The Magnetic Moments of the Neutron and the Deuteron*

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The magnetic moments of the neutron and the deuteron have been measured in terms of the proton moment. The radiofrequency resonance technique of Alvarez and Bloch was used to determine the Larmor frequency of neutron precession in a magnetic field in which the frequency of proton precession had been determined by the method of Purcell, Torrey, and Pound. The ratio of the magnetic moment of the neutron to the magnetic moment of the proton is 0.68479 ± 0.0004. The ratio of the deuteron moment to the proton moment is 0.30702 ± 0.0001, in agreement with the Columbia value. Taking the magnetic moment of the proton as 2.7896 nuclear magnetons, the neutron moment is -1.9103 ± 0.0012 and the deuteron moment 0.85647 ± 0.0003 nuclear magneton. These values lead to a prediction of an admixture of 4.0 percent $^3D_1$ state in the ground state of the deuteron, in agreement with the value 3.9 percent found by Rarita and Schwinger from the magnitude of the electric quadrupole moment of the deuteron. The need for further theoretical investigation of the deuteron magnetic moment is pointed out.

INTRODUCTION

THE discovery of the electric quadrupole moment of the deuteron by Kellogg, Rabi, Ramsey, and Zacharias[1] showed that the ground state of the deuteron cannot be exclusively the $^3S_1$ state it had previously been assumed to be, since S states possess spherically symmetrical charge distributions. It became necessary to assume that Russell-Saunders coupling does not strictly apply, and so spin and orbital angular momentum are not separately conserved. By assuming the existence of a tensor interaction between the neutron and proton, Rarita and Schwinger[2] found it possible to account for the observed magnitude of the quadrupole moment by adjusting the constants of the interaction. In this way they found that the ground state of the deuteron must be $^3S_1$ 96.1 percent of the time, and $^3D_1$ 3.9 percent of the time.

Now, the admixture of $^3D_1$ state means that part of the time the neutron and proton have a relative angular momentum of two units, and accordingly, the magnetic moment of the deuteron should no longer be expected to be the algebraic sum of the proton and neutron moments, even on the assumption that the proton and neutron retain, when combined in the deuteron, the intrinsic moments they possess when free. Rarita and Schwinger calculated the dependence of the deuteron moment on the percentage admixture of $^3D_1$ state. Using the values for the proton and deuteron moments then current, namely, 2.785 and 0.855 nuclear magnetons,[3] they calculated that the magnetic moment of the neutron should be -1.911 nuclear magnetons. This is different from the algebraic difference -1.930 to be expected in the absence of a quadrupole moment. With the latest values for the proton and deuteron moments ($\mu_p = 2.7896 \pm 0.0008$, $\mu_d = 0.8565 \pm 0.0004$[4]), the predicted value becomes -1.9108 and the algebraic difference -1.9331.

The magnetic moment of the neutron has been measured by Alvarez and Bloch,[5] who obtained the value -1.935 ± 0.03 nuclear magnetons. This value is not inconsistent with the value predicted by Rarita and Schwinger; clearly, however, greater precision in the measurement is desirable, in order to see whether the simple mechanism postulated leads to a correct prediction of the neutron moment.

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1 J. M. B. Kellogg, I. I. Rabi, N. F. Ramsey, and J. R. Zacharias, Phys. Rev. **57**, 677 (1940).
2 W. Rarita and J. Schwinger, Phys. Rev. **59**, 436 (1941).
3 J. M. B. Kellogg, I. I. Rabi, N. F. Ramsey, and J. R. Zacharias, Phys. Rev. **56**, 728 (1939).
4 S. Millman and P. Kusch, Phys. Rev. **60**, 91 (1941).
5 L. W. Alvarez and F. Bloch, Phys. Rev. **57**, 111 (1940); Phys. Rev. **57**, 352 (1940).

Recently, Purcell, Torrey, and Pound,[6] and Bloch, Hansen, and Packard[7] have described two closely related and simple radiofrequency methods for measuring the gyromagnetic ratio, g, of a nucleus. These methods are based on the observation of the resonant frequencies of precession in a magnetic field, which are given by the relation

$$h\nu = gH.$$

The techniques involved are simpler than those used in the molecular beam methods. Radio techniques are employed, and the nuclei are contained in a solid or liquid sample.

Alvarez and Bloch measured the neutron moment by observing the dip in transmission of a polarized beam of neutrons through a ferromagnet, when the neutrons traversed a fixed magnetic field upon which was superposed a radiofrequency magnetic field in resonance with their precession frequency; this method is directly analogous to the molecular beam resonance method of Rabi and his collaborators. We have now remeasured the neutron moment by the same method, using a magnetic field in which the frequency for proton resonance was also observed by the method of Purcell, Torrey, and Pound. Thus we have in effect measured the ratio of the neutron and proton moments. As a partial check on the results, we have also measured the ratio of the deuteron moment to the proton moment. The results, it will be seen, confirm the predictions of Rarita and Schwinger. In neither the experiment by Alvarez and Bloch, nor our experiment, is the sign of the magnetic moment of the neutron determined.

APPARATUS

Neutron Moment

In order to measure the neutron moment three electromagnets were arranged as shown in Fig. 1. The air gap in each magnet was $\frac{3}{4}$ inch; the spacing between the rectangular pole-pieces of successive magnets was adjustable, and was made one-half inch or less. The pole-pieces of the polarizing and analyzing magnets were 3 by 3 inches; the central magnet had 7 by 3 inch pole-pieces. Since all the fields were in the same direction, there were no depolarizing weak field regions for the neutrons to traverse between the polarizing and analyzing iron blocks. Because of the proximity of adjacent magnets, the magnetic field in the central gap depended upon the currents through the exciting coils of the outer magnets. Accordingly, the currents in all three magnets were electronically regulated, and it was possible, by suitable adjustment of the currents in the outer magnets, to reduce considerably the inhomogeneity of the magnetic field in the central gap. This was feasible because the degree of polarization of the neutron beam is not too strongly dependent upon the current when saturation of the magnets is approached. The electronic stabilizers used held the current constant to within 0.01 percent over long periods of time. The magnets were air-cooled.

The polarizing and analyzing blocks were of Armco iron, $1\frac{3}{16}$ inches long in the direction of the beam and $\frac{3}{4}$ by $1\frac{1}{8}$ inches in cross section. They were set near the outer edges of the pole faces. The thermal neutron beam from the Argonne heavy water pile was defined by cadmium slits, and was 1 inch high and $\frac{3}{8}$ inch wide. The detector was a BF_3 proportional counter, heavily shielded. The beam is so well collimated and thermalized that a thin sheet of cadmium placed anywhere along it reduces the counting rate by a factor of well over a thousand. No background corrections were necessary.

The coil used for flipping the neutrons was rectangular in cross section, and large enough to pass the neutron beam. For the first four runs a

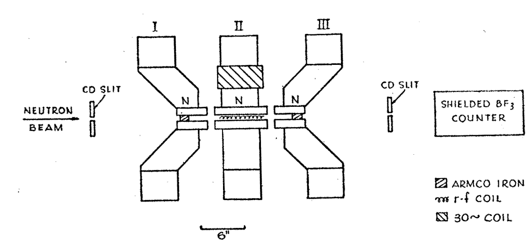

FIG. 1. Arrangement of apparatus for observing neutron resonances. A collimated thermal neutron beam is polarized by passage through a block of saturated iron in the gap of the first magnet. A coil in the center magnet induces transitions which depolarize the beam when the r-f field of the coil oscillates at the Larmor frequency. The depolarization is detected by the change in transmission of the beam through the second block of iron.

[6] E. M. Purcell, H. C. Torrey, and R. V. Pound, Phys. Rev. **69**, 37 (1946).

[7] F. Bloch, W. W. Hansen, and M. Packard, Phys. Rev. **69**, 127 (1946); *ibid.* **70**, 474 (1946); F. Bloch, Phys. Rev. **70**, 460 (1946).

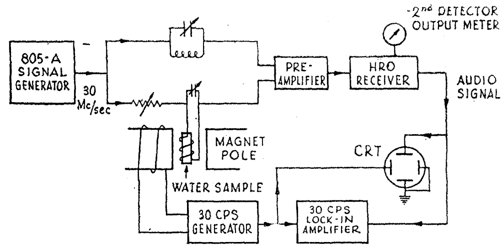

FIG. 2. Apparatus for observing the proton and deuteron moments. Balance in an r-f bridge circuit is detected by means of a receiver. One arm of the bridge contains the water sample in a coil in a magnetic field. At resonance absorption of energy by the induced transitions unbalances the bridge. Unbalance is detected by (a) an increase in the second detector output, (b) a pip on the cathode-ray tube screen each time the magnetic field goes through resonance, and (c) a reading on the output meter of the lock-in amplifier.

coil 4 inches long was used; in the remaining runs a 3-inch coil was used despite the fact that the neutron resonances were slightly widened because of the shorter time spent in the r-f field. In the first four runs different coils were used for the proton and neutron resonances. The shorter coil was used in later runs for both neutron and proton resonances in order to improve the homogeneity of the field in which the flipping took place and to insure its identity for neutrons and protons.

For the neutron resonances, a self-excited oscillator with an output of about six watts was used.

Since there are small fluctuations in the neutron intensity of the heavy water pile, the neutron counting rates were normalized by means of a monitor chamber in the thermal beam. Counting rates in the BF_3 detector were about 30,000 counts per minute, and about 40,000 counts were taken per point.

Proton Moment

For the proton resonances a water sample was inserted in the coil of a resonant circuit forming one arm of a bridge as in Fig. 2.*** The bridge was excited by a General Radio 805-A signal generator, and the residual signal at balance detected by a National HRO receiver with a preamplifier with enough gain to give an observable current from rectified noise in the second detector. Balancing was accomplished by adjusting phase and amplitude in each arm. With care, the residual signal could be made 50 to 60 db below the signal transmitted by each arm. Input signals between 0.01 and 1 volt were used. The signal-to-noise ratio could be improved by narrowing the i-f band width of the receiver with the quartz crystal filter provided in the receiver; the improvement possible is limited by the frequency drift of the signal generator.

Proton resonances were detected in one or more of three ways. At resonance, the protons absorb energy, decreasing the amount of signal passing through one arm of the bridge and thereby unbalancing it; a meter in the output of the second detector in the receiver accordingly shows an increased reading. However, resonances are more easily located, and sensitivity increased if an a.c. field is superposed on the constant magnetic field. For this purpose a small additional coil was added to one arm of the central magnet, and was excited by 30-cycle a.c. from a power amplifier fed by a synchronously driven permanent-magnet generator. If the a.c. field is made 30 to 50 gauss (peak-to-peak), a cathode-ray oscilloscope, whose horizontal sweep is obtained from the 30-cycle generator and whose vertical plates are connected to the receiver output, shows a pip each time the magnetic field sweeps through resonance, twice each cycle. When the resonance has been located, the a.c. field can be gradually reduced and the fixed field adjusted until the resonance is exactly centered, and occupies the entire sweep; the a.c. field is then less than one gauss, and can be turned off without affecting the fixed field.

Finally, a very large factor in sensitivity is gained by using a tuned audio amplifier sensitive only to a narrow band of frequencies centered at 30 cycles. By suitable restriction of the alternating field amplitude, the field can be made to vary over a small portion of the resonance curve, and the tuned "lock-in" amplifier then indicates the amount of 30-cycle component present in the audio output of the receiver. This technique was found unnecessary in the proton measurements because sensitivity was adequate without it; however, it was useful in initially locating the much weaker deuteron resonance.

Radiofrequencies were measured by means of a 1-Mc crystal oscillator provided with frequency

*** We are indebted to Dr. E. M. Purcell for private communication of the improvements in technique here described.

dividers and harmonic generators which gave signals at 10-kc intervals of adequate intensity over the entire frequency range used. A vernier was added to the 500-division dial of the National receiver so that readings could be made to 0.1 division directly. Frequencies were thus read directly to the nearest 10 kc, and the next place obtained by interpolation. In many cases, the band-spread features of the receiver could be used to facilitate frequency measurements.

The deuteron resonances were measured in the same way as the proton resonances. The frequencies were, of course, much lower. Heavy water was used; the resonance of the deuteron in heavy water has an apparent width of the same order of magnitude as that of the proton in ordinary water.

PROCEDURE

The first step in doing the experiment was to determine the strength of the fields in the outer magnets that would give enough polarization of the neutron beam to give a dip in neutron intensity at neutron resonance whose minimum could be accurately determined. It was found that fields of about 8000 gauss in the outer magnets, and 7000 gauss in the center magnet, resulted in the most uniform field in the center magnet (less than 0.05 percent inhomogeneity) and gave neutron resonance dips of 5 to 8 percent. The increase of transmission through either block of iron when it was magnetized and the other block when it was unmagnetized was about 8 percent, indicating that after passing through the first block of iron 69 percent of the neutrons had their moments aligned in one direction, and 31 percent were oppositely aligned. When both blocks of iron were magnetized, the increase of intensity of the neutron beam was 22 to 24 percent. This indicated that some of the neutrons had been depolarized in the iron and that a maximum dip of $23-(2\times 8)$ percent, or about 7 percent could be expected.

The 805-A signal generator was set to a frequency which would give proton resonance at about 7000 gauss, the coil containing water was placed in the center magnet, the bridge was balanced for that frequency, the 30-cycle a.c. component of the field in the center magnet was set at about 30 gauss, and the current to the center magnet was then varied until the pips indicating proton resonance appeared on the oscilloscope. The 30-cycle a.c. field was then decreased in steps, the central field being readjusted each time to center the pips on the oscilloscope screen, until the 30-cycle field was zero. This method of setting the central field was checked by the output meter on the HRO receiver and the lock-in amplifier, and found to give the same result in all cases. The frequency of the signal generator was then measured. In the first four runs the box containing the coil and the water sample was then removed from the center magnet and replaced by a similar box, having a coil 4 inches long and with a cross section large enough to pass the neutron beam. Power from the r-f oscillator was then fed into this second coil and

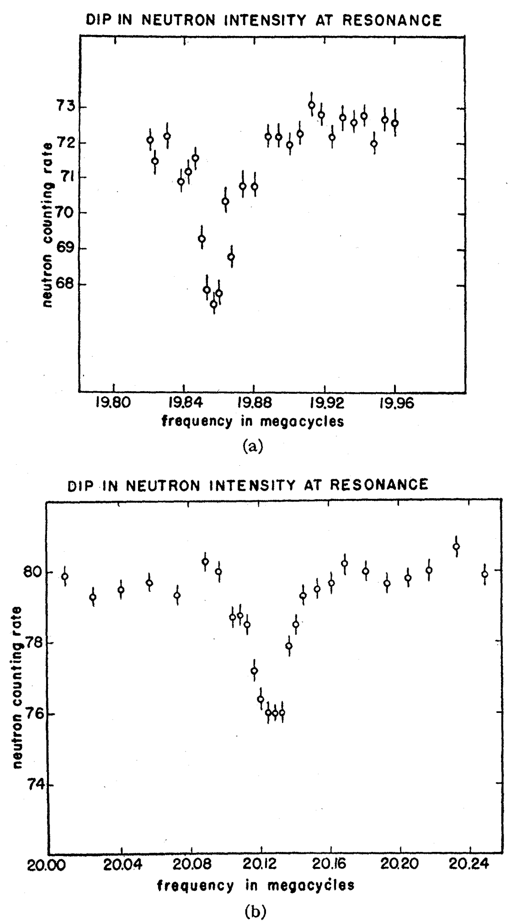

FIG. 3. Typical neutron resonance dips. (a) with 4-in. flipping coil, (b) with 3-in. flipping coil. The dip is wider in (b) than in (a) because of the shorter time spent in the field by the neutrons.

TABLE I. Results of neutron runs.

Run	Neutron frequency f_n Mc/sec.	Proton frequency f_p Mc/sec.	Ratio f_n/f_p	Statistical weight
1	19.820	28.936	0.68496	2
2	19.513	28.491	.68488	1
3	19.507	28.496	.68455	2
4	19.647	28.701	.68454	2
5	19.487	28.452	.68491	4
6	19.520	28.520	.68442	4
7	20.180	29.474	.68468	4
8	20.122	29.402	.68438	4
9	20.125	29.375	.68511	4
10	19.964	29.157	.68470	4
11	19.891	29.038	.68498	4
12	19.174	27.997	.68486	4
13	19.107	27.898	.68489	4
14	19.856	28.997	.68475	8
15	19.858	28.993	.68492	8
16	19.854	28.989	.68488	8
17	19.850	28.987	.68479	8

Weighted average $\mu_n/\mu_p = 0.68479 \pm 0.0002$

a neutron beam sent through the apparatus as shown in Fig. 1. In later runs a new coil 3 inches long was used from which the water sample was removed to permit the neutrons to pass through the same coil that was used to obtain the proton resonance. The frequency of the oscillator was then changed in small steps, and the intensity of the neutron beam coming from the analyser was measured for each frequency until the neutron intensity had increased to its original value again after passing through the dip indicating resonance. The frequency corresponding to the center of the dip was then measured. The signal generator frequency was then changed slightly and the next run made. Figure 3a shows a neutron dip using the 4-inch coil; Fig. 3b shows a similar dip obtained with the 3-inch coil.

Since the r.m.s. deviations in frequency measurements in the first four values were about equal to the uncertainty in reading the HRO dial, a vernier was added to the dial to give greater accuracy, and used in the remaining 13 runs.

Although the currents in the magnets, as measured by a Leeds and Northrup type K potentiometer, remained practically constant over the 30 to 40 minutes between the measurements of the proton and neutron resonance frequencies, and although partial saturation of the iron of the magnets made the change in field with changing current even smaller, an experiment was performed to determine the change in the field of the center magnet with time. This was done by holding the currents to the magnets constant, and observing the change in the proton resonance frequency with time. This showed a continued systematic decrease of frequency of about 3 kc per hour with good air cooling, and up to 10 kc per hour with poor air cooling (at 30 Mc). This change in field is probably caused by the change in dimensions and permeability of the iron in the magnets as they grow warmer.

The last four runs were made to check on the corrections made to the first 13 runs, and were all made without changing the currents to any of the magnets. The values for the proton frequencies shown for the last four runs in Table I are interpolated between the values for proton resonance found before and after each neutron run, and they indicate the drift in the field that occurred during the $3\frac{1}{2}$ hours it took to make the four runs.

Deuteron Moment

The procedure in measuring the ratio of the deuteron magnetic moment to the magnetic moment of the proton was to set the signal generator to a frequency of about 4.3 megacycles, set the 30-cycle a.c. field to about 25 gauss, and, with a coil containing about 20 grams of D_2O in the arm of the bridge that is in the magnetic field, to balance the bridge. The current to the center magnet was then varied until the resonance pips appeared on the oscilloscope screen. The 30-cycle field was then reduced slowly to zero with simultaneous adjustments of the current to the magnet to keep the ever-widening resonance pips centered on the screen. The signal generator frequency was then measured. (In three measurements its frequency was held to zero beat with a harmonic of the crystal oscillator.) The current to the magnet was then held constant while a coil containing ordinary water was substituted for the deuterium coil, and the signal generator set to the necessary frequency, as calculated from the known ratio of the magnetic moments. The 30-cycle field was turned up to 25 gauss, the bridge was balanced, and the pips for proton resonance appeared on the oscilloscope. The 30-cycle field was then reduced to zero in small steps; this time the necessary changes to keep the pips centered were made in the signal generator frequency. When zero 30-cycle field was reached, the signal generator frequency was measured. Hysteresis from the a.c. cycling of the magnetic field was checked and found not to change the field measurably.

The deuteron resonance was obtained first, since the resonance pips for deuterium were only about 4 times the noise level. The proton pips were about 30 times noise and thus much easier to center, especially when the bridge was some distance off balance.

The values for the proton resonance frequency were adjusted to correct for the small drift in the field, the correction being about three times the r.m.s. deviation between the values obtained in the four runs.

RESULTS

The results of seventeen runs on the neutron moment are summarized in Table I. This table includes every run taken except the very first one. This trial run was discarded because about four hours elapsed between the setting of the magnetic field on the proton resonance and the location of the neutron resonance. The value of the neutron moment obtained in this trial run was lower than the average of the remaining runs by more than three times the standard deviation of the other runs; there were other reasons to suspect that the magnetic field might have changed during that interval.**** After the trial run it was known where to look for the neutron resonance, and the neutron run could be started at 30 to 50 kc off resonance. All runs for which data are given required less than an hour between setting on the proton resonance and passing through the neutron resonance; in most cases the time was about one-half hour.

After the first four runs, the technique of frequency measurement was somewhat improved by the addition of the vernier to the dial of the receiver. The second run was cut short by a power failure before the completion of the neutron resonance dip. Also, in the first four runs different coils were used for the proton and neutron resonances. In runs 1 to 4 considerable heating of the magnet coils occurred; in all later runs better air-cooling reduced the heating of the magnets. After run 13, technique had improved to the point where it was possible to leave the magnetic field fixed after determining the neutron resonance, and to find the proton resonance again by varying the frequency of the signal generator. This is difficult because the bridge must be rebalanced at each frequency—a tedious procedure unless the proton resonance is quite strong. The last four runs were taken without changing the magnetic field at all, and taking proton and neutron resonances alternately. The observed drift in the magnetic field could thus be corrected.

By making repeated observations on the proton resonance frequency as a function of time, it was possible to determine approximately the amount of drift in the magnetic field under the conditions obtaining in runs 1 to 13. The observed proton resonance frequencies have been corrected for this drift. The correction is always in such a direction as to decrease the proton frequency; in runs 1 to 4 it is 5 kc and in runs 5 to 13, 3 kc. Thus the correction is never as great as 0.02 percent. The drift in magnetic field is caused by the heating of the magnets, not by the electronic stabilization, which shows no such drift in the magnet current.

In view of all the above, statistical weights have been arbitrarily assigned to the various runs in accordance with our judgment as to their relative merits. Runs 1–4 are assigned the weight 2, with the exception of the incomplete run No. 2 which is assigned the weight 1. Runs 5–13, taken under improved conditions, are assigned the weight 4, and runs 14–17, in which the magnetic field drift was observed directly, the weight 8. An unweighted average differs from the weighted average by less than the assigned error.

Table II shows the results of four runs on the deuteron-proton moment ratio. Small corrections for the magnetic field drift have been applied in

TABLE II. Results of deuteron runs.

Run	Deuteron frequency f_d Mc/sec.	Proton frequency f_p Mc/sec.	Ratio f_d/f_p
1	4.2900	27.948	0.153499
2	4.3035	28.031	.153526
3	4.4000	28.662	.153513
4	4.4100	28.728	.153509
		Average	0.153512 ± 0.00001
		$\mu_d/\mu_p = I_d f_d / I_p f_p =$	0.307023 ± 0.00002

**** It is interesting to note that if a correction for drift of the magnetic field is applied, the value obtained becomes concordant with later data.

all cases. Since the deuteron moment has previously been measured with precision as great as our own, no further runs were taken. Our concordant measurement of the deuteron-proton ratio with the same equipment makes it less likely that there are unknown systematic errors in the neutron-proton ratio, and has the further merit of providing all the information necessary to a check on the Rarita and Schwinger calculations. The agreement of the observed deuteron moment with the results of Kellogg *et al.*[3] gives us more confidence in the precision of the neutron value.

The ratio of the neutron resonance frequency to the proton resonance frequency is the ratio of their gyromagnetic ratios. The spin of the proton is one-half, and while there is no direct measurement of the neutron spin, the overwhelming weight of the evidence is that it, too, has a spin of one-half. The ratio of the resonance frequencies is thus the ratio of the magnetic moments. In the case of the deuteron, the ratio of the deuteron resonance frequency to the proton resonance frequency must be doubled to obtain the ratio of the magnetic moments, since the spin of the deuteron is one.

PRECISION OF RESULTS: SOURCES OF ERROR

The standard deviation of the seventeen runs on the neutron-proton ratio is 0.0002. This quantity can be decreased indefinitely by taking more and more observations; the final estimate of the probable error must therefore include other considerations.

Accidental Errors

The accidental errors of the experiment include the following:

1. Errors in Frequency Measurement

We estimate that this error seldom exceeds 2 kc. Better frequency determination is readily possible; however, no suitable equipment was available. This error is as likely to be in one direction as the other.

2. Errors in Determining the Center of the Resonance Peaks

Insofar as the peaks are symmetrical this error is accidental. The proton resonance peaks were visually symmetrical; so were the deuteron peaks. There was no consistent indication of asymmetry in the neutron resonance dips.

3. Statistical Errors in Counting

These amount to 0.3 percent for each neutron point. The dips in intensity at resonance were 5 to 8 percent.

Systematic Errors

Accidental errors are decreased by taking many runs. More important, of course, are systematic errors, both known and unknown.

The known systematic errors are these:

1. Drift in the Magnetic Field

This was measured and corrected for, as described above.

2. Inhomogeneity in the Magnetic Field

This was measured by means of a fluxmeter. It was found possible to reduce the inhomogeneity to less than 0.02 percent over the volume occupied by the r-f coil; the residual inhomogeneity consisted of a slight decrease in the field near the ends of the coil. To a first approximation, such an inhomogeneity tends to broaden the resonances asymmetrically on the low frequency side and could thus shift the peak. The shift is in each case proportional to frequency, however, and thus vanishes to a first approximation in the ratio. In addition, as remarked above, no definite asymmetries were observed.

3. Difference Between the Magnetic Fields for the Neutron and the Proton

In runs 1–4, where different coils were used, this error could be larger than in subsequent runs where the same coil was used. Some of this error might still persist, however, since the water sample occupied the geometric volume of the coil, while neutron flips might be induced outside the ends of the coil. Since the inhomogeneity is slight in any case, and since no systematic difference between the first four and later runs is found, no correction is made.

4. Difference Between the Magnetic Fields Actually Present at the Proton and Neutron Because the Protons are in Water

The diamagnetic susceptibility of water is less than 10^{-6}, and thus of no concern in itself. The diamagnetic effect of the hydrogen atom[4] is likewise too small to matter (3×10^{-5}). The magnetic field at one proton caused by the magnetic moment of a neighboring proton is of the order of one gauss; the true width of the resonance is due to the average effect of such fields. It is difficult to see how the resonance might be shifted, but such a possibility cannot be excluded. Accordingly, the frequencies of proton resonances in tap water, distilled water, benzene, paraffin, and a 1.5 percent solution of $FeCl_3$ were compared in a fixed magnetic field. They could not be distinguished from one another, and were all within 0.01 percent. The paraffin resonance is considerably wider than the others, which are of about equal width. This width, *ca.* 0.6 gauss, may not yet be the true width, since we could not have detected inhomogeneities in the magnetic field appreciably less than this. Polystyrene shows a resonance even wider than that of paraffin. While this experiment does not exclude the possibility of a systematic shift, it makes it unlikely that it should be large compared to 0.01 percent.

5. *Doppler Shift of Resonance Frequency Caused by Neutron Motion*

The thermal neutrons traverse the r-f coil with a mean speed of about 2.5×10^5 cm/sec. The quantity v/c is thus of the order 10^{-5}. It is clear that any symmetric periodicity or change in the apparent frequency seen by the neutrons in entering and leaving the coil can only change the resonance curve symmetrically. It is difficult to see how there can be an asymmetric change in the frequency seen by the neutrons. The observed widths of the neutron dips are in agreement with values computed from the time they spend in the coil; the dips are narrower with the 4-inch coil than with the 3-inch coil.

Similar arguments apply to the Doppler effect of the thermal agitation of the protons. This is, however, random, and no shift is to be expected.

6. *Systematic Errors in Frequency Determination*

These can be caused by drift of the local oscillator of the receiver, or by backlash in the dial. They can be readily checked by means of the crystal oscillator, and were found to be negligible. The crystal itself was checked against transmissions from WWV; its error was far smaller than the random errors of frequency determination.

7. *Shift of Resonant Frequency by the Radio-frequency Magnetic Field*

Bloch and Siegert[8] have shown that if H_1 is the amplitude of the oscillating field and H_0 the magnitude of the fixed field, a shift in the resonant frequency whose magnitude is given, to the first order, by $H_1^2/16H_0^2$ is to be expected. In our neutron runs, H_1 was about 7 gauss, H_0 7000 gauss; the shift is thus of the order 10^{-7} and thus completely negligible. In the proton runs, the oscillating field is smaller by a factor of 10^2 or more.

Determination of the deuteron-proton ratio is subject to similar errors. While fewer runs were taken, they gave a smaller statistical spread.

It is, of course, impossible to make any reasonable allowance for unknown systematic error.

FINAL RESULTS

In view of the possible sources of error quoted, we think it reasonable to state the **final results** with their probable errors as follows, **on the basis of a proton moment of 2.7896 nuclear magnetons:**

$$\boldsymbol{\mu_n/\mu_p = 0.68479 \pm 0.0004}$$
$$\boldsymbol{\mu_n = -1.9103 \pm 0.0012} \textbf{ nuclear magnetons}$$
$$\boldsymbol{\mu_d/\mu_p = 0.30702 \pm 0.0001}$$
$$\boldsymbol{\mu_d = 0.85647 \pm 0.0003} \textbf{ nuclear magneton.}$$

Taking into account the probable error 0.0008 in the proton moment, the **absolute values for the neutron and deuteron moments** become

$$\boldsymbol{\mu_n = -1.9103 \pm 0.0013} \textbf{ nuclear magnetons}$$
$$\boldsymbol{\mu_d = 0.85647 \pm 0.0004} \textbf{ nuclear magneton.}$$

Rabi and his coworkers[3] give for the deuteron-proton ratio 0.30703 ± 0.0001, and for the absolute value of the magnetic moment of the deuteron 0.8565 ± 0.0004.

DISCUSSION OF RESULTS

The value of the magnetic moment of the free neutron which is predicted by the theory of Rarita and Schwinger, using the latest determinations of the proton and deuteron moments of the Columbia investigators, is -1.9108 nuclear

[8] F. Bloch and A. Siegert, Phys. Rev. 57, 522 (1940).

magnetons. Our observed value is -1.9103 ± 0.0012. The prediction is based upon the value of 3.9 percent admixture of $^3D_1$ state in the ground state of the deuteron, a value obtained from the observed value of the quadrupole moment. Using only the relative values of the neutron, proton, and deuteron moments obtained in this experiment, we find the value 4.0 percent for the $^3D_1$ admixture more directly and with fewer assumptions. The possibility that the deuteron moment is simply the algebraic sum of the proton and neutron moments is far outside the limits of error.

The theory developed by Rarita and Schwinger thus predicts correctly the magnetic moment of the deuteron, provided that the magnetic moments of the proton and neutron and the electric quadrupole moment of the deuteron are given. In order to do this, certain assumptions are made concerning the range of the nuclear forces and the shape of the potential well. However, it appears that the successful prediction cannot be taken as proof of the correctness of the assumed range and potential distribution, since the simple theory used omits certain considerations which must now, because of the high precision of the measurements, be taken into account.

These considerations concern the assumption that the proton and neutron retain, when combined in the deuteron, the magnetic moments they possess when free. This assumption is certainly true to a high degree of approximation, as the numerical values show. It is, however, now necessary to examine it somewhat more critically, before assuming it to be correct to a numerical precision of less than one part in one thousand, which the experimental values have now attained.

There are at least two reasons for doubting the validity of the assumption of unaltered intrinsic moments to this degree of precision. One is that magnetic dipole moments, like other electromagnetic quantities, are subject to relativistic correction. From the point of view of the proton or neutron, the magnetic moment of the other nucleon in the deuteron should appear to be somewhat different from its rest value because of the relative motion of the two particles; the same is true of an observer stationary with respect to the deuteron. In fact, a particle of rest moment μ should appear to possess an electric dipole moment of $\mu p/m_0c$. Theories of the relativistic correction to the magnetic moment of the deuteron, caused by the relative motion of the neutron and proton, have been proposed by Margenau[9] and by Caldirola.[10] While neither of these theories can be accepted as final, it appears that some such correction is necessary. The corrections proposed by Margenau and Caldirola are of the order of 0.005 nuclear magneton, a value considerably larger than the experimental error.

In addition, a meson theory of nuclear forces of any currently fashionable type will in general imply an alteration of the magnetic moments of two nucleons as close to each other as are the neutron and proton in the deuteron; this is because of the interaction of the meson fields. No detailed calculations of this nature have, so far as we know, yet been made; the subject has, however, been investigated by Pais[11] and by Serpe,[12] who find the usual divergences. In principle, when the relativistic correction to the deuteron moment has been made, it might be possible to use the experimental data to help decide among the various forms of meson theory.

It may be concluded, then, that gratifying as the agreement of the present results with the Rarita-Schwinger theory may be, it is not yet final. The fundamental correctness of the approach seems to be verified. However, the value of 4.0 percent for the $^3D_1$ admixture cannot be taken too literally until further detailed theoretical investigations have been made.

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[9] H. Margenau, Phys. Rev. **57**, 843 (1940).
[10] P. Caldirola, Phys. Rev. **69**, 608 (1946).
[11] A. Pais, Physica **9**, 407 (1942).
[12] J. Serpe, Thesis, Univ. of Liège, 1943; also in Mémoires de la Soc. Roy. des Sciences de Liège, [1], **1**, No. 2, 251–322.