TABLE I.

Z	Isotope	A	$B\rho$	Energy (Mev)
51	Antimony	124	3494	0.654
73	Tantalum	182	2905	0.499
74	Tungsten	185	3572	0.675
77	Iridium	192	3354	0.617

TABLE II.

Isotope	Reported half-life	Reported energy (Mev)	Method	Present value energy (Mev)
Antimony	60 days	0.74, 2.45	Cl. Ch[2]	0.654
Tantalum	97 days	0.53	Spec. m.[3]	0.499
Tungsten	77 days	0.67	Abs. mes.[4]	0.675
Iridium	60 days	0.67	Spec. m.[5]	0.617

For each of the isotopes, plots were made of N against $B\rho$, and N/B against $B\rho$. These two different plots represent the natural and the normalized energy spectra. They indicate the end points of the distribution curves from which the maximum disintegration energies of the radioactive isotopes may be readily obtained. These distribution curves, however, need certain corrections. The corrected curves with an analysis will be presented later. The maximum $B\rho$ of the end points and the corresponding energy values in Mev are given in Table I. The present measured energy values of the end points are given along with values previously reported in Table II for easy reference and comparison.

The results of experiments with ${}_{51}Sb^{124}$ indicate two spectra, the end point of the low energy component being 0.654 Mev, the end point of the high energy component being beyond the upper limit of the spectrograph. It can be seen from Table II that the energy of the upper end point of Tungsten^{195} agrees with the value previously reported. It may be pointed out that the sample of ${}_{77}Ir^{102}$ was not pure, and was mixed with ${}_{76}Os^{103}$, so that the spectrum of ${}_{77}Ir^{192}$ overlaps that of ${}_{77}Os^{193}$.

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Recurrence Formulas for Coulomb Wave Function

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IN a short note[1] under the same title, L. Powell derives the recurrence formulas for the Coulomb wave functions. These formulas, derived by me[2] as particular cases of the factorization method some seven years ago, have been used since in the discrete and continuous spectrum.[3] For their derivation the explicit form of the solution is entirely unnecessary, and the recurrence formulas follow at once from the structure of the differential equation itself.

Indeed the differential equation,

$$\frac{d^2F^L}{d\rho^2}+\left\{1-\frac{2\eta}{\rho}-\frac{L(L+1)}{\rho^2}\right\}F^L=0, \quad (1)$$

can be written in *either* of these two forms:

$$\mathcal{H}^{(L+1)+}\mathcal{H}^{(L+1)-}F^L=F^L, \quad (2a)$$

or

$$\mathcal{H}^{L-}\mathcal{H}^{L+}F^L=F^L, \quad (2b)$$

where

$$\mathcal{H}^{L\pm}=\left(1+\frac{\eta^2}{L}\right)^{-\frac{1}{2}}\left(\frac{L}{\rho}+\frac{\eta}{L}\pm\frac{d}{d\rho}\right). \quad (2c)$$

From (2) we have the recurrence formulas

$$\mathcal{H}^{(L+1)-}F^L=F^{L+1}, \quad \text{and} \quad \mathcal{H}^{L+}F^L=F^{L-1}. \quad (3)$$

The meaning of (2a) is: go one step up the ladder, then one step down the ladder, and you will arrive at the original function. Similarly, (2b) leads you to the original function by going one step down and one step up. This is only a particular application of the general theory of factorization.[4]

[1] L. Powell, Phys. Rev. **72**, 626 (1947).
[2] L. Infeld, Phys. Rev. **59**, 737 (1941); compare p. 743 and 744; also theorem II on p. 738.
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[4] A paper on new types of differential equations on which this factorization method can be used is in preparation.

Relative Nuclear Moments of H^1 and H^2

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IT was pointed out that nuclear induction is highly suited as a method for comparing, with high precision, the magnetic moments of the neutron, the proton, and the deuteron, and progress of these measurements at Stanford has been announced.[1]

We have just concluded a satisfactory series of experiments on the relative moments of proton and deuteron. In view of their separate interest, particularly in connection with the ratio of the hyperfine structure of H^1 and H^2,[2] we want to report these data while those on the neutron moment are still under careful re-examination.

The method[3] was essentially the same as that used for the relative moments of H^1 and H^3. Both isotopes were contained in about equal amounts in the sample, with the frequencies ν_1 for the proton and ν_2 for the deuteron superimposed in the transmitter and the receiver tuned to simultaneous response for both frequencies by two coupled resonant circuits. Using an r-f amplification stage for ν_2, comparable signals were obtained for the two isotopes, and proper choice of ν_1 and ν_2 resulted in their superposition on the screen at a common value H_0 of the resonance field.

In contrast to H^3 with a moment so close to that of H^1 that their respective signals have practically identical shapes, the considerably smaller moment of H^2 causes the relaxation time of the deuterons to be much longer than that of the protons in the same sample. The shape of their signals in a common modulated field is therefore rather different, and they cannot be brought to complete superposition (or cancellation). By adjusting the observation to the proper phase, components of the signals (denoted in reverence 1 by the symbol "v") both can, however, be made to be symmetrical on both sides of the resonance. Proper setting of ν_1 and ν_2, which corresponds to a common value of the resonance field for both isotopes, is then ascertained with high accuracy by the fact that the resultant signal from both isotopes appears again with a common point of symmetry on the screen.

The measurements were carried out with 1.5 cc of a 0.1 molar solution of $MnSO_4$ in water, consisting of about as much H_2O as D_2O. This concentration of the paramagnetic salt was chosen in order to avoid an excessive broadening of the proton signal, caused by too short relaxation times, and yet at the same time to keep the relaxation time for the deuterons well below the modulation period of 1/60 second. Sharp resonance lines were observed in the center of a well shimmed magnet of 8-in. pole diameter with fields in the neighborhood of 10,000 gausses. A relative departure of 1/100,000 from the proper frequency setting caused a noticeable asymmetry of the resultant signal; this determined the accuracy of setting ν_1 and ν_2 to common resonance field.

Taking into account the spin ratio $\frac{1}{2}:1$ for H^1 and H^2, and denoting by $\gamma_{1,2}$ and $\mu_{1,2}$ their respective gyromagnetic ratios and magnetic moments, one has then

$$\mu_1/\mu_2=\gamma_1/2\gamma_2=\nu_1/2\nu_2. \tag{1}$$

The proton transmitter was operating on the 6th harmonic of a master oscillator whose beat frequency,

$$\Delta=\nu_1/6-\nu_2, \tag{2}$$

with the deuteron transmitter was measured together with the frequency ν_2 of the latter. One obtains then from (1) and (2),

$$\mu_1/\mu_2=3(1+\Delta/\nu_2). \tag{3}$$

The following data give the results of many runs obtained for three different values of the resonance field.

ν_2	Δ	μ_1/μ_2
6.35000 Mc	0.544392±0.00006	3.257193±0.00003
6.74000 Mc	0.577807±0.00007	3.257184±0.00003
7.22000 Mc	0.619010±0.00007	3.257206±0.00003

While earlier determinations gave[4] 3.2571+0.001, we can thus state with considerably higher accuracy that the ratio of the moments of proton and deuteron is given by

$$\mu_1/\mu_2=3.257195\pm0.00002.$$

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On the Symmetrical Vector Meson Pair Theory of Nuclear Interaction

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SOME features in recent experimental investigations of cosmic-ray mesons[1] may indicate that at least an important part of the nuclear forces is due to interaction between pairs of mesons and the nucleons.[2] As the experiments give rather strong evidence for neutral mesons, it might, therefore, be of interest to extend the previous meson pair theories,[3] where only charged mesons were used, to the case of both charged and neutral mesons, a possibility which was recently suggested for the scalar meson pair theory by Rosenfeld at a discussion in Copenhagen. The authors have now made the corresponding generalization of the charged vector meson pair theory, previously investigated by Klein and the authors.[3] If we denote by U^c and U^n the field vectors of the charged and neutral mesons and by σ_A and τ_A the spin and isotopic spin vectors of the nucleon A, we find that the generalization means that, as an example, the interaction term of the charged pair theory,

$$i\gamma\frac{e^2}{\mu c^2}\Sigma_A(U_A{}^{c*}\times U_A{}^c)\sigma_A,$$

will be replaced by

$$i\frac{e^2}{\mu c^2}\Sigma_A\sigma_A\Bigg[\left(\gamma_1\frac{1+\tau_A{}^{(3)}}{2}+\gamma_2\frac{1-\tau_A{}^{(3)}}{2}\right)(U_A{}^{c*}\times U_A{}^c)$$
$$+\gamma_3\frac{\tau_A{}^{(1)}-i\tau_A{}^{(2)}}{2}(U_A{}^{c*}\times U_A{}^n)+\gamma_3\frac{\tau_A{}^{(1)}+i\tau_A{}^{(2)}}{2}(U_A{}^{n*}\times U_A{}^c)$$
$$+\left(\gamma_4\frac{1+\tau_A{}^{(3)}}{2}+\gamma_5\frac{1-\tau_A{}^{(3)}}{2}\right)(U_A{}^{n*}\times U_A{}^n)\Bigg],$$

and corresponding expressions for the other possible Lorentz-invariant terms. With a suitable choice of the γ:s, it is possible to make these interactions symmetrical with respect to charged and neutral mesons, such as to give a charge-independent interaction between nucleons. The result of the perturbation calculation in the weak coupling case is that the interaction between two nucleons will be the same as in the charged meson pair theory, only that it is multiplied by a factor $(a^2+b^2\tau_1\tau_2)$, where a and b are arbitrary constants, a statement which holds also in the scalar meson pair theory. The investigation shows that it is possible to combine the different interaction terms in such a way that the force between two nucleons will get the general features necessary to account for the scattering experiments and the ground state and quadrupole moment of the deuteron. The forces are in this symmetrical pair theory, of course, of the exchange type in contrast to the force deduced from the charged meson pair theories.

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