the first subgroup of II and the second sub-group of V. Recently uranium and rhenium, groups VI and VII respectively, have been reported to be superconducting. The other connection is that the superconducting elements occupy a rather well-defined region in the atomic volume *versus* atomic number graph.[1] In a search for other empirical relations, a relation, shown in Fig. 1, between the superconducting threshold temperature (T_s) and the Debye characteristic temperature (Θ_D) was found. This relation is presented with the thought that it may prove to be of heuristic value.

The Debye temperatures used in Fig. 1 were re-computed in order to include recent data occurring in the literature. In order that the graph be self-consistent, the Debye temperatures were determined for a range of values of T/Θ_D from 0.2 to 0.8. In certain cases a better fit to the specific heat data was obtained by a combination of a Debye curve and a small correction term directly proportional to the absolute temperature. On the assumption that the linear term represented the electron contribution, the Debye curve fitted in this manner was used

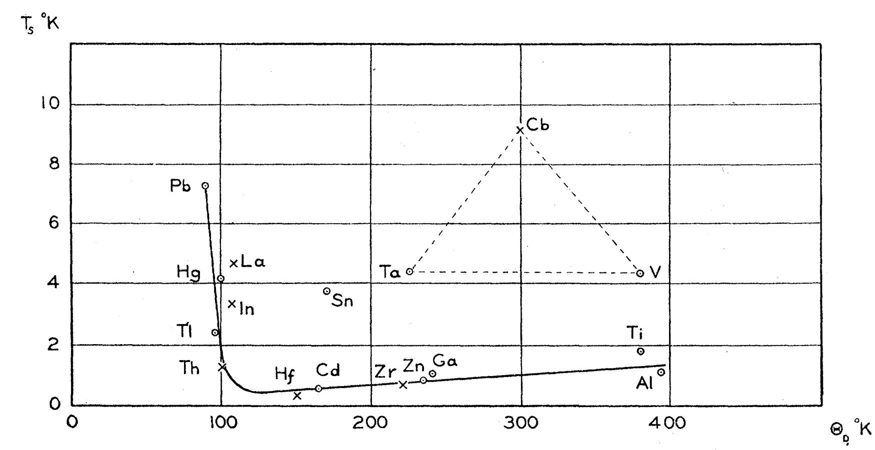

FIG. 1. A plot of superconducting threshold temperature (T_s°K) *versus* Debye characteristic temperatures (Θ_D°K) for the seventeen known superconducting elements in groups II to V. Circles indicate those elements whose characteristic temperatures are derived from specific heat data. Crosses indicate those elements whose characteristic temperatures are estimated by means of other standard methods.

[1] For example: E. F. Burton, H. Grayson Smith and J. O. Wilhelm, *Phenomena at the Temperature of Liquid Helium* (Reinhold Publishing Corporation, New York, New York, 1940), articles 29–30.

for determining the characteristic temperature. Where specific heat data were insufficient for a determination of the Debye temperature, other standard methods were employed. In plotting Fig. 1, those elements whose Debye temperatures were determined by these other methods are distinguished by the use of crosses instead of circles.

In Fig. 1 it is seen that the superconducting elements fall into two major groups: namely, the electro-negative group V (V, Cb, Ta) and the electro-positive group. These electro-negative elements have threshold temperatures well above the electro-positive elements possessing corresponding values of Debye temperature. Only when the Debye temperature is of the order 100°K do the threshold temperatures for the electro-positive group become comparable with those of the electro-negative. Uranium and rhenium are not included in Fig. 1 as they would have determined only a single point in their respective groups.

In addition to the above, the distribution of the elements in Fig. 1 indicates the following:

A. For the electro-positive group those elements lying near the vertical part of the curve have larger atomic volumes than those lying near the horizontal portion of the curve. The range of atomic volumes along the vertical part of the curve is 14.2 to 22.6 cm^3/mole. The range of atomic volumes along the horizontal portion of the curve is 9.2 to 14.2.

In examining those elements which have not been tested for superconduction, this relationship between atomic volume and Debye characteristic temperature for the known superconducting elements may be an additional restriction on the occurrence of superconduction.

B. Within the electro-positive group, there is almost a clean-cut division in the threshold temperature range occurring at a Debye temperature of the order 120°K. This suggests that in the electro-positive group, threshold temperatures in excess of 2°K would not be expected for those elements having Debye temperatures larger than 120°K.

C. The horizontal portion of the electro-positive curve has a slight positive slope.

D. The one serious exception to the regularity shown by the electro-positive group is tin. This may be connected with the fact that tin is an allotropic element.

E. In drawing the horizontal portion of the curve, more weight was given to Al than to Ti. The value of threshold temperature (1.8°K) for Ti has been questioned by Schoenberg[2] whose work suggests a much lower value.

[2] D. Schoenberg, Proc. Camb. Phil. Soc. **36**, 84 (1940).

Assuming the empirical relationship of Fig. 1 to be valid, it is interesting to apply it, in conjunction with the atomic volume *versus* atomic number graph, to those elements of groups II to V which have not been explored in order to see what predictions can be made. Scandium, yttrium, actinium and the rare earths (numbers 58 to 71) of the electro-positive group, and protoactinium of the electro-negative group V are elements satisfying the rule concerning the occurrence of superconduction in the periodic table.

Scandium has not been isolated. It is estimated that its atomic volume is 18 and that its Debye temperature is 330°K. On the basis of these values, it would appear that scandium is not among the superconducting elements. It lies far out of the region occupied by the superconducting elements in the atomic volume *versus* atomic number graph. In addition, from *A* we do not expect to find such large atomic volumes in the region characterized by Debye temperatures of the order of 330°K. Only if the actual atomic volume of scandium proves to be less than the estimated value of 18 (say, 9 to 14) would there be reason to suppose that it would be superconducting, with a threshold temperature of the order of 1°K.

Yttrium has an atomic volume of 16.1 and a Debye temperature of the order of 230°K. The atomic volume of yttrium exceeds the limit set by both empirical rules. From this one would not expect yttrium to be a superconductor.

Actinium has an atomic volume somewhere in the range 22 to 38 and a characteristic temperature of the order of 100°K. Thus, if its atomic volume should prove to be of the order of 22, it would satisfy the empirical conditions and would be expected to fall on the vertical part of the curve in Fig. 1.

Data available on the rare earths (numbers 58 to 71) of the electro-positive group limit the discussion to cerium, praseodymium, neodymium and samarium. Cerium, praseodymium and neodymium have atomic volumes of the order of 20 and Debye temperatures, derived from the melting point formula, of the order of 100°K. These rare earths, therefore, are probably superconductors and should lie on the vertical part of the curve. Cerium has been tested to 1.4°K. However, from Fig. 1, the possibility that it may be superconducting at a lower temperature is not excluded. The data on samarium give an atomic volume of 19.4 and indicate a Debye temperature in excess of 130°K. Although the atomic volume, 19.4, would lie within the region on the atomic volume *versus* atomic number graph, because of the relatively large value of Debye temperature, greater than 130°K, it is not likely that samarium is a superconductor.

Protoactinium occupies a position in the periodic table indicating an atomic volume in the neighborhood of 16 and a Debye temperature of the order of 150°K. Protoactinium belongs to the electro-negative group for which the data in Fig. 1 is very sketchy. The best that can be inferred from the data available is that there is some likelihood that protoactinium is superconducting and will have a transition temperature within reach of helium cryostats.

It would be interesting to see if something could be done along these lines with the superconducting compounds and alloys.

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On the Kinematics of Uniformly Accelerated Motions and Classical Electromagnetic Theory

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In continuation of earlier work a study is made of the 4-dimensional conformal group of transformations in space-time as the extension of the Lorentz group permitting the introduction of uniformly accelerated reference frames into relativity theory. The problem of the motion of a particle is discussed, as well as the implications for the classical-type electron theory developed by Dirac.

1. INTRODUCTION

THE problem of extending the special theory of relativity to permit the introduction of euclidean systems of uniformly accelerated reference axes has been shown[1-4] to depend on the generalization from the group of inhomogeneous Lorentz transformations L_4 to the group of conformal transformations C_4 in space-time. This group is characterized by a line-element of the form[5]

$$ds^2=\lambda^2(d\tau^2-dx^2-dy^2-dz^2)=\lambda^2\eta_{ij}dx^idx^j, \quad (1)$$

in which the function λ is determinable from the group properties of C_4. The mathematical characterization of this group was first given by Lie[6] who showed that it consists of a 15-parameter family, within which L_4 forms a 10-parameter subgroup. In the earlier discussion by the writer,[4] the detailed proof of the association of C_4 with uniformly accelerated motions was established only for the one-dimensional case, that for motion in three dimensions being obtained by generalization only. In the present paper the complete solution of this problem will be given.

The interest for physical theory in this extension of the special theory of relativity rests on three main foundations. In the first place, it supplies a direct procedure for the study of uniformly accelerated motions, in a relativistic sense, by their reduction to analytical coordinate transformations. Secondly, it has been known for a number of years from the work of Cunningham[7] and of Bateman[8] that C_4 is the general symmetry group of point transformations of the Maxwell-Lorentz field equations. The association with the kinematical interpretation of uniformly accelerated motions provides a direct approach to the study of the radiations from accelerated charged

[1] L. Page, Phys. Rev. **49**, 254 (1936).
[2] H. P. Robertson, Phys. Rev. **49**, 755 (1936).
[3] H. T. Engstrom and M. Zorn, Phys. Rev. **49**, 701 (1936).
[4] E. L. Hill, Phys. Rev. **67**, 358 (1945).
[5] The notation here is $x^0=\tau=ct$, $x^1=x$, $x^2=y$, $x^3=z$, with $\eta_{ij}=0$ if $i\neq j$, $=+1$ if $i=j=0$, $=-1$ if $i=j=1, 2, 3$.
[6] S. Lie, *Theorie der Transformationsgruppen* (Teubner, Leipzig, 1930).
[7] E. Cunningham, Proc. Lond. Math. Soc. [2]**8**, 77 (1910).
[8] H. Bateman, Proc. Lond. Math. Soc. [2]**8**, 223 (1910).