

In the cases where information on annealing is given it is clear that the ratio of the resistance at helium temperatures to that at 0°C (R_n/R_{273}), the transition temperature and the initial critical field slope are strongly affected by the heat treatment. In view of this, the poor characteristics of the sample of Keesom and Désirant² are probably explained as being due to severe strains in their specimen. The influence of kinking the tantalum wire⁷ and reducing the wire diameter⁸ on the superconducting characteristics are probably similarly explained.

The influence of current on the critical fields of the annealed and outgassed specimen is shown in Fig. 6. The critical fields are 50 to 70 gauss lower for the higher current, an amount considerably in excess of what would be expected from the hypothesis of Silsbee.¹⁴

The author is particularly indebted to Professor C. T. Lane for his frequent advice and encouragement. He is also indebted to Dr. Henry A. Fairbank and Mr. William Fairbank for their assistance in the collection of data.

¹⁴ F. B. Silsbee, *Sci. Pap. Bur. Stand.* **14**, 301 (1917).

Ultrasonic Velocity and Absorption in Liquid Helium*

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Measurements are given on the velocity and attenuation of ultrasonic energy in liquid helium at a frequency of 15 megacycles per second as a function of temperature from 1.57°K to 4.5°K. The velocity at 15 Mc/sec. was found to agree with results obtained at 1.3 Mc/sec. by previous investigators, and hence there is no dispersion in this frequency range. The attenuation measurements exhibit three important features: (a) in the upper temperature range of He I the measurements agree very well with classical theory, (b) at the λ -point the attenuation coefficient rises abruptly, presumably to infinity, indicating complete absorption of the ultrasonic energy, (c) just below the λ -point the attenuation (first sound in He II) has its smallest value and with lowering temperature the attenuation increases.

I. INTRODUCTION

THE experiments described in this paper give the velocity and attenuation of sound in liquid helium at a frequency of 15 megacycles per second as a function of temperature from 1.57°K to 4.5°K. The object of these experiments is twofold: (a) to check experimentally the theory of sound absorption for a monatomic liquid, and (b), to probe into the nature of liquid helium in its low temperature phase and at the transition point. The experiments were made possible by two recent developments: the ultrasonic pulse technique which grew out of radar

research¹ and the Collins Cryostat² which can produce relatively large amounts of liquid helium.

II. EXPERIMENTAL

The special advantages and capabilities of the pulse technique originally developed for radar have made measurements of the type conducted in this research possible. By using short pulses absorption measurements may be conducted at relatively high frequencies (15 Mc/sec. in this case), for which the attenuation is sufficient to



FIG. 1. Transducer-reflector system.

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¹ Radiation Laboratory, Massachusetts Institute of Technology Report 963, March 1946.

² S. C. Collins, *Rev. Sci. Inst.* **18**, 3 (1947).

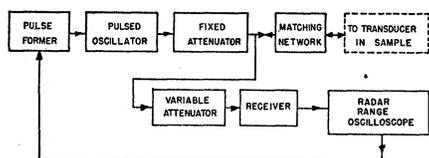


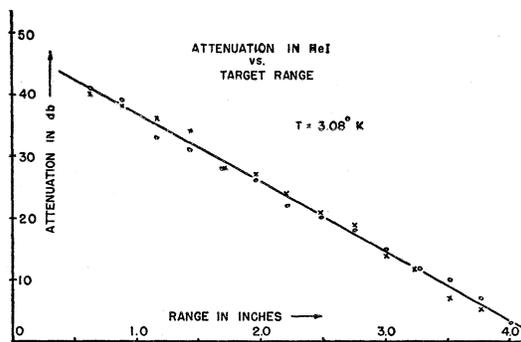
FIG. 2. Block diagram.

make quite accurate measurements possible. As will be seen, the method is direct and straightforward both for obtaining velocity and attenuation; absorption measurements thus obtained³ are usually superior to those of older, continuous wave methods.

The scheme is essentially to use the liquid sample as a "storage medium" for the short sound pulses and to measure the time delay and attenuation undergone by the sound in traveling a known path length. The acoustical pulses, after being generated from electrical pulses by the transducer, travel through the liquid to a plane reflector (shown purely schematically in Fig. 1) and back again to re-excite the crystal at a later time.

The transducer-reflector distance is variable, so that the increased delay produced by an increase in path length is a direct measure of wave velocity. The attenuation which must be removed from the electrical circuit to balance the acoustical losses in the additional distance provides a measure of absorption.

The timing sequence is as follows (Fig. 2). A crystal-controlled circuit in the oscilloscope starts the scope sweep and simultaneously sends a

FIG. 3. Sample plot of receiver attenuation *versus* target range at constant temperature.

³ J. R. Pellam and J. K. Galt, *J. Chem. Phys.* **14**, 608 (1946).

trigger to the pulse generator. The pulse formed by the generator passes through an impedance-matching network to the transducer. When the resulting sound pulse returns to the transducer after its round trip within the liquid, the delayed electrical pulse is formed and returns through the matching network. Since the transmitter is off when the echo signal arrives, very little leakage loss occurs and the main portion travels through the variable attenuator to the receiver.

Both triggering and time measurements are accomplished by means of the crystal-controlled circuit in the oscilloscope (DuMont type 256B A/R range scope), so that automatic synchronization occurs. In addition to setting off the pulse generator and starting the sweep, this circuit also provides marker pips spaced every 10,000 yards of radar range. An expanded sweep with variable delay is used for examination of the front edge of the echo signal. Crystal control is necessary primarily for accurate timing of the marker pips; one pip is developed for each oscillation, and since the crystal frequency is accurate to better than 1 part in 10,000 the same holds for range marker positions.

Concerning measurement procedures, velocity is obtained directly from the slope of *distance travelled* plotted *versus pulse delay*, and absorption is obtained from the slope of *compensating electrical attenuation versus distance*. Since the method depends only upon observing the effects of differences in acoustical path length, knowledge of exact distance traveled by the sound is unnecessary.

Velocity readings are taken for helium at 10,000-yard (radar) range intervals by placing the returned signal pip at definite positions with respect to successive scope-range markers, and recording the transducer-reflector distance. By adjusting the scope properly these markers appear as very short (dark) breaks in the sweep, and, if consistent criteria are adopted, accurate range difference readings result. Since the echo signal is adjusted to a predetermined level on the scope for each range position (by means of attenuators, without disturbing the gain of the remainder of the system), absorption data are obtained directly. Figure 3 gives an example of excellent data for attenuation in liquid helium at 3.08°K; the attenuation is plotted against the

transducer-reflector distance, and the slope of the straight line is a measure of absorption.

Results obtained by such measurements may be regarded as physically equivalent to those obtained by continuous wave methods. It may be shown that differences in behavior between pulsed sound and continuous sound in liquids should produce negligible effects. For example, no significant dispersion in liquids has been detected in the 15 Mc/sec. range (nor pulse distortion) so that the group velocities measured by pulsing should not differ measurably from the phase velocities ordinarily determined. Similarly, although attenuation within liquids varies as frequency squared, the band width associated with the pulses is too narrow for a measurable effect to result.

The velocity of ordinary sound in liquid helium is about 2×10^4 cm/sec., corresponding to a wave-length at 15 Mc/sec. of roughly 1.3×10^{-3} cm. Since the crystal radiating and receiving surface is about 1 cm across, this diameter corresponds to about 750 wave-lengths. Accordingly, the pulses travel out within a very narrow beam (0.1° when computed on the basis of Fraunhofer diffraction to obtain an order of magnitude). Therefore, the pulses retain nearly their exact size and shape throughout their travel, so that any geometrical attenuations caused by spreading are completely overshadowed by true liquid absorption. Therefore, attenuation data may be converted directly to absorption coefficients by applying a numerical factor; thus from the linearity of the curve of Fig. 3 one obtains db/inch = 11.1, and $\alpha = 11.1/44.3 = 0.25$ is the absorption coefficient, in (cm^{-1}) .

In the classical theory of sound⁴ the α is commonly referred to as the coefficient of pressure attenuation and enters into the solution of the wave equation as

$$P = P_0 e^{-2\pi i\nu[t-(x/v)] - \alpha x}, \quad (1)$$

where P is the pressure, ν the frequency, v the wave velocity, and x the coordinate of distance along which the wave travels. In Fig. 3 the exponential decay is experimentally verified, and it is understood that the α determined from the slope of data such as in Fig. 3 is the composite

attenuation caused by viscosity and heat conduction. For liquids containing polyatomic molecules an additional attenuation involving internal molecular vibrations would also be included.

In discussing the equipment for these experiments no detailed information (such as circuit diagrams) is included concerning the standard electronic elements of pulse power, transmitter, and the receiver. The ultrasonic equipment in the bath of liquid helium requires some discussion and is illustrated in Fig. 4. The entire ultrasonic equipment is placed in a Collins Cryostat² (A) where as much as 5 liters of liquid helium may be produced for experiments. The ultrasonic equipment and its associated helium vapor pressure thermometer, (T), pumping tubes, (P), and vacuum jacket, (V), underwent

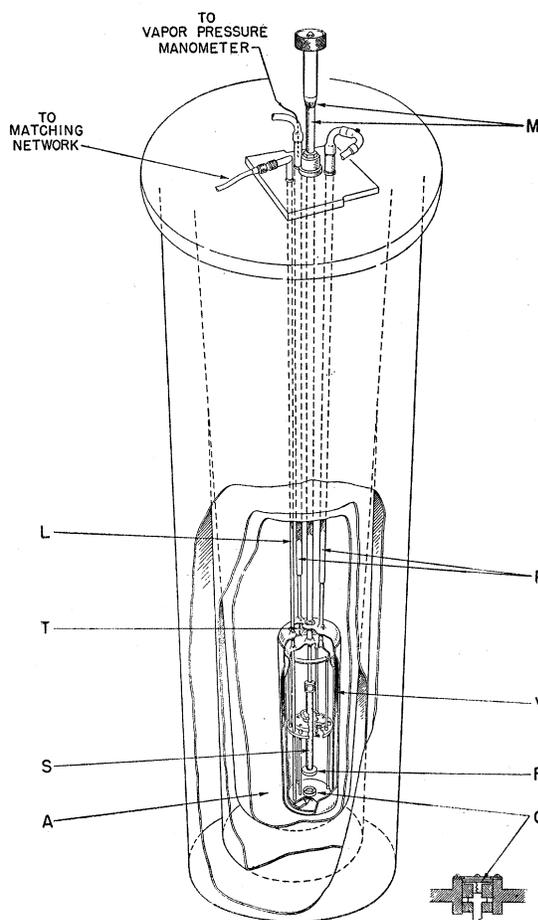


FIG. 4. Composite isometric view of transducer-reflector equipment within cryostat.

⁴ Lord Rayleigh, *Theory of Sound* (Dover Publications, New York, 1945), Vol. 2.

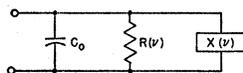


FIG. 5. Equivalent matching network.

continuous development* during the course of the research program. The crystal (C) was driven by electrical pulses, arriving through the coaxial line (L) at a frequency of 15 Mc/sec. The resulting sound waves traveled out from the crystal to the reflecting target (R). This mirror could be placed at any desired range by means of the screw (S). This precision screw was driven by a steel rod extending from the top of the equipment, and was equipped with a micrometer vernier scale (M) for precise distance readings. Liquid helium was allowed to condense in the innermost container until it covered the crystal transducer to a height of approximately 15 centimeters. The intermediate chamber (V) was then pumped out to a high vacuum so that the liquid in which the measurements are made was thermally insulated. Except at the very lowest

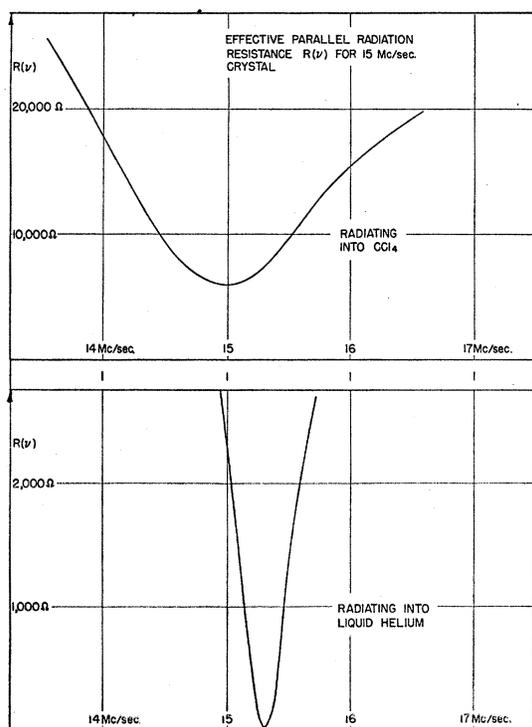


FIG. 6. Radiation resistance of crystal versus frequency.

* An earlier version of low temperature sound gear is described by Dr. J. Galt in a paper "Sound Absorption and Velocity in Liquefied Argon, Oxygen, Nitrogen, and Hydrogen" to be published shortly.

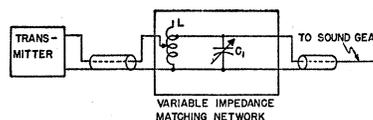


FIG. 7. Impedance-matching network.

temperatures (below 2°K) the liquid in the outer cryostat was kept at a slightly lower temperature than the liquid covering the crystal. These conditions permitted excellent temperature control and avoided rapid boiling which could have been detrimental to accurate readings.

The transducer consisted of a thin x cut quartz crystal one centimeter in diameter. Such a crystal undergoes thickness vibrations when an oscillating voltage is applied across its thin dimension. The crystal was cut to the proper thickness (about 0.018 cm) for its fundamental resonance to occur at approximately 15 Mc/sec.

The upper (and radiating) surface was completely silvered and was grounded to the housing cartridge. On the other hand, the silver plating on the lower surface did not extend quite to the crystal circumference, thus forming an insulated electrode to which the r-f signal could be fed from the pulser. This transducer had two separate effects on the electrical properties of the system. First of all the geometry of the crystal provided a certain fixed capacity between its electrodes, amounting to about $25 \mu\text{f}$.

A more important effect of the transducer upon the system was a marked narrowing of the band width, especially for the case when the crystal was in contact with liquid helium. This may be illustrated by means of the equivalent transducer circuit, Fig. 5. This particular equivalent form for representing the crystal has been chosen as convenient for the present discussion. Here C_0 is the fixed crystal capacity ($25 \mu\text{f}$) in parallel with a frequency-dependent radiation resistance $R(\nu)$ and a frequency-dependent reactance $X(\nu)$.

The band width of the transducer may be determined by measuring the radiation resistance $R(\nu)$ vs. frequency and obtaining the half-power points. Sample Q -meter measurements of $R(\nu)$, given in Fig. 6 for CCl_4 and liquid helium, illustrate the sensitivity of the electrical properties of the crystal to the medium into which it radiates. It is noted that, whereas for CCl_4 the band width $\Delta\nu$ is about 1.7 Mc/sec.,

the band width for liquid helium is only about 0.2 Mc/sec.

The characteristic impedance of most ordinary liquids such as CCl_4 is about one-tenth that for crystalline quartz, and this provides sufficient damping (through radiation) to reduce the crystal Q to the neighborhood of 10 or less. On the other hand, the characteristic acoustical impedance for liquid helium being only about one-thousandth that for quartz, the band width of 0.2 Mc/sec. corresponds to a Q of about 75, since

$$Q = v/\Delta v = 15/0.2 \approx 75. \quad (2)$$

The band width $\Delta v = 0.2$ Mc may be converted directly to the corresponding acceptable pulse length $\Delta \tau$ by the relationship

$$\Delta \tau = (\Delta v)^{-1} = 5 \times 10^{-6} \text{ sec.} \quad (3)$$

It is therefore evident that pulse-lengths of at least 5 $\mu\text{sec.}$ duration were required. This corresponded to about 75 complete oscillations per pulse, also numerically equal to the Q of the system. These are merely quantitative ways of expressing the fact that helium is a most difficult liquid in which to generate sound pulses.

It is necessary to match the impedance presented by the crystal system to that of the coaxial line reading from the pulser. At resonance the impedance presented by the crystal is complex, and ordinarily the impedance level is different from that of the line. Therefore a matching network was essential, and was placed directly outside the cryostat. This is illustrated in Fig. 7.

The effect of the fixed capacity of the crystal (25 $\mu\mu\text{f}$) plus the capacity of the internal coaxial line (100 $\mu\mu\text{f}$) was tuned out by means of the coil L . The fixed tap from the coaxial line into this coil was positioned to provide somewhat more inductance than actually necessary for this purpose. The excess was available for adjustment during operation (using the echo signal as a guide) by tuning out with the variable capacity C_1 .

The input signal from the transmitter entered coil L a *via* variable tap T . A sliding contact was provided to make the turns ratio continuously variable, and the setting was obtained by maximizing the returned echo signal.

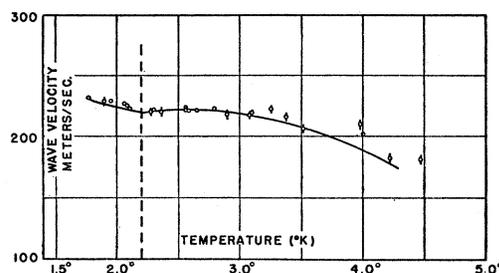


FIG. 8. Ultrasonic wave velocity *versus* temperature in liquid helium. — Toronto data (1.3 Mc/sec.); ○○○ Massachusetts Institute of Technology data (15 Mc/sec.)

III. RESULTS

The velocity of first sound (meter/sec.) is plotted *versus* temperature (degrees Kelvin) in Fig. 8. The circles represent individual values at 15 Mc/sec. and are determined by the pulse method described. These are compared with results of previous investigators⁵ obtained interferometrically at 1.3 Mc/sec. and indicated by the solid line. The general correlation shows the absence of any dispersive effects within this frequency range. All of the above data refer to liquid helium under its own vapor pressure. The investigation by the Toronto Laboratory⁵ revealed a discontinuity in velocity at the λ -point under conditions of increased pressure. The conclusion was that whatever discontinuity existed

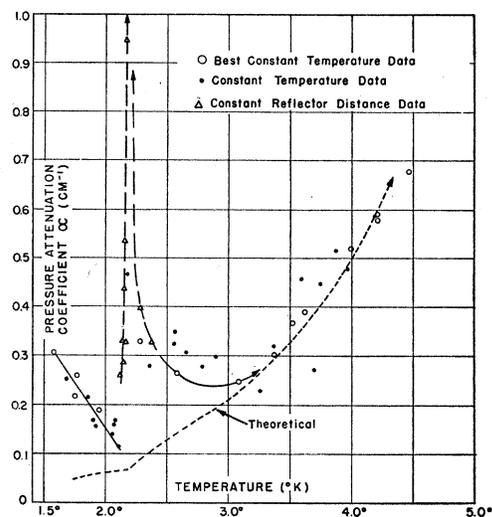


FIG. 9. Ultrasonic pressure attenuation coefficient α *versus* temperature.

⁵ Findlay, Pitt, Grayson Smith, and Wilhelm, Phys. Rev. 54, 506 (1938); *ibid.* 56, 122 (1939).

TABLE I. Values of α , the attenuation coefficient and the ratio of c_p/c_v from which α was calculated.

Temperature	$\gamma-1$	$\alpha_{\text{viscosity}}$	α_{thermal}	Total $\alpha_{\text{theoretical}}$	Total $\alpha_{\text{experimental}}$
5.10 \pm 0.1°K					(1.78 cm ⁻¹)
4.47 \pm 0.01					0.68
4.22 \pm 0.01	1.35	0.244	0.382	0.62	0.58
4.00 \pm 0.01	1.14	0.200	0.260	0.46	0.52
3.62	0.80	0.131	0.198	0.33	0.39
3.52	0.72	0.122	0.184	0.30	0.37
3.38	0.58	0.114	0.164	0.28	0.30
3.08	0.38	0.103	0.122	0.22	0.25
2.58	0.14	0.087	0.055	0.14	0.27
2.28	0.03	0.079	0.015	0.09	0.33
λ -point (2.19)	0	0.077	0	0.077	(∞)
2.11		0.071	0	0.071	0.11
1.94		0.061	0	0.061	0.15
1.76		0.050	0	0.050	0.24
1.57		0.042	0	0.042	0.30

under its own vapor pressure conditions was too small for measurement by that method. Similarly, the present technique was incapable of detecting such an effect.

Attenuation

The pressure attenuation coefficient α in cm⁻¹ is plotted as a function of temperature in degrees Kelvin in Fig. 9 and represents the essentially new information resulting from this investigation. Each point represents a separate series of electrical attenuation *versus* range readings at constant temperature. These db *versus* range data were graded according to linearity of the plots, and the most reliable results from this standpoint indicated by circles. The results of remaining runs contributing significant but less reliable values are represented by dots. The solid curves of Fig. 9 represent the composite average for the measurements. In the region above 3°K the solid curve is omitted since it merges with the theoretical curve represented by the dotted line.

It is evident from Fig. 9 that the absorption changes abruptly in the neighborhood of the λ -point, so that for this temperature region it proved impractical to conduct db *versus* range measurements. Accordingly, an alternative procedure was adopted of holding the transducer-reflector distance fixed and noting receiver signal strength *versus* temperature as the λ -point was passed. Although this technique is inadequate for detailed measurements in the low attenuation ranges, it is suitable for mapping out large scale, rapid changes. This is represented by the triangles and dashed line of Fig. 9. Such a pro-

cedure yields only differences in α after one has obtained the shape of the curve; the absolute magnitude of α is adjusted to fit the true values of the solid line curve of Fig. 9.

Several significant features are revealed. Beginning with the highest temperatures in the He I domain, the attenuation falls steadily as temperature is decreased. A minimum is reached in the 2.8°K region below which the attenuation increases, finally to rise sharply in the immediate vicinity of the λ -point. It was not possible to determine whether α actually became infinite, but on the basis of signal strengths at temperatures of known attenuation it is clear that values of at least 3.5 cm⁻¹ occur for α .

Lowest attenuations occur immediately below the λ -point where values of about 0.1 cm⁻¹ were observed. However, at lower temperatures the attenuation again increases in a steady manner. This trend appears to continue even to the lowest temperature at which measurements were conducted (1.57°K).

Numerical values of attenuation coefficient α , including one at 5.1°K, are given in Table I for runs indicated by circles in Fig. 9. Numerical values for the computed α are given at corresponding temperatures where data exist for such computations. The ratio of the specific heat capacities $\gamma = c_p/c_v$ can be computed from data in Keesom's recent book⁶ by means of the thermodynamic relationship

$$\gamma - 1 = - \frac{T}{\rho^2 c_v} \left(\frac{\partial P}{\partial T} \right)_\rho \left(\frac{\partial \rho}{\partial T} \right)_P,$$

where ρ is the density, P the pressure, and T the temperature. Values of $(\gamma - 1)$ are included with the theoretical and experimental results of Table I.

IV. DISCUSSION

He I

One of the revealing features of the attenuation measurements is the manner in which classical theory completely accounts for the absorption in He I at temperatures above about 3°K. Values computed on the basis of classical theory are represented by the dotted curve of Fig. 9, which

⁶ W. H. Keesom, *Helium* (Elsevier Publishing Company, Inc., Amsterdam, 1942), p. 244.

above 3°K also represents the best experimental curve. This is significant, since for ordinary liquids (with the exception of Hg) the measured values consistently exceed⁷ the amounts expected on the basis of classical viscosity and heat-conduction losses. This is presumably attributable to the monatomic nature of helium, which precludes the type of relaxation phenomena associated with the inner degrees of vibration of polyatomic molecules. Moreover, He I presents a situation in which the relative contributions to absorption between viscosity and heat conductivity vary over a large range as temperature is varied. Numerical values for these two classical contributions to absorption,

$$\alpha = \alpha_{\text{viscosity}} + \alpha_{\text{thermal}}, \quad (4)$$

are given in Table I and were computed according to the well-known formulas⁸

$$\alpha_{\text{viscosity}} = \frac{8\pi^2}{3} \frac{\eta}{\rho c_1^3} \nu^2, \quad (5)$$

$$\alpha_{\text{thermal}} = \frac{2\pi^2(\gamma-1)K}{\rho c_1^3 c_p} \nu^2. \quad (6)$$

In the above, ν is the ultrasonic frequency in cycles per second, c_1 the ultrasonic velocity in cm/sec., η the coefficient of viscosity in poise, ρ the liquid density in g/cc, K the heat conductivity coefficient in cal./deg. cm sec., c_p the specific heat at constant pressure in cal./g deg., and γ the ratio of specific heats. With the exception of heat conductivity K , each of these quantities is a known function of temperature. In the case of K results are published⁹ only for 3.3°K, so that this value of 6.0×10^{-5} cal./cm sec. was of necessity used throughout the temperature range of the He I calculations. It is noteworthy that the magnitude of the ultrasonic absorption places He I in the category of relatively high absorption liquids (in spite of its low viscosity). This behavior is accounted for by

the occurrence in the denominator of Eqs. (5) and (6) of ρ and c_1^3 , both of which are extremely low for helium.

In the general region of 3°K the experimental results of α begin to exceed considerably the classical theoretical values. Finally, the sharp peak occurs at the λ -point, whereas the predicted value for that temperature is only 0.07 cm^{-1} . Clearly, the inadequacy of the classical theory points to some other mechanism. Possibly a discontinuity in velocity such as suggested by the Toronto investigations might account for this effect. Another more general explanation of the complete absorption of ultrasonic energy at the λ -point is possible. The energy of transition from He II phase to He I phase is vanishingly small as the system approaches the λ -point (because the phase transition is of the second order—i.e., no latent heat at the λ -point). In just this region such transitions might be forced by the ultrasonic energy.

He II

The mechanism and formulas of classical absorption are not directly applicable to the case of He II since this liquid obeys a special system of complex hydrodynamics. Discussion of attenuation is rendered difficult because the complex hydrodynamics has been developed only to the first approximation. The main characteristic of He II is the possibility of a heat transfer as a first-order hydrodynamical process. The only dissipative process considered so far in the literature is the damping of an oscillating disk through viscosity. It appears that the classical formula (5) may be applied below the λ -point as long as the appropriate value of η is used. It turns out that this proper quantity is the effective viscosity, η_n referred to the normal fluid only (η_s for superfluid is taken as zero). Since for first sound, which we are considering, the two fluid components vibrate in phase with essentially identical particle excursions, the fluids are effectively locked together. Therefore, in setting up a wave equation the differential acceleration force on a thin layer involves the total density ρ , exactly as for the classical case (see footnote of Eq. (5)). However, the viscous retarding forces opposing the expansion and contraction of the layer are dependent upon the only existent vis-

⁷ The discrepancy ranges to an extreme factor of 800 in the case of liquid H₂S.

⁸ The expression for $\alpha_{\text{viscosity}}$ may be derived from the wave equation $\rho \ddot{\xi} - \rho c^2 \nabla^2 \xi = (4/3) \eta \nabla^2 \dot{\xi}$, where the last term involves viscosity (here ξ is the particle displacement). A similar formulation involving heat flow leads to (6); L. Bergman, *Ultrasonics* (John Wiley and Sons, Inc., New York, 1939), p. 128.

⁹ W. H. Keesom, *Physica* 5, 281 (1938).

cosity present, namely, η_n for the normal fluid component. Accordingly, for He II this becomes

$$\alpha_{\text{viscosity}} = \frac{8\pi^2}{3} \frac{\eta_n}{\rho c_1^3} \nu^2, \quad (7)$$

which is the quantity plotted in Fig. 9 for points below the λ -point. The curve falls only gradually with lowering temperature, since the viscosity coefficient η_n is given by a smooth extension^{††} of η for He I into the He II region. The effect of heat conductivity on ultrasonic absorption is more complicated. The ordinary diffusion type of heat transfer must exist also in He II, but is completely masked by the first-order (reversible) heat transfer. The latter manifests itself in the so-called second sound,^{10,11} which has been verified experimentally by Peshkov¹² and by Lane.¹³ The α -thermal of Eq. (6) would be of the same order of magnitude in He II as it is in He I close

to the λ -point and would not essentially change the total theoretical α plotted in Fig. 9 below the λ -point.

One might expect other dissipation effects which are peculiar to He II and which would give ultrasonic absorption of the relaxation type. Tisza¹⁰ has suggested non-adiabatic transitions between the normal and superfluid component induced by ultrasonic waves. Other relaxation mechanisms such as between the ultrasonic energy and the zero-point energy may also appear. Finally, the coupling between first and second sound might give rise to dissipation effects. It does not seem possible at present to compute these effects without making arbitrary assumptions.

V. ACKNOWLEDGMENTS

Professor S. C. Collins has given this research a great deal of enthusiastic and technical support, and the authors are most pleased to acknowledge his generosity. There have been many fruitful discussions with Professor J. C. Slater and with Professor J. A. Stratton. Finally, the authors wish to express their thanks for the interest shown in this work by Professor F. Bitter and especially to Professor L. Tisza for many valuable discussions concerning the theory of liquid helium.

^{††} Viscosity measurements in He II made by use of the oscillating-disk method⁶ were originally taken to indicate a coefficient falling sharply with temperature from the λ -point. A subsequent reinterpretation of the viscosity data by Tisza leads to a normal value of η for the normal fluid and an essentially zero value of η for the superfluid.

¹⁰ L. Tisza, *J. de phys. et rad.* **8**, 1 (1940), *Phys. Rev.* **72**, 838 (1947).

¹¹ L. Landau, *J. Phys. U.S.S.R.* **5**, 71 (1941).

¹² Peshkov; *J. Phys. U.S.S.R.* **8**, 381 (1944).

¹³ C. T. Lane, H. Fairbank, and W. Fairbank, *Phys. Rev.* **71**, 9 (1947).