PHYSICAL REVIEW VOLUME 72, NUMBER 2 JULY 15, 1947

The Transition Effect for Large Bursts of Cosmic-Ray Ionization—II

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(Received April 4, 1947)

Observations were made of the bursts of cosmic-ray ionization in a chamber covered by thin plates of lead, iron, and magnesium. From the observed multiplications of the numbers of rays in the shield, it is possible to separate the bursts of ionization caused by large atmospheric showers from bursts caused by heavily ionizing particles ejected from the chamber walls. Knowledge of the number of atmospheric showers allows the energy spectrum of the cosmic-ray electrons in the high atmosphere to be calculated. The integral spectrum is found to follow a power law with the exponent -3.1 in the region near 2×10^{15} electron volts.

INTRODUCTION

In a previous paper[1] of the same title, some observations were reported on the change in the number of large bursts of ionization as the ionization chamber was moved from a location under a very thin roof to one under a roof of the equivalent thickness of about 1 meter of water and to a place under more than 3 meters equivalent thickness of earth. Observations were made with and without a flat lead plate of 1 cm thickness over the chamber. Under the layer of earth, the size-frequency distribution curves with and without the lead were parallel if plotted on log-log paper. Thus the increase in the number of bursts was interpreted as an increase in the size of the bursts by additional cascade multiplication in the lead plate. The amount of multiplication observed was in agreement with the predictions of the cascade theory. This simple situation did not exist, however, for the observations taken under the very thin roof. The size-frequency distribution curves with and without the lead were not parallel, and the multiplication for the bursts of small size was much less than expected from the cascade theory. Since the size-frequency distribution curve of bursts with no material over the ionization chamber is almost a direct measurement of the primary cosmic-ray energy spectrum in the region of very high energies, it was desirable to investigate this seeming discrepancy more completely. The observations[2] were therefore extended to include measurements with other materials of several thicknesses over the chamber.

OBSERVATIONS

The ionization chamber and recording system have been described previously.[3] The chamber was spherical, 40 cm in diameter with walls of magnesium 1 cm thick. The shielding materials covered a square area 41 cm on a side and were placed directly above the chamber. The chamber contained tank nitrogen at a pressure of 14.6 atmos. at 20°C. If a specific ionization of 60 ion pairs per cm at atmospheric pressure is assumed, a burst of 10^6 ion pairs corresponds to 60 rays through the chamber. A correction for the lack of saturation has been included.

TABLE I. The integral size-frequency distributions.

(The upper figure is the number per hour $R(n)$ of bursts of size greater than the specified ionization; the lower figure is the number of bursts observed.)

Ion pairs $\times 10^{-6}$	Number of rays, n	1 cm Mg	10 cm Mg	20 cm Mg	1 cm Fe	2 cm Fe	$\frac{1}{4}$ cm Pb	$\frac{1}{2}$ cm Pb	1 cm Pb
1.0	60	1.22	1.11	1.94	1.98	3.43	2.14	3.40	4.27
		420	98	218	554	847	867	841	910
1.1	66	0.60	0.68	1.16	1.30	2.50	1.17	2.25	3.17
		205	60	130	364	617	502	556	676
1.2	72	0.40	0.44	0.73	0.80	1.28	0.74	1.56	2.40
		139	39	83	223	316	317	387	511
1.3	78	0.25	0.32	0.48	0.50	0.95	0.50	0.86	1.76
		87	28	54	141	235	217	213	374
1.5	90	0.13	0.22	0.35	0.34	0.55	0.27	0.54	1.11
		57	20	39	96	136	117	133	237
2.0	120	0.061	0.10	0.17	0.17	0.20	0.12	0.27	0.53
		26	9	19	47	50	50	67	113
2.5	150	0.044	0.09	0.10	0.10	0.13	0.08	0.16	0.34
		19	8	11	28	32	36	41	72
3.0	180	0.028	0.07	0.07	0.05	0.08	0.06	0.11	0.24
		12	6	8	15	20	26	28	51
4.0	240	0.019	0.02	0.04	0.04	0.06	0.04	0.05	0.10
		8	2	5	10	15	19	12	21
6.0	360	0.005	0	0.04	0.02	0.04	0.02	0.01	0.06
		2	0	4	5	11	10	3	12

[1] C. G. Montgomery and D. D. Montgomery, Phys. Rev. **56**, 640 (1939).

[2] A preliminary report of these observations was made to the American Physical Society, Phys. Rev. **59**, 471 (1941).

[3] C. G. Montgomery and D. D. Montgomery, Phys. Rev. **47**, 429 (1935).

TABLE II. Integral distributions of air showers and disintegration bursts.

Ion pairs $\times 10^{-6}$	Number of rays	Density of rays meter^{-2}	$R_0(n)$	$R_D(n)$
1.0	60	478	0.37	0.85
1.1	66	526	0.30	0.30
1.2	72	574	0.25	0.15
1.3	78	621	0.20	0.05
1.5	90	717	0.15	
2.0	120	956	0.06	
2.5	150	1220	0.04	
3.0	180	1430	0.03	
4.0	240	1910	0.02	
6.0	360	2870	0.005	

The numbers and rates of occurrence of the bursts observed are given in Table I. For each shield, between one and two hundred hours of records were taken. The accuracy of the observed rates can be estimated from the number of bursts observed for showers of large or moderate size. The rates of occurrence of the smallest bursts are somewhat less accurate than would be estimated from the number, since for these bursts the accuracy in the measurement of the burst size is important. In a steeply falling distribution curve, a small error in size produces a much larger error in the rate.

ANALYSIS OF THE OBSERVATIONS

These observations show the same salient characteristic that was evident from the previous ones. The multiplication produced by the shield is smaller for the small bursts than for the large ones and less for the small bursts than expected from the cascade theory. This suggests that at least two agencies are operating to produce the burst of ionization. One agent, which predominates at large sizes, consists of cascade showers from the atmosphere which are multiplied in numbers of rays by the shield. The other agent, predominating at the smallest sizes measured, is not multiplied by the shield. It appears likely that these small bursts of ionization are the result of nuclear disintegrations produced by the cosmic radiation in which heavily ionizing particles are emitted, the "stars" found in cloud chambers and in photographic plates. If this were the case, the shield should have little effect on the number or size of such bursts.

The data can then be analyzed on this basis. Let the differential size-frequency distribution of the disintegration bursts be $r_D(n)$, the distribution of cascade showers from the air be $r_0(n)$, and the total distribution be $r(n)$. Then

$$r(n)=r_D(n)+r_0(n).$$

The air showers can be represented over the range of sizes in question by the power law

$$r_0(n)=An^{-S}.$$

This is evident from the curve for 1 cm of lead plotted in Fig. 1 of reference 1. Let us assume that the presence of the shield multiplies the number of rays in the air shower by a factor π and does not change r_D. Consequently, if $r'(n)$ is the total rate with the shield

$$r'(n)=r_D(n)+A\pi^S n^{-S}.$$

If the integral rates are represented by capital letters, that is

$$R(n)=\int_n^\infty r(n)dn,$$

then

$$R'(n)=R_D(n)+\pi^S R_0(n).$$

The integral rates given in Table I were, therefore, analyzed graphically into one distribution which was independent of the shield plus another of the power-law form with the constant term, but not the exponent, a function of the shield thickness and material.

It was found possible to fit all the observations with sufficient accuracy with a value of the parameter $S=3.5$ and with the distributions $R_D(n)$ and $R_0(n)$ given in Table II. It is evident that $R_D(n)$ is negligible for the larger showers of rays. This method of analysis is not a very accurate one. Several trials indicated that R_D could be found to within 10 to 20 percent. This

TABLE III. Comparison of observed and calculated multiplications.

Substance	Thickness in Arley's rad. units	Critical energy Mev	Observed multiplication	Calculated multiplication
Air	—	98	—	—
¼ cm Pb	0.70	7	1.27	1.2
½ cm Pb	1.39	7	1.53	1.7
1 cm Pb	2.78	7	1.88	2.9
1 cm Fe	0.78	25	1.36	1.1
2 cm Fe	1.56	25	1.64	1.2
10 cm Mg	—	56	1.12	—
20 cm Mg	1.86	56	1.30	1.1

results in a possible error in R_0 perhaps as large as 40 percent for the smallest sizes.

This decomposition of the observations also results in values of the multiplication π for the various shields, and these values of π may be compared with the predictions of the cascade theory. Table III lists the observed and calculated values of π, as well as pertinent parameters of the cascade theory. The cascade calculations were made in the following way. In an air shower there are rays both above and below the critical energy for multiplication in air. Since the energy distribution of the rays below the critical energy is not well known, a mean energy for an air-shower electron was chosen and used to estimate the factor π. The calculations of Arley[4] were employed because they appear to be the most accurate available for small thicknesses of material.

This method of calculation is admittedly only approximate. Besides the use of a mean energy, the circumstance that the shower rays pass through the shield at an angle has been neglected. Moreover, for showers incident at a large angle from the zenith, the shield does not completely cover the chamber. The production of electrons by photons in the air shower does not appear explicitly in the method of calculation. It is believed, however, that the values obtained in this manner are perhaps accurate to within 30 percent. A more significant error occurs in the neglect of the 1-cm wall of the chamber. This error is most important for lead which has a critical energy of only 7 Mev. Since 1 cm of magnesium will stop electrons of energies less than about 4 Mev, the values of the multiplication calculated in this way will be too large. The observed values of π agree perhaps as well as can be expected with the calculated values. For an accurate check of this point, both a better theoretical treatment and improved experiments with the shielding in a spherical shape surrounding the chamber are needed.

COMPARISON WITH OTHER OBSERVERS

Several other experiments have been reported in the literature which provide more information on the rate of occurrence $R_0(n)$ of air showers.

[4] N. Arley, Proc. Roy. Soc. **A168**, 519 (1938).

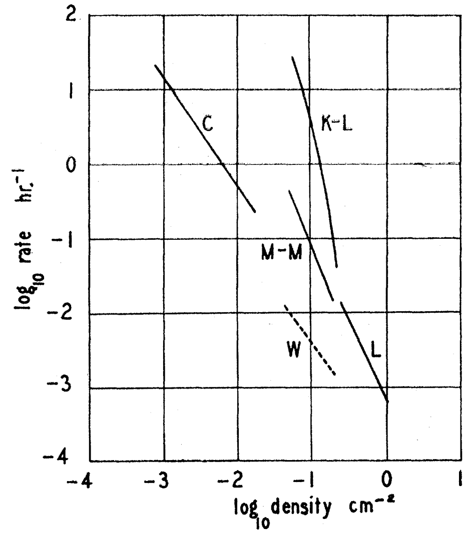

FIG. 1. The integral rates of occurrence as a function of the density of rays in air showers. Curve C represents the counter data of Cocconi, K-L the ionization-chamber data of Kingshill and Lewis, L the observations of Lapp corrected for multiplication, and M-M the corrected observations described in the present paper. The curve W represents Wolfenstein's calculation of the showers expected from Euler's electron-energy distribution.

Summaries of the previous work have recently been made by Lapp[5] and by Kingshill and Lewis.[6] These authors do not attempt to separate bursts produced by cascade showers from bursts produced by nuclear disintegrations within the ionization chamber. Information is lacking in most cases to allow this separation to be made. Lapp has reported, however, observations made in a steel ionization chamber of diameter 35 cm with walls 1.25 cm thick with no shielding material present. Showers with a density of rays larger than our own observations were recorded. For these large density showers, the contribution of nuclear bursts should be negligible. A correction should be introduced for the multiplication in the thick iron wall. If the observed value of the multiplication π given in Table III for 1 cm of iron is used, Lapp's rates of occurrence should be divided by a factor of π^S or about 3. Figure 1 shows a double logarithmic plot of these data in terms of the frequencies of bursts of a density of rays greater than a given value. Although the observations reported here do not cover the same

[5] R. E. Lapp, Phys. Rev. **69**, 321 (1946).
[6] K. L. Kingshill and L. G. Lewis, Phys. Rev. **69** 159 (1946).

range of densities, the agreement with Lapp's data corrected for multiplication is satisfactory.

The observations of Kingshill and Lewis are also plotted in Fig. 1. Their rates lie considerably above our observations and have a steeper slope. It seems likely that most of the bursts measured by these authors are the result of disintegrations within the ionization chamber. Kingshill and Lewis used a chamber 35 cm in diameter, but filled with argon to only slightly above atmospheric pressure. The amount of ionization produced by a shower of electrons of a given density is therefore much smaller than the number of ion pairs that would be produced in a high pressure chamber such as used in Lapp's experiments and in the present observations. In fact, the range of burst sizes observed by Kingshill and Lewis is from 10^5 to $6\cdot 10^5$ ion pairs, scarcely larger than the range of ionization of natural α-particles from the radioactive contamination in the chamber walls. This point of view is confirmed by the steep slope of the frequency curve which is also found in the values of R_D in Table II.

If this interpretation is correct, it is of course improper to plot the data of Kingshill and Lewis in terms of the density of rays in the shower. It would be more nearly correct to compare their observations with the values of R_D on the basis of the total ionization produced in the chamber. Such a comparison would not be valid, however, since the ranges of the particles and the consequent effect of pressure is not known. Moreover, in our experiments, it is doubtful that the ionization produced by a heavily ionizing particle would all be recorded because of extensive recombination present at high pressures in nitrogen.

Air showers have also been observed by counter techniques. The recent experiments of Cocconi and his collaborators[7] can be directly compared with the present observations. The rates of occurrence can be expressed as a power-law variation with the density of the rays in the shower. In Fig. 1, Cocconi's observations are plotted. These observations cover a range of densities smaller than those observed by ionization-chamber methods. A definite discrepancy appears to exist between observations made by the two methods. An investigation of the region of densities between 0.01 and 0.1 per cm^2 by both methods would be very desirable.

THE PRIMARY ENERGY SPECTRUM

The importance of the determination of the number of air showers is that the energy spectrum of the cosmic radiation at very high energies can be found. It may be assumed that electrons of very high energies are present in the upper atmosphere which originate either from some process that produces them there or from a process in the depths of space. The cascade theory can be used to calculate the multiplication in the atmosphere and the energy distribution of the electrons can be derived. Such a calculation, using our earlier data, was presented in the previous paper.[1] The theoretical expression for the lateral spreading of the showers which was used in these calculations has since been shown to be incorrect. More accurate calculations have since been made by Cocconi[7] and by Wolfenstein.[8] These authors, however, attack the problem from the other end; they assume an energy spectrum of electrons at the top of the atmosphere, and calculate the number of air showers at sea level. The dashed curve in Fig. 1 shows the results of Wolfenstein's calculations based on the primary spectrum proposed by Euler,[9] namely,

$$F(>E) = 170(10^8\,\text{ev}/E)^{1.8}\ \text{cm}^{-2}\ \text{sec}^{-1}.$$

Cocconi used the same primary spectrum. Wolfenstein's calculation may be used to find what energy spectrum is necessary to produce the observed number of air showers, since there is a rough one-to-one correspondence in the energy and the density of shower rays. From Fig. 1 for showers of a density of 0.1 cm^{-2}, the calculated value is too small by a factor of 34. The power law calculated is -1.5 and the observed value -2.5. The observed exponent for showers corresponds[1] to an exponent of -3.1 in the integral primary energy spectrum. The required integral spectrum becomes, therefore,

$$F(>E) = 1.4\times 10^{-9}(10^{15}\,\text{ev}/E)^{3.1}\ \text{cm}^{-2}\ \text{sec}^{-1}.$$

This spectrum should be valid near 10^{15} ev. The accuracy of these estimates, both theoretical and experimental, leave much to be desired.

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[7] G. Cocconi, A. Loverdo, and V. Gongiorgi, Phys. Rev. **70**, 841, 846 (1946).

[8] L. Wolfenstein, Phys. Rev. **67**, 238 (1945).

[9] H. Euler, Zeits. f. Physik **116**, 73 (1940).