The results for iridium may be summarized:

1. $\sigma_{\text{thermal}} = [(14\pm6)+(64\pm2)E^{-\frac{1}{2}}]$.
2. For the first resonance:
   $E_0(0.64\pm0.015)$ ev,
   $\sigma_0\Gamma^2 = (5 \text{ to } 20)\times10^{-24}$ (ev)^2 cm^2/atom,
   $\sigma_0 \geqslant 4500$ (probably). $\Gamma \leqslant 0.07$ ev (probably).
3. For the second resonance:
   $E_0 = (1.27\pm0.04)$ ev,
   $\sigma_0\Gamma^2 = (5 \text{ to } 20)$,
   $\sigma_0 \geqslant 4000$ and $\Gamma \leqslant 0.07$ ev (probably).
4. $\sigma = (25\pm4)$ for 2.2 ev $\leqslant E \leqslant$ 3.5 ev.
5. For the third resonance
   $E_0 = (5.2\pm0.2)$ ev,
   $\sigma_0\Gamma^2 = 55$ roughly.
6. For the fourth resonance:
   $E_0 = (8.7\pm0.3)$ ev,
   $\sigma_0\Gamma^2 = 50$ roughly.
7. $\sigma = (18\pm3)$ near $E = 14$ ev.
8. For the "fifth" resonance
   $E_0 = (25\pm5)$ ev.

9. The shape of the transmission curve for $E > 25$ ev indicates that probably more levels are present at higher energies so $\sigma_0\Gamma^2$ has not been calculated for this "level."

10. Iridium has the isotopes Ir^{191} and Ir^{193} of 38.5 percent and 61.5 percent abundance, respectively. The above values of σ_0 and $\sigma_0\Gamma^2$ for the resonances are uncorrected values for the natural element. Depending on which isotope is responsible for a given level, the values should be increased by a factor somewhat greater or somewhat less than two to give the true values for the responsible isotope.

Conclusion

Since the measurements were made which have been described in two previous papers[1,2] slow neutron transmission *vs.* time of flight measurements have been made for a large number of elements and the slow neutron spectrometer system has been considerably improved. In this first of the new series of papers describing these measurements, the results for Cd, Ag, Sb, Mn, and Ir, have been presented and other results will be presented soon in subsequent papers. This work represents, in all, several thousand hours of cyclotron running time. We wish to thank those who have made this work possible and assisted in the measurements.

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Angular Momentum of Photons

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The theory of elementary particles permits one to attribute well-defined angular momentum expressions to a given radiation field. A few simple examples are discussed, which involve systems of two photons, and of one atom and one photon, and which permit one to account for the conservation of total angular momentum. Conservation of total angular momentum, including radiation, is, however, of little practical importance, because spatial orientation of a given system requires, in general, the use of external magnetic fields, which perturb the angular momentum balance.

1. INTRODUCTION

According to the field theory of elementary particles, angular momentum expressions can be attributed to a given particle field, which, in general, can be divided into an orbital angular momentum and a spin part.[1] In the particular case of electromagnetic radiation, however, no gauge invariant separation between spin and orbital momentum is possible.[2]

[1] F. J. Belinfante, Physica 6, 887 (1939); W. Pauli, Rev. Mod. Phys. **13**, 203 (1941).
[2] L. Rosenfeld, Mém. Acad. Roy. Belg. (Cl. Sc.) **18**, 562 (1942).

TABLE I. Quantum states.

	$M=2$	1	0	-1	-2
$L=0$			$\frac{1}{\sqrt{3}}\Gamma_{00}(\nu_1\nu_2)+\frac{1}{\sqrt{3}}\Gamma_{1\,-1}(\nu_1\nu_2)+\frac{1}{\sqrt{3}}\Gamma_{-1\,1}(\nu_1\nu_2)$		
$L=1$		$-\frac{1}{\sqrt{2}}\Gamma_{10}(\nu_1\nu_2)+\frac{1}{\sqrt{2}}(\Gamma_{01}\nu_1\nu_2)$	$\frac{1}{\sqrt{2}}\Gamma_{1\,-1}(\nu_1\nu_2)-\frac{1}{\sqrt{2}}\Gamma_{-1\,1}(\nu_1\nu_2)$	$-\frac{1}{\sqrt{2}}\Gamma_{-1\,0}(\nu_1\nu_2)+\frac{1}{\sqrt{2}}\Gamma_{0\,-1}(\nu_1\nu_2)$	
$L=2$	$\Gamma_{11}(\nu_1\nu_2)$	$\frac{1}{\sqrt{2}}\Gamma_{10}(\nu_1\nu_2)+\frac{1}{\sqrt{2}}\Gamma_{01}(\nu_1\nu_2)$	$\left(\frac{2}{3}\right)^{\frac{1}{2}}\Gamma_{00}(\nu_1\nu_2)-\frac{1}{6^{\frac{1}{2}}}\Gamma_{1\,-1}(\nu_1\nu_2)-\frac{1}{6^{\frac{1}{2}}}\Gamma_{-1\,1}(\nu_1\nu_2)$	$\frac{1}{\sqrt{2}}\Gamma_{-1\,0}(\nu_1\nu_2)+\frac{1}{\sqrt{2}}\Gamma_{0\,-1}(\nu_1\nu_2)$	$\Gamma_{-1\,-1}(\nu_1\nu_2)$

The angular momentum operator of an electromagnetic field is given by

$$\mathbf{J}=\frac{1}{8\pi c}\int \mathbf{r}\times[\mathbf{E}^{\dagger}\times\mathbf{H}-\mathbf{H}^{\dagger}\times E]d\tau,$$
$$\mathbf{E}=\sum_{s,\,k} a_s(k)\mathbf{E}_s(k),\quad \mathbf{H}=\sum_{s,\,k} a_s(k)\mathbf{H}_s(k), \tag{1}$$

where $a_s^{\dagger}(k)$ and $a_s(k)$ represent, respectively, emission and absorption operators, $\mathbf{k}$ the wave vector, $\mathbf{E}_s$ and $\mathbf{H}_s$ field vectors belonging to an orthogonal set of solutions of Maxwell's equations, normalized to represent single photons.

Equation (1) permits one to attribute to the angular momentum of a radiation field a matrix representation, which, in the case of a single photon present, reduces to

$$\mathbf{J}_{ik}=\frac{1}{8\pi c}\int \mathbf{r}\times[\mathbf{E}_i^*\times\mathbf{H}_k-\mathbf{H}_k\times\mathbf{E}_i^*]d\tau. \tag{2}$$

Diagonal terms of (2) were already known in classical theory and led M. Abraham to attribute to a circular polarized dipole wave $J_x=J_y=0$; $J_z=\pm W/2\pi\nu$.

2. ANGULAR MOMENTUM MATRICES

It is well known that the use of a set of spherical solutions of Maxwell's equations is adequate to our problem. We shall use the solutions given by Born,[3] distinguishing between electric (E) and magnetic (M) waves, normalized by

$$(lm\,|\,W(kk')\,|\,l'm')=\frac{1}{8\pi}\int[\mathbf{E}^{*l'm'}(k')\cdot\mathbf{E}^{lm}(k)$$
$$+\mathbf{H}^{*l'm'}(k')\cdot\mathbf{H}^{lm}(k)]d\tau$$
$$=h\nu\delta_{kk'}\delta_{ll'}\delta_{mm'}. \tag{3}$$

[3] M. Born, *Optik* (Julius Springer, Berlin, 1933), p. 278.

In the case of a single photon present in the field, the angular momentum matrices become, both for (E) and (M) waves,

$$(lm\,|\,J_z\,|\,l'm')=\hbar m\delta_{ll'}\delta_{mm'},$$
$$(lm\,|\,J_x\pm iJ_y\,|\,l'm')=\hbar[(l\pm m)(l\mp m+1)]^{\frac{1}{2}}$$
$$\times\delta_{ll'}\delta_{m\mp 1m'}, \tag{4}$$

while the transition elements $(E)\leftrightarrow(M)$ vanish.

In the case of an arbitrarily large number of photons, only those matrix elements do not vanish, for which no transitions between states of different frequency ν and different angular momentum l occur, and in which, within the mentioned limitations, the total number of photons is conserved and no more than one photon shifted from one state, i, to another, f.

J_z turns out to be a diagonal matrix

$$(n_1\cdots\,|\,J_z\,|\,n_1\cdots)=\hbar\sum n_i m_i, \tag{5}$$

while

$$(n_1\cdots n_i\cdots n_f\cdots\,|\,J_x\pm iJ_y\,|\,n_1\cdots n_i-1\cdots n_f+1)$$
$$=\hbar[n_i(n_f+1)(l\pm m_i)(l\mp m_i+1)]^{\frac{1}{2}} \tag{6}$$

with $m_f=m_i\pm 1$.

W. Heitler has proved that the matrices (5) and (6) satisfy the well-known commutation relations between angular momenta.[4]

3. COMPOSITION OF ANGULAR MOMENTUM OF TWO PHOTONS

The matrices (5) and (6) permit us, immediately, to compose angular momenta of two photons, which may have different frequencies, $\nu_1\neq\nu_2$, but which may both belong to dipole waves of angular momentum quantum numbers m_1, m_2. In particular, we shall denote the field eigenfunctions of this configuration by $\Gamma_{m_1m_2}(\nu_1\nu_2)$.

[4] W. Heitler, Proc. Camb. Phil. Soc. **32**, 112 (1936).

TABLE II. Angular intensity distribution.

	$M=2$	1	0	-1	-2
$L=0$			$K=\frac{C}{r^2}(\nu_1+\nu_2)$		
$L=1$		$K(1+\frac{1}{2}\sin^2\theta)$	$K(1+\cos^2\theta)$	$K(1+\cos^2\theta)$	
$L=2$	$K(1+\cos^2\theta)$	$K(1+\frac{1}{2}\sin^2\theta)$	$K(\frac{2}{3}+\sin^2\theta)$	$K(1+\frac{1}{2}\sin^2\theta)$	$K(1+\sin^2\theta)$

Transforming the total angular momentum to principal axes, we obtain nine different states belonging to the total angular momentum quantum numbers $L=0, 1, 2;\ -L\leqslant M\leqslant +L$, corresponding to the following linear combinations which are analogous to the eigenfunctions of two material particles, obeying Bose-Einstein statistics. As in the case of material particles, the solutions belonging to $L=1$ vanish, if we put $\nu_1=\nu_2$.

The linear combinations of Table I permit us immediately to determine the angular intensity distribution of the two photon fields (Table II) which is identical with that one would obtain in classical theory by superposing the corresponding dipole waves and averaging over the relative phases. This last remark permits us to account immediately for the polarization state of the field in any direction.

It is a characteristic feature of the radiation field, that no coordinates can be attributed to a photon. The localization of a photon cannot be made, therefore, unless we refer explicitly to the type of position measurement we have in mind, and it will depend, in general, on the particular method adopted for position determination. Measuring, e.g., the position of our two photons by the coordinates of two atoms in which they produce photo-effects, we can easily study the relative angular distribution of our two photons, which shows interference phenomena similar to those we find for the relative angular distribution of two material particles, obeying Bose-Einstein statistics.[5]

4. RELATIONS WITH THE ZEEMAN EFFECT

Applying our considerations to a system consisting of a photon and a (Schrödinger) electron, we can easily transform the total angular momentum to principal axes. The linear combinations which appear, for a p electron and a dipole photon, are the same as those contained in Table I if, now, the Γ's are understood to be eigenfunctions of the system: electron-photon. As a matter of fact, conservation of total angular momentum should hold if an emission process takes place in free space, without the influence of an external field. Orientating the initial states of an atom by a magnetic field, in order to have a well-defined initial angular momentum, and removing the magnetic field afterwards in order not to perturb the subsequent emission process, the total angular momentum of photon and electron after emission has to be equal to that of the initial state. It can be shown that the linear combinations resulting from these considerations are identical with the ones which one obtains from the Zeeman transition probabilities in a vanishing magnetic field. In a finite magnetic field, however, we can determine individually the angular momentum both of the electron and the photon in the final state while the total angular momentum is not transferred to principal axes, influenced by the external field. In principle, the conservation of total angular momentum should hold for a weak magnetic field, i.e., not one sufficiently strong for the decoupling of the angular momenta of photon and electron. The coupling energy between the two mentioned momenta can be computed by a second-order perturbation calculation and turns out to be equivalent to an extremely weak magnetic field (of the order of 10^{-18} gauss in the case of dipole radiation).

It is a pleasure to express my indebtedness to Professor G. Beck for having suggested this investigation and for many helpful discussions and advice.

[5] A more detailed account of this point will be given in a paper to be published in the *Revista de la Unión Matemática Argentina*.