Least Squares Fit of the Fundamental Mass Doublets

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October 21, 1947

ALL calculations of nuclear masses are dependent on three or four "anchor" values which are usually determined by mass spectroscopic comparisons with the ultimate standard $^{16}O = 16.000000$. Commonly, the three fundamental mass doublets $^{1}H_2 - ^{2}D$; $^{2}D_3 - ^{12}C^{++}$; $^{12}C^{1}H_4 - ^{16}O$, which form a closed ring, are used to obtain the secondary standards ^{1}H, ^{2}D, ^{12}C. These three values are then considered as established anchor points for computing mass values from other doublets and from reaction energies. Unfortunately, this procedure permits of no external check on the consistency of the anchor values. By extending the ring to include the doublets $^{12}C^{1}H_2 - ^{14}N$; $^{12}C^{1}H_3 - ^{14}N^{1}H$; $^{12}C^{1}H_4 - ^{14}N^{1}H_2$; $^{14}N^{1}H_2 - ^{16}O$; $^{14}N^{1}H_3 - ^{16}O^{1}H$; $^{14}N_2 - ^{12}C^{16}O$, we obtain an overdetermined system which does permit an external consistency check, and which provides us with an additional reference point.

From these doublets we obtain an overdetermined system of six equations in four unknowns which can be solved by the usual methods of least squares. The six equations are the condensation of the nine actual doublets quoted. Least squares methods have been previously used to varying extents, Bönisch and Mattauch,[6] Bainbridge and Jordan,[10] and others. The input values of the doublets used in the present determination are, in units of 10^{-4} MU:

$$^{1}H_2 - ^{2}D = 15.39 \pm 0.021,^{2-4, 6, 9}$$
$$^{2}D_3 - ^{12}C^{++} = 422.30 \pm 0.19,^{2-4, 6, 9}$$
$$^{12}C^{1}H_4 - ^{16}O = 363.69 \pm 0.21,^{1, 2, 4-6, 8, 9}$$
$$^{12}C^{1}H_{2+n} - ^{14}N^{1}H_n = 125.63 \pm 0.07,^{1, 2, 4-7, 9}$$
$$^{14}N^{1}H_{2+m} - ^{16}O^{1}H_m = 237.80 \pm 0.32,^{1, 4}$$
$$^{14}N_2 - ^{12}C^{16}O = 112.22 \pm 0.40.^{1, 4}$$

The numerical values used in these equations are the weighted means of all reliable data. The weights were based on the reported errors listed by each experimenter.

The above system yields for the output values of the masses

$$^{1}H = 1.0081284 \pm 0.0000027,$$
$$^{2}D = 2.014718 \pm 0.000005,$$
$$^{12}C = 12.003847 \pm 0.000016,$$
$$^{14}N = 14.007539 \pm 0.000015.$$

Furthermore, as a result of the least squares adjustment these mass values can no longer be considered as observationally independent quantities, but are interrelated through the "correlation coefficients": $r_{HD} = 0.92$, $r_{HC} = -0.30$, $r_{HN} = 0.004$, $r_{DN} = 0.05$, $r_{DC} = -0.24$, $r_{NC} = 0.86$. The "correlation coefficient" used here is defined in such a manner that the probable error of a linear function $ax + by$ is given by:

$$[a^2 \delta_x^2 + 2abr_{xy}\delta_x\delta_y + b^2\delta_y^2]^{\frac{1}{2}},$$

where δ_x is the probable error in x, δ_y the probable error in y, and r_{xy} the "correlation coefficient" between x and y.

In this manner we then obtain the least squares best values for the doublets:

$$^{1}H_2 - ^{2}D = 15.390 \pm 0.021,$$
$$^{2}D_3 - ^{12}C^{++} = 422.30 \pm 0.19,$$
$$^{12}C^{1}H_4 - ^{16}O = 363.60 \pm 0.18,$$
$$^{12}C^{1}H_2 - ^{14}N = 125.64 \pm 0.08,$$
$$^{14}N^{1}H_2 - ^{16}O = 237.96 \pm 0.14,$$
$$^{14}N_2 - ^{12}C^{16}O = 112.32 \pm 0.19.$$

The comparison of external to internal consistency, following Birge and using his notation, gives $R_e/R_i = 0.35$. Apparently this indicates either that the errors ascribed by the experimenters to the individual data were too pessimistic or that our selection of the data has been somewhat too critical. The discrepancy can in part be ascribed to the quotation of "limits of error" by some authors in place of "probable errors".

Work is now in progress which is intended to extend the mass table from ^{1}n to ^{20}Ne inclusively, based on these secondary standards and the best experimental information available at the present time.

We are grateful to Dr. William R. Smythe for the helpful interest he has shown in this work.

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Effects of Space Charge on the Detection of High Energy Particles by Means of Silver Chloride Crystal Counters*

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October 14, 1947

SILVER chloride crystals operating at liquid N_2 temperatures are being used for detection of high energy protons produced by recoil and exchange reactions in the neutron beam emitted by the 184-in. cyclotron. Crystals of dimension $\frac{1}{8}'' \times 1'' \times 1''$ are annealed and coated by the usual processes;[1] mountings, amplifiers, scalers, and power supplies are more or less conventional. With this apparatus, γ-rays from radium give pulses of about four times the input-stage noise level of 50 microvolts; in the neutron beam, pulses up to 20 times this noise level are observed.

The chief difficulty encountered at any useful counting rate, with either form of radiation, is the gradual decrease in observed pulse height as counting proceeds, because of neutralization of the electric field by the space charge accumulating in the crystal. After perhaps 10,000 pulses, all pulses have noticeably decreased in height and frequency (as observed on a synchroscope); beyond 50,000, the rate and size are but a few percent of initial. This space charge is found to persist for long periods of time, up to 48 hours in one case.

As observed by Woolridge, Ahearn, and Burton[2] in diamond, upon subsequently grounding the high voltage electrode, pulses of reverse polarity and of height equal to those originally observed are now seen; the pulse rate is also fully as great as that originally measured. As the space-charge field is depleted by the ionizing radiation, these "reverse" pulses correspondingly decrease in amplitude and frequency, until they likewise have vanished. The original potential may then by reimpressed, and another complete counting cycle is possible. This process has been repeated several hundred times with various crystals; to insure consistent results, the applied voltage is actually completely reversed after, say, one thousand counts. Over such a small interval the change in pulse height is entirely negligible.

In line with results of Street[3] and Hofstader,[4] pulse rise times were found to be masked by amplifier rise time; in our best tests this was about 0.2 microsecond, with a decay time of the same order.

The silver chloride crystal counter is thus a fast, high density counter of convenient dimension, well suited for detection and discrimination of particles of range upwards of the crystal thickness. In the coincidence equipment now under construction, the crystal voltage will be synchronously reversed between successive cyclotron beam pulses, to preclude missing counts caused by interference of amplifier blanking with beam pulses, as would otherwise be the case; the pulse spectrum will also be more clearly displayed by locking in the synchroscope sweep.

We wish to express our appreciation of the interest and encouragement of Professor Ernest O. Lawrence and the cooperation of the Laboratory staff.

* This letter is based on work performed at the Radiation Laboratory under Contract No. W-7405-Eng-48 with the Atomic Energy Commission.
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Geophysics and the Radioactivity of Potassium

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October 13, 1947

THE recent publications of Gleditsch and Gráf[1] and of Bleuler and Gabriel[2] lead to a reduction of the effective half-period of K^{40} from 15.7×10^8 year to 2.4×10^8 year, and an increase of the present rate of energy production from about 5×10^{-6} cal. per year to 38×10^{-6} cal. per year per g of (ordinary) potassium. As Gleditsch and Gráf make clear, the importance of this revision for geophysics can hardly be overestimated. A few of the major problems which must be re-examined, if these changes are accepted, are the following:

1. The thermal history of the Pre-Cambrian periods.
2. The origin of the moon and of lunar topography.
3. The chemical and mineralogical differentiation of the silicate shell, and the segregation of the radioactive elements.
4. The formation of continents and ocean basins.
5. The origin of folded mountain ranges.

It seems possible, at first glance, that the new constants for K^{40} may aid in resolving some of the difficulties now encountered with all of these studies. On the other hand, certain new problems arise. For example, the most reliable radioactive age determinations[3] agree in giving ages over 1×10^9 and up to 2×10^9 year for a number of rocks and minerals. With the new values for heat production from potassium, however, it seems doubtful whether a permanent crust could have existed for more than 10^9 or at most perhaps 1.5×10^9 year. Prior to this time, repeated remelting or flooding of the crust must have occurred. Neither the data nor our understanding of the processes are sufficiently exact so that a flat contradiction may be said to exist; but there is at least a suggestion here that the new half-life may be too short. The purpose of this letter is thus to reiterate the importance for geophysical theory of a redetermination of the constants for K^{40} and, in particular, of the period of β-decay upon which the other results depend.

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The Role of Exchange Interaction in Paramagnetic Absorption

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October 23, 1947

ACCORDING to theory,[1–3] the paramagnetic absorption per ion in a system of ionic spins should be inversely proportional to the magnetic interaction between the ions, characterized by the so-called internal field H_i.[4] This theory has in general been confirmed by Volger[3, 5] for a number of hydrated chromic, manganous, ferric, and gadolinium salts. However, the paramagnetic absorption in a number of anhydrous compounds of the same ions and in a few hydrated cupric salts (notably in $CuCl_2 \cdot 2_2HO$) is considerably higher than expected.

The absorption in the first-mentioned group of hydrated salts vanishes in accordance with the theoretical expectation[2] when a constant perpendicular field of the order of H_i is applied. In the anhydrous compounds and in the copper salts, on the other hand, the absorption already disappears in much smaller perpendicular fields.

Recently Zavoisky[6] and Cummerow and Halliday[7] have succeeded in observing an absorption band corresponding to the Larmor frequency in large perpendicular fields. In contrast with theoretical expectation, however, the width of this band in anhydrous compounds and copper salts appears to be considerably smaller than the internal field H_i.

From all this it appears that, in the anhydrous and copper salts, the effects of magnetic interaction (characterized by H_i) have been reduced as a consequence of exchange interaction[8, 9] between the ions.