Least Squares Fit of the Fundamental Mass Doublets

E. Richard Cohen and William F. Hornyak
Department of Physics, California Institute of Technology, Pasadena, California
October 21, 1947

ALL calculations of nuclear masses are dependent on three or four "anchor" values which are usually determined by mass spectroscopic comparisons with the ultimate standard $^{16}O=16.000000$. Commonly, the three fundamental mass doublets $^1H_2-^2D$; $^2D_3-^{12}C^{++}$; $^{12}C^1H_4-^{16}O$, which form a closed ring, are used to obtain the secondary standards ^1H, ^2D, ^{12}C. These three values are then considered as established anchor points for computing mass values from other doublets and from reaction energies. Unfortunately, this procedure permits of no external check on the consistency of the anchor values. By extending the ring to include the doublets $^{12}C^1H_2-^{14}N$; $^{12}C^1H_3-^{14}N^1H$; $^{12}C^1H_4-^{14}N^1H_2$; $^{14}N^1H_2-^{16}O$; $^{14}N^1H_3-^{16}O^1H$; $^{14}N_2-^{12}C^{16}O$, we obtain an overdetermined system which does permit an external consistency check, and which provides us with an additional reference point.

From these doublets we obtain an overdetermined system of six equations in four unknowns which can be solved by the usual methods of least squares. The six equations are the condensation of the nine actual doublets quoted. Least squares methods have been previously used to varying extents, Bönisch and Mattauch,[6] Bainbridge and Jordan,[10] and others. The input values of the doublets used in the present determination are, in units of 10^{-4} MU:

$$^1H_2-^2D=15.39\pm0.021,^{2-4,6,9}$$
$$^2D_3-^{12}C^{++}=422.30\pm0.19,^{2-4,6,9}$$
$$^{12}C^1H_4-^{16}O=363.69\pm0.21,^{1,2,4-6,8,9}$$
$$^{12}C^1H_{2+n}-^{14}N^1H_n=125.63\pm0.07,^{1,2,4-7,9}$$
$$^{14}N^1H_{2+m}-^{16}O^1H_m=237.80\pm0.32,^{1,4}$$
$$^{14}N_2-^{12}C^{16}O=112.22\pm0.40.^{1,4}$$

The numerical values used in these equations are the weighted means of all reliable data. The weights were based on the reported errors listed by each experimenter.

The above system yields for the output values of the masses

$$^1H=1.0081284\pm0.0000027,$$
$$^2D=2.014718\pm0.000005,$$
$$^{12}C=12.003847\pm0.000016,$$
$$^{14}N=14.007539\pm0.000015.$$

Furthermore, as a result of the least squares adjustment these mass values can no longer be considered as observationally independent quantities, but are interrelated through the "correlation coefficients": $^rHD=0.92$, $^rHC=-0.30$, $^rHN=0.004$, $^rDN=0.05$, $^rDC=-0.24$, $^rNC=0.86$. The "correlation coefficient" used here is defined in such a manner that the probable error of a linear function $ax+by$ is given by:

$$[a^2\delta_x{}^2+2abr_{xy}\delta_x\delta_y+b^2\delta_y{}^2]^{\frac{1}{2}},$$

where δ_x is the probable error in x, δ_y the probable error in y, and r_{xy} the "correlation coefficient" between x and y.

In this manner we then obtain the least squares best values for the doublets:

$$^1H_2-^2D=15.390\pm0.021,$$
$$^2D_3-^{12}C^{++}=422.30\pm0.19,$$
$$^{12}C^1H_4-^{16}O=363.60\pm0.18,$$
$$^{12}C^1H_2-^{14}N=125.64\pm0.08,$$
$$^{14}N^1H_2-^{16}O=237.96\pm0.14,$$
$$^{14}N_2-^{12}C^{16}O=112.32\pm0.19.$$

The comparison of external to internal consistency, following Birge and using his notation, gives $R_e/R_i=0.35$. Apparently this indicates either that the errors ascribed by the experimenters to the individual data were too pessimistic or that our selection of the data has been somewhat too critical. The discrepancy can in part be ascribed to the quotation of "limits of error" by some authors in place of "probable errors".

Work is now in progress which is intended to extend the mass table from ^1n to ^{20}Ne inclusively, based on these secondary standards and the best experimental information available at the present time.

We are grateful to Dr. William R. Smythe for the helpful interest he has shown in this work.

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Effects of Space Charge on the Detection of High Energy Particles by Means of Silver Chloride Crystal Counters*

Louis F. Wouters and Richard S. Christian
Radiation Laboratory, Department of Physics, University of California, Berkeley, California
October 14, 1947

SILVER chloride crystals operating at liquid N_2 temperatures are being used for detection of high energy protons produced by recoil and exchange reactions in the neutron beam emitted by the 184-in. cyclotron. Crystals of dimension $\frac{1}{8}''\times1''\times1''$ are annealed and coated by the usual processes;[1] mountings, amplifiers, scalers, and power supplies are more or less conventional. With this apparatus, γ-rays from radium give pulses of about four times the input-stage noise level of 50 microvolts; in the neutron beam, pulses up to 20 times this noise level are observed.

The chief difficulty encountered at any useful counting rate, with either form of radiation, is the gradual decrease in observed pulse height as counting proceeds, because of neutralization of the electric field by the space charge accumulating in the crystal. After perhaps 10,000 pulses, all pulses have noticeably decreased in height and frequency (as observed on a synchroscope); beyond 50,000, the rate and size are but a few percent of initial. This space charge is found to persist for long periods of time, up to 48 hours in one case.