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Long Range Tensor Forces and the Magnetic Moment of the Deuteron

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A theoretical study has been made of relativistic effects in the magnetic moment of the deuteron. It is concluded that the present unsatisfactory state of the theory of nuclear forces makes it impracticable to deduce the amount of D state admixture in the wave function of the deuteron ground state from the experimental value of $\mu_d-(\mu_p+\mu_n)$. The possibility that nuclear tensor forces have ranges considerably longer (and magnitudes considerably smaller) than the central forces is also discussed.

THE recent precise measurements of the magnetic moments of the (free) neutron, deuteron, and proton μ_n, μ_d, μ_p give[1] (in nuclear magnetons) $\mu_d-(\mu_p+\mu_n)=0.8565-(2.7896-1.9103)=-0.0228$. This result receives an exact quantitative explanation if it is assumed that:[2]

I. The proton and neutron retain, when combined in the deuteron, the magnetic moments they possess when free and moving slowly.

II. The central and tensor forces operative in the deuteron between the proton and neutron have the same range[3] and magnitudes (square-well depths) appropriate to the deuteron's binding energy and quadrupole moment. As a result, the ground state of the deuteron has a 3.9 percent $^3D_1$ admixture with a (proton orbital) magnetic moment of $-\frac{3}{2}(\mu_p+\mu_n-\frac{1}{2})(0.039)=-0.0222$, a value differing by only 0.0006 from the experimentally observed $\mu_d-(\mu_p+\mu_n)$.

Now assumption I is subject to failure because of:

(A) a field-theoretic non-additivity of the magnetic moments of the proton and neutron in the deuteron; this effect may exist but cannot be quantitatively estimated at present.

(B) a relativistic variation of the magnetic moments of the proton and neutron due to their comparatively large velocities in the deuteron.

Further, assumption II suffers from the rather arbitrary choice of equal ranges for the central and tensor forces; this choice leads to particular difficulties with the binding energies of H^3 and He^4,[4] since a common range for the central and tensor forces implies a tensor force not much smaller than the central force. This latter force is then considerably smaller than the central force required to bind the deuteron in the absence of the tensor force. Now, because of its spin dependence, the tensor force is most effective in the triplet ground state of the deuteron; in the H^3 and He^4 nuclei, however, the spins of the nucleons are either partly or wholly paired up so that the tensor force is relatively ineffective, and the corresponding comparatively small central force is insufficient to give the observed binding.

In view of these difficulties and of the fact that the relativistic variation of the proton and neutron magnetic moments within the deuteron is certainly present and is even of the right order of magnitude[5] to account for the whole experimental value of $\mu_d-(\mu_p+\mu_n)$, a calculation has been made to estimate the relativistic effect. In this calculation, the motion of the proton and neutron in the deuteron is treated as a relativistic two-body problem by approximate methods involving elimination of the small components of the 16 component Dirac wave function of the two nucleons,[6] these nucleons being supposed to satisfy a Dirac type wave-equation with additional terms of the Pauli type to describe their anomalous magnetic moments.[7] In addition, the

[1] W. R. Arnold and A. Roberts, Phys. Rev. **70**, 766 (1946); S. Millman and P. Kusch, *ibid.* **60**, 91 (1941); J. M. B. Kellogg, I. I. Rabi, N. F. Ramsey, and J. R. Zacharias, *ibid.* **56**, 728 (1939).

[2] W. Rarita and J. Schwinger, Phys. Rev. **59**, 436 (1941).

[3] This range is taken in reference 2, in accordance with the postulate of the charge independence of nuclear forces, as 2.8×10^{-13} cm, the value giving the best square-well fit to the proton-proton scattering.

[4] E. Gerjuoy and J. Schwinger, Phys. Rev. **61**, 138 (1942).

[5] H. Margenau, Phys. Rev. **57**, 83 (1940); and P. Caldirola, *ibid.* **69**, 608 (1946), estimate the order of magnitude of the relativistic variation of the magnetic moment of a nucleon (proton or neutron) in the deuteron by supposing it to move in a static force field (e.g., a short range square-well) arising from the other nucleon.

[6] G. Breit, Phys. Rev. **51**, 248 (1937); S. Share and G. Breit, *ibid.* **52**, 546 (1937).

[7] Similar calculations with generally similar results have been made by R. G. Sachs. I wish to express my thanks to Dr. Sachs for the opportunity of seeing the manuscript of his paper, "On the Magnetic Moment of the Deuteron," prior to its publication. *Note Added in Proof:* A model in which the proton and neutron move in the same (properly adjusted) central force field but do not interact with each other has just been used by G. Breit (Phys. Rev. **71**, 400,

tensor force between the nucleons, which is required to explain the quadrupole moment of the deuteron, is assumed (in accordance with a suggestion by E. Feenberg) to have a much longer range and (hence) smaller magnitude than the central force; this assumption removes the major portion of the above mentioned difficulty with the binding energies of H^3 and He^4, is in agreement with the fact that the deuteron quadrupole moment is much smaller than the square of is effective radius, and is even not inconsistent with a possible field theoretic formulation.[8]

With regard to the results of this calculation, it is found that the magnitude of the relativistic variation of the proton and neutron magnetic moments, as determined by the calculated value of $\mu_d-(\mu_p+\mu_n)$, (neglecting the effect of tensor force), is of the form

$$\alpha\left[\frac{\text{average kinetic energy of the nucleons in the deuteron}}{\text{rest energy of the nucleons}}\right], \qquad (1)$$

where α is a numerical constant of the order of unity. The exact value of α and even its sign are, however, strongly dependent upon the covariance properties of the nuclear force fields (e.g., scalar or vector or linear combination of the two). Thus, with a central force between the nucleons, arising from a not unreasonable combination of scalar and vector fields and of range $\cong 1.0\times 10^{-13}$ cm, one can account for the whole experimental value of $\mu_d-(\mu_p+\mu_n)$ on the basis of the relativistic correction to the deuteron magnetic moment given in Eq. (1); on the other hand, if the central force has a range of 2.8×10^{-13} cm, as in reference 3, the kinetic energy factor in Eq. (1) is so small that (unless a rather improbable combination of scalar and vector fields is chosen) the relativistic correction is comparatively unimportant—perhaps 10 to 20 percent of the experimental value of $\mu_d-(\mu_p+\mu_n)$. Again, neglecting the relativistic effects and solving, by a crude variational method, the Schrödinger equation (with the strong short range central force and the weak long range tensor force) for the $^3S_1$ and $^3D_1$ wave-functions, one finds (consistent with the observed quadrupole moment and more or less independently of the exact value of the ranges) a 1 percent $^3D_1$ admixture and a proton orbital moment of only -0.006; thus if the proton-neutron tensor force is actually weak and long range (as is perhaps indicated by the binding energies of H^3 and He^4 —see above), the relativistic effect of Eq. (1) must account for the major portion of the experimental value of $\mu_d-(\mu_p+\mu_n)$, so that the central force must have a rather short range.

The preceding discussion makes it clear that the absence of any satisfactory field theory of nuclear forces, with the resultant uncertainty regarding the field-covariance properties, ranges, and magnitudes of both the central and tensor forces, renders impossible an exact quantitative estimate regarding the magnitude of the relativistic variation of the magnetic moments. It is therefore not feasible to deduce unequivocally from the experimental value of $\mu_d-(\mu_p+\mu_n)$ the magnitude of the deuteron ground state $^3D_1$ admixture; in particular the 3.9 percent admixture of reference 2 must be accepted with reserve in spite of the agreement with the experimental $\mu_d-(\mu_p+\mu_n)$, since it involves the additional assumptions of a very small relativistic magnetic moment correction (presumably achieved by the relatively long range of the forces) and of a comparatively large tensor force of range equal to the central force (which leads to the above mentioned H^3, He^4 difficulties).

In conclusion it might be added that the just published experimental value of the magnetic moment of the H^3 nucleus[9] does not agree with theoretical predictions made on the basis of assumptions analogous to I and II above, coupled with the additional assumption that the $^2S_{\frac{1}{2}}$ ground state of H^3 has only a $^4D_{\frac{1}{2}}$ admixture.[10] The discrepancy may arise, at least in part, from an appreciable relativistic variation of the magnetic moments of the nucleons within the H^3. However, Sachs has shown[11] that the experimental result may be accounted for by supposing that the ground state of H^3 has an appreciable $^2P_{\frac{1}{2}}$ and $^4P_{\frac{1}{2}}$ admixture.

(1947)) to estimate relativistic corrections to the deuteron's magnetic moment. Breit finds that the correction is quite small when the range of the force field is taken as 2.8×10^{-13} cm, and is, in general, quite sensitive to the detailed assumptions regarding the nuclear force fields. Similar conclusions are obtained below.

[8] L. Hulthen, Rev. Mod. Phys. **17**, 263 (1945).

[9] H. L. Anderson and A. Novick, Phys. Rev. **71**, 372 (1947). F. Bloch, A. C. Graves, M. Packard and R. W. Spence, Phys. Rev. **71**, 373 (1947); **71**, 551 (1947).

[10] R. G. Sachs and J. Schwinger, Phys. Rev. **70**, 41 (1946).

[11] R. G. Sachs, Phys. Rev. **71**, 457 (1947).