## Solar Absorption Lines between 2950 and 2200 Angstroms

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SOLAR spectrograms were taken by the Naval Research Laboratory on the V-2 rocket flights of October 10, 1946[1] and March 7, 1947. The spectra obtained at 55 km and 75 km on the respective dates have been studied to identify the Fraunhofer absorption lines in the region between 3000 and 2200A. Some 300 observable absorption features were compared to a master finding list of 1100 of the principal classified lines of the arc and spark spectra of elements 1 to 30. The heavier elements will be added as time permits. The finding list was prepared in collaboration with Dr. Charlotte Moore-Sitterly, who generously made available the unpublished multiplet lists which are being compiled as an extension of the Revised Multiplet Table[2] to cover the rocket ultraviolet region. Entries included laboratory intensities, assigned multiplet numbers, and, in many cases, her solar intensity predictions.

At the available resolution of about 1A, nearly every observed line is a blend; as many as 10 possible contributors to a single line have been found. Likely contributors have been assigned to nearly every observed feature, and work is under way to estimate their relative importance.

As in the previously known region, Fe *I* and Fe *II* are dominant and clearly contribute to a majority of the lines. In Fig. 1, the matching of many of the solar lines (indicated by dots) to the Fe arc spectrum is apparent. In the regions just below 2750, 2630, 2550, and 2490A there is a piling up of intense iron lines. This causes the whole level of the solar radiation to fall off sharply as shown in Fig. 1. In addition, many strong single Fe lines are found throughout the spectrum.

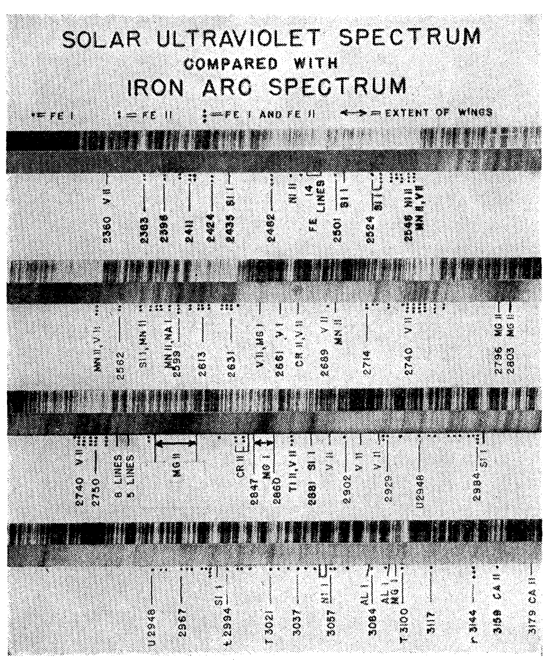


Fig. 1.

The great Mg *II* lines at 2803 and 2796A are of particular interest. They appear as two bright emission lines in the center of a great absorption band running from 2775 to 2825A.

Dr. Menzel of the Harvard Observatory offered the above explanation of the observed spectra and suggested the following explanation: A strong eruption of hydrogen occurred about 1 hour before the rocket flight, and the Mg emission may have originated in the prominence. However, similar emission lines, unresolved, may be inferred from the October 10 spectra, at which time no important prominences are known to have occurred.

Twelve lines of Si *I* of great intensity were found. The one at 2882 and the group between 2507 and 2529A showed strong wings. The existence of wings on the group between 2208 and 2219A could not be proved. A strong line of C *I* was found at 2478A.

In addition to the above elements one or more lines have been assigned as follows: definite, V *I*, V *II*, Cr *II*, Mn *II*, Co *I*, and Al *II*; probable, Na *I*, Ni *I*, Ni *II*, Cr *I*, Co *II*, Be *I*, and Al *I*; possible, P *I*, and Cu *I*. More definite assignments will be made taking into account multiplet intensity and the relative abundances of the elements.

There appear to be regions of general absorption between 2886 and 2893A and between 2442 and 2472A. The finding list contains few lines in these regions; the absorption may, therefore, be molecular.

A complete report of the analysis will be published later.

[1] W. A. Baum, F. S. Johnson, J. J. Oberly, C. C. Rockwood, C. V. Strain, and R. Tousey, Phys. Rev. **70**, 781 (1946).
[2] Charlotte E. Moore, "A revised multiplet table of astrophysical interest," Princeton Observatory.

## Lower Bounds for Eigenvalues

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BARTA[1] has given a method for finding lower bounds for the "ground state" eigenvalue of a vibrating membrane. He does not claim that his method is as nice as the Rayleigh-Ritz method for finding upper bounds, but in a numerical example he shows that fair accuracy is obtainable.

The purpose of this note is threefold: (1) We extend Barta's method to include the Schrödinger equation, (2) the boundary conditions of Barta are relaxed, for instead of $u=0$ on the boundary we have $u\geq 0$, and this may be easier to apply in practice, (3) we give a very simple, but mathematically rigorous, proof. Barta makes use of the fact that the ground state eigenfunction is positive; the proof of this fact is very long and sophisticated.

Consider a continuous real function, $\psi$, which vanishes on the secondary of a region, $R$, and satisfies the $n$-dimensional Schrödinger equation, $\Delta\psi+(\lambda-V)\psi=0$. For simplicity we assume that $V$ is a finite function. Let $u$ be any function with bounded second derivatives satisfying $u\geq\epsilon$ for a positive constant, $\epsilon$. The continuous function $r=\psi/u$ vanishes on the boundary and may be assumed to have a positive maximum in $R$ (otherwise take $-\psi$). At a maximum point the first derivatives of $r$ vanish, and the second derivatives are negative or zero. Adding the second derivatives gives $\Delta r=(u\Delta\psi-\psi\Delta u)/u^2\leq 0$. Thus $\Delta u/u\geq\Delta\psi/\psi=V-\lambda$ so that $\lambda\geq V-\Delta u/u$. *A fortiori*, $\lambda\geq\min.(V-\Delta u/u)$ for all points of the region. This gives the theorem: *The eigenvalues of Schrödinger's equation satisfy the inequality* $\lambda\geq\min.(V-\Delta u/u)$ *where u is a positive function in the region.*

The rest of this note is mainly of mathematical interest and proves that on the boundary, $S$, it is only necessary to assume $u\geq 0$. In case the region is finite, we suppose that $S$ is sufficiently regular so that there is a function, $g$, which satisfies $\Delta g+(h-V)g<0$ on $S$ and $g\geq 1$ in $R$, where $h=\min.(V-\Delta u/u)$. If $w=u+\epsilon g$, then $\lambda\geq\min.(V-\Delta w/w)$ by what has just been shown. Clearly, $V-(\Delta u+\epsilon\Delta g)/(u+\epsilon g)\geq V+(hu-Vu-\epsilon\Delta g)/(u+\epsilon g)=h+\epsilon(Vg-hg-\Delta g)/(u+\epsilon g)$. In a neighborhood of the boundary the last term is positive, and in the remainder of the region it must vanish with $\epsilon$, so that $\lambda\geq h$, Q. E. D.

In an infinite region we take $g=1$ in the above proof, that is, $w=u+\epsilon$. Assume, moreover, that outside a sufficiently large sphere $h-V<0$. Thus $V-\Delta w/w\geq h+\epsilon(V-h)/(u+\epsilon g)$. Outside the sphere the last term is positive, and inside the sphere it vanishes with $\epsilon$, so that $\lambda\geq h$.

The problem discussed here could be generalized for an equation in which $\lambda$ is multiplied by a positive function. On the other hand, the writer has been unable to treat by similar methods the eigenvalues of the acoustic equation, the equation of the clamped plate, or Maxwell's equations.

[1] J. Barta, Comptes rendus, **204**, 472 (1937).

## On a Connection between the Fountain Effect, Second Sound, and Thermal Conductivity in Liquid Helium II

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WE have investigated the possible phenomenological relations between the observed properties of liquid helium II. We assume that in the absence of a constriction the internal forces responsible for the fountain effect are still present but produce an internal momentum density **M** such that

$$\mu\nabla T=\nabla p=-d\mathbf{M}/dt, \tag{1}$$

where $T$ is the temperature and $p$ is the fountain effect pressure difference. We then apply the first law of thermodynamics to a heat transported with the momentum

$$\nabla\cdot\mathbf{h}=\nabla\cdot(\epsilon\mathbf{M}/m)=-\rho C_v dT/dt. \tag{2}$$

We solve (1) and (2) simultaneously to give a wave equation for momentum and temperature similar to second sound. The phase velocity is

$$V_s=(\epsilon\mu/m\rho C_v)^{\frac{1}{2}}, \tag{3}$$

where $\mu$ is the fountain effect coefficient,[1,2] $\rho C_v$ is the specific heat[3] per $cm^3$, and $\epsilon$ is the heat associated with a mass $m$ transported with the momentum. Empirical values of $\mu$, $V_s$ (second sound velocity),[4,5] and $\rho C_v$ below 1.8°K yield values of $\epsilon/m$ which fit a monotonically increasing curve from zero at 0°K to the heat content at the lambda point ($T_\lambda$).

From the equation

$$d(\epsilon\rho_x/m)/dT=\rho C_v \tag{4}$$

we calculated the concentration $\rho_x/\rho$ of the liquid taking part in the momentum. The curve connecting these values is approximately equal to $(T/T_\lambda)$,[6] going therefore through zero at 0°K and unity at $T_\lambda$, which it should do from general considerations.

The observed finite heat conductivity in liquid helium requires a wave equation having a relaxation (damping) term added to (1), namely,

$$\nabla\cdot\nabla T=\left\{\frac{d^2T}{dt^2}+\frac{1}{\tau}\frac{dT}{dt}\right\}\Big/V_s^2. \tag{5}$$

Combined with the data on thermal conductivity[3] this gives a time of relaxation $\tau$ of the order $10^{-3}$ to $10^{-4}$ sec., going to zero at $T_\lambda$. Equations (5) and (1) with the damping term lead to an alternative form of the heat current as a convection process:

$$\mathbf{h}=\rho Q\mathbf{u}_x, \tag{6}$$

where $\rho Q$ is the heat content per $cm^3$ of liquid helium, and $\mathbf{u}_x$ is the velocity associated with the steady momentum density as limited by the relaxation term. The data on thermal flow give $\mathbf{u}_x$ as a function of $T$ and $dT/dx$, leading to values practically identical with those of the super critical velocities through slits.[2]

The relaxation causes an absorption and dispersion of second sound near the lambda point, formally analogous to the transmission of electromagnetic waves in a conducting medium. This may be responsible for the rapid approach to zero of the observed phase velocity as $T$ approaches $T_\lambda$, and leads to the expectation that under ideal conditions (small amplitude and high frequency) the velocity of second sound should rise up to the immediate neighborhood of $T_\lambda$ just as the maximum in the curves of the heat conductivity and the fountain effect shift towards the lambda point under appropriate conditions.[2]

We are studying the physical meaning of these relations.

[1] H. London, Proc. Roy. Soc. London **A171**, 484 (1939).
[2] L. Meyer and J. H. Mellink, Physica, 's-Gravenhage (to be published).
[3] W. H. Keesom, *Helium* (Amsterdam, 1942).
[4] V. P. Peshkov, Vestnik Akademii Nauk **4**, 117 (1945); Nature **157**, 200 (1946).
[5] H. Fairbank, W. Fairbank, and C. T. Lane, Phys. Rev. **71**, 477(A) (1947).