Diffraction Pattern of a Circular Aperture at Short Distances

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The diffraction pattern of electromagnetic waves was studied at distances from zero to five wave-lengths from a circular aperture. Microwaves of 12.8-cm wave-length were employed. The aperture was an iris diaphragm that could be varied between one and six wave-lengths in diameter. The beam was incident normally upon the plane of the aperture from a 4 ft parabolic reflector 24 feet away. The sharpest diffraction patterns were in the plane of the aperture. Measurements were made of the intensity of radiation in the electric and magnetic planes through the axis. Individual plots were made of intensities along the diameter and along lines parallel to the diameter at steps of quarter wave-lengths from the diameter. Measurements were also made of the intensities at fixed points along the axis as the diameter of the iris was varied from one to six wave-lengths. It was observed that Fresnel zone theory could be employed as a rule of thumb in predicting the intensity along the axis even in the illogical case of predicting the intensity at the center of the aperture. Checks were made against Kirchhoff's theory.

INTRODUCTION

THE wartime development of tools and techniques for the production of continuous microwaves has provided a supplement to the study of diffraction of x-rays and light.

The study of the diffraction of electromagnetic waves by apertures has been of four kinds:

1. Fraunhofer diffraction for which the source and the field to be studied are both effectively an infinite distance from the aperture.

2. Fresnel diffraction for which the source and field to be studied are at large distances from the aperture compared to a wave-length but at small enough distances to require consideration of the effect of phase difference between secondary wavelets from the different points in the aperture, even when the source and field to be considered are on a normal to the screen.[1]

3. Diffraction by apertures of small dimensions compared to a wave-length.[2]

4. Diffraction by apertures of the order of a wave-length in diameter observed in and near the aperture.

When the second and third types of diffraction are treated theoretically, approximations can be made in the solution of Maxwell's equations or in Kirchhoff's semi-empirical equation. The theory of the fourth type of diffraction must be general and may, when developed, add to the understanding of the other two.

Most theoretical determinations of diffraction patterns since the time of Fresnel have been based on the simple but necessarily blind assumption that the field over the surface of the aperture is identical with that of the unperturbed incident wave.

When a sufficiency of precise experimental data has been taken of fields in and near the aperture, a sound theory of diffraction may be built on the basis of experimentally known boundary conditions and Maxwell's equations. Bethe has suggested[2] that it may be possible to determine the boundary conditions by rigorous theory such as he applied to small holes. However, the generalization of his method of determination of the boundary conditions appears so difficult that it would not be too unsportsmanlike to make experimental observations first.

At present the only self-consistent theory of diffraction of electromagnetic waves[3] is that for the case in which the boundary surface is a perfect reflector. The condition of perfect reflectivity is closely approximated for microwaves incident upon a metal surface.

APPARATUS

The diffraction pattern of electromagnetic waves was studied at distances from zero to five

[1] S. A. Shelkunoff, *Electromagnetic Waves* (D. Van Nostrand Company, Inc., New York, 1943), p. 365.
[2] H. A. Bethe, Phys. Rev. 66, 163 (1944).
[3] B. Baker and E. Copson, *Huygens Principle* (Oxford University Press, 1939), p. 152.

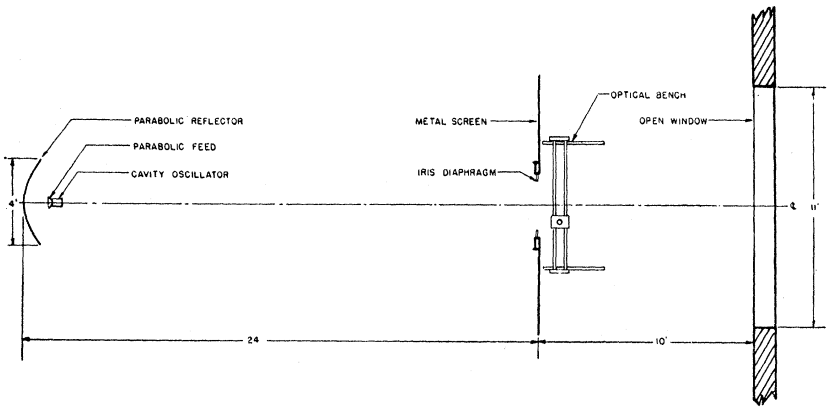

FIG. 1. Arrangement of apparatus for the study of diffraction by a circular aperture at short distances from the aperture.

wave-lengths behind a circular aperture. Figure 1 is a sketch of the horizontal cross section of the arrangement. Microwaves of 12.8 cm were employed. The oscillator was two coaxial, coupled, resonant cavities with a General Electric disk-seal 2C39 triode as an integral part of the cavities. The parabolic reflector, four feet in diameter, was fed by a small paraboloid, with a focal length of a quarter wave and diameter of one and one-half wave-lengths, having a dipole at its principal focus. The three-foot iris diaphragm was mounted in a 6 ft. by 6 ft. sheet iron "barn door" which could be rolled to one side when the undisturbed intensity was to be measured. To supplement the sheet iron screen, a wire-mesh screen was mounted around it making the total screen 12 ft. by 12 ft. The top of the optical bench was 18 in. below the bottom of the iris. A vertical wooden dowel 3 ft. long supported the receiving dipole antenna so that it could be moved in a horizontal plane through the axis of the diaphragm.

As shown in Fig. 2, the dipole was at the end of a 2-ft. coaxial line of silver tubing in the other end of which was mounted a silicon crystal detector. A quarter-wave choke, coaxial with the silver tubing, was mounted behind the dipole. A shielded line extended from the detector to a 50-microammeter outside the field. The combined crystal and meter were calibrated with a wave-guide attenuator. The rectified current through the crystal was proportional to the square of the potential across it and, therefore, to the received power and to the intensity of the radiation in the region of the dipole.

Both the dipole of the transmitter and of the receiver could be rotated to the vertical or horizontal plane. When they were vertical, the diffraction pattern in the H or "magnetic plane" could be measured. When they were horizontal, the pattern in the E or "electric plane" could be measured.

To prevent reflections, the iris was set before the center of an open window 11 ft. wide and 8 ft. high. The intensity at the edges of the window and along the floor, ceiling and walls was negligible. Standing waves were observed, of course, between the transmitter and screen. Care had to be taken that the opening and closing of the iris did not react on the oscillator to change its output. It was found that when the antenna was coupled sufficiently loosely to the oscillator, the plate current of the tube remained constant as the iris was opened and closed.

RESULTS

In the following set of graphs, all distances are expressed in wave-lengths. The intensities are relative to the intensity of the undisturbed beam when the iris and screen were removed from the region.

Figures 3a, b, c, d, and e show the intensities in the plane of the iris along the diameter in the

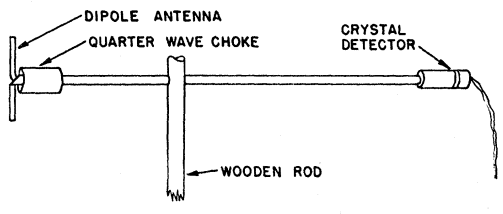

FIG. 2. Probe for exploring the field.

H plane when the diaphragm was one, two, three, four, and five wave-lengths in diameter. Figures 3a′, b′, c′, d′, and e′ were of intensities along the diameter in the E plane for the same sizes of diameter. To one accustomed to employing Fresnel's zone theory or Kirchhoff's theorem in optics, with the assumption of constant intensity over the aperture, it was startling to find that the sharpest diffraction pattern in the whole field was in the plane of the aperture. Measurements were made of the intensity beyond the edge of the iris. In these cases, the antenna was kept at a distance of about two millimeters from the surface of the leaves of the iris. In the E plane the beam was broader than in the H plane. Moreover, it was noted that the intensity in the E plane was not symmetrical with respect to the center. This was possibly caused by the fact that contact resistance between leaves of the iris diaphragm was variable so that the surface currents were not the same on the two sides.

Figures 4a, b, c, d, e, and 4a′, b′, c′, d′, and e′ are for a diaphragm one wave-length in diameter, and give the intensities in the H and E planes, respectively, along lines parallel to the plane of the iris, at various distances f from the iris as shown.

Figure 5 gives a similar set of graphs of the diffraction pattern of an aperture two wave-lengths in diameter. Curves a through k, and a' through k', give the intensities in the H and E planes, respectively, taken along lines parallel to a diameter and at distances f from the iris as indicated on each graph.

Figure 6 similarly shows the diffraction pattern of an aperture three wave-lengths in diameter. Following down the column of graphs the peaks of intensity could be seen receding and rising. To make it simpler to visualize the intensities in the H plane, the three-dimensional graph, of Fig. 7 was plotted. The broken lines are lines of constant intensity over the surface. The dis-

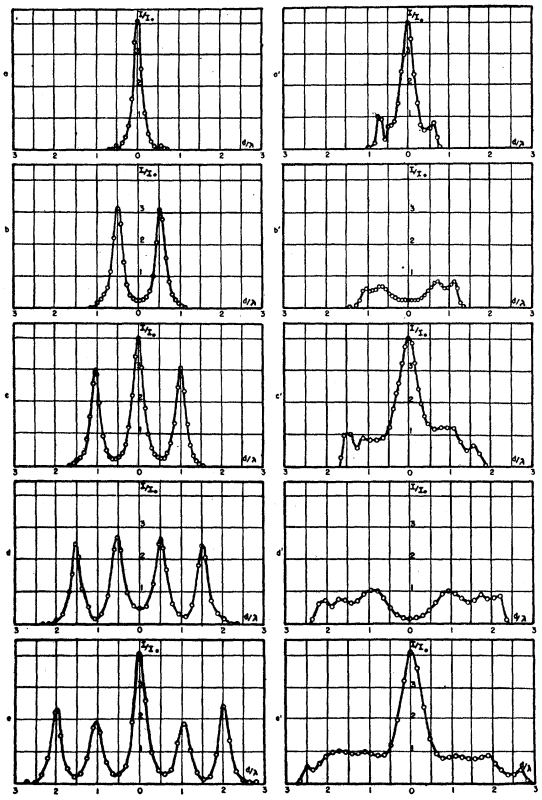

FIG. 3. Intensities along the diameters of the aperture in the H and E planes for diameter of from one to five wave-lengths.

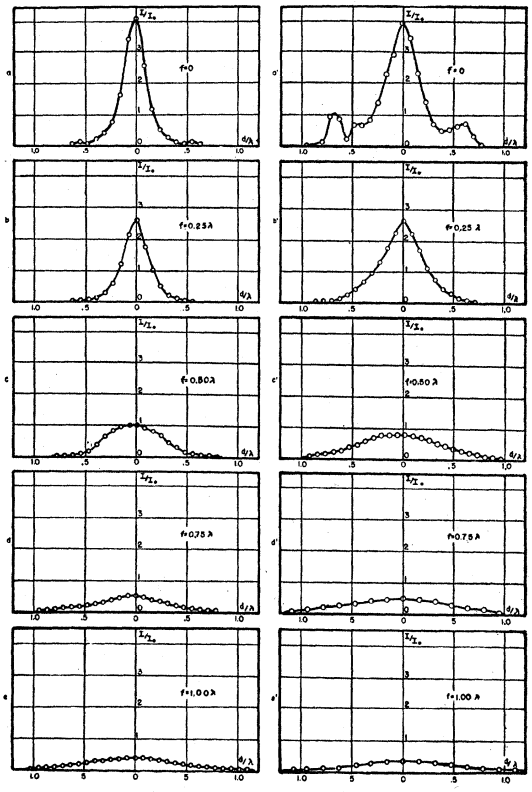

FIG. 4. Intensities in the H and E planes taken along lines parallel to the plane of the aperture when the aperture was one wave-length in diameter.

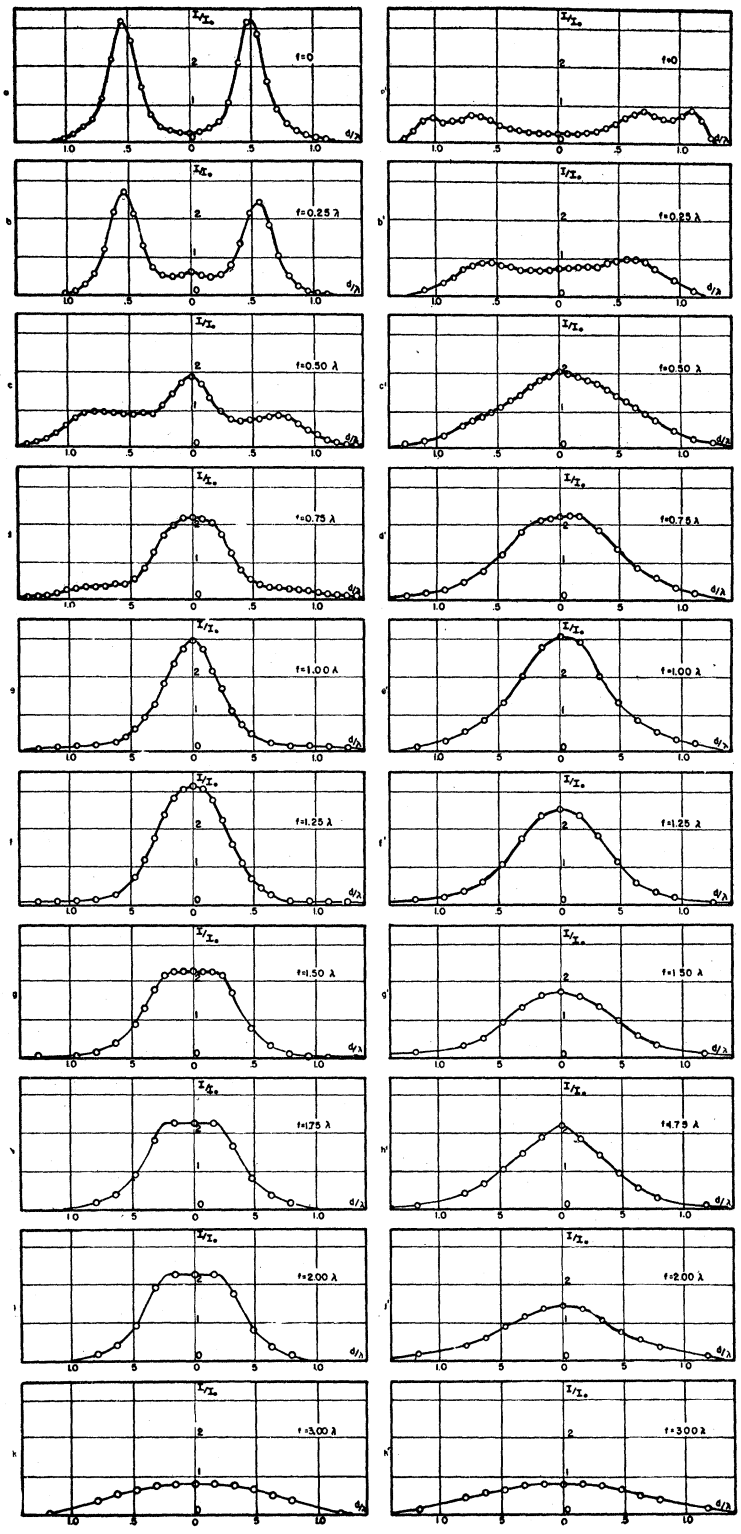

FIG. 5. Intensities in the H and E planes taken along lines parallel to the plane of the aperture when the aperture was two wave-lengths in diameter.

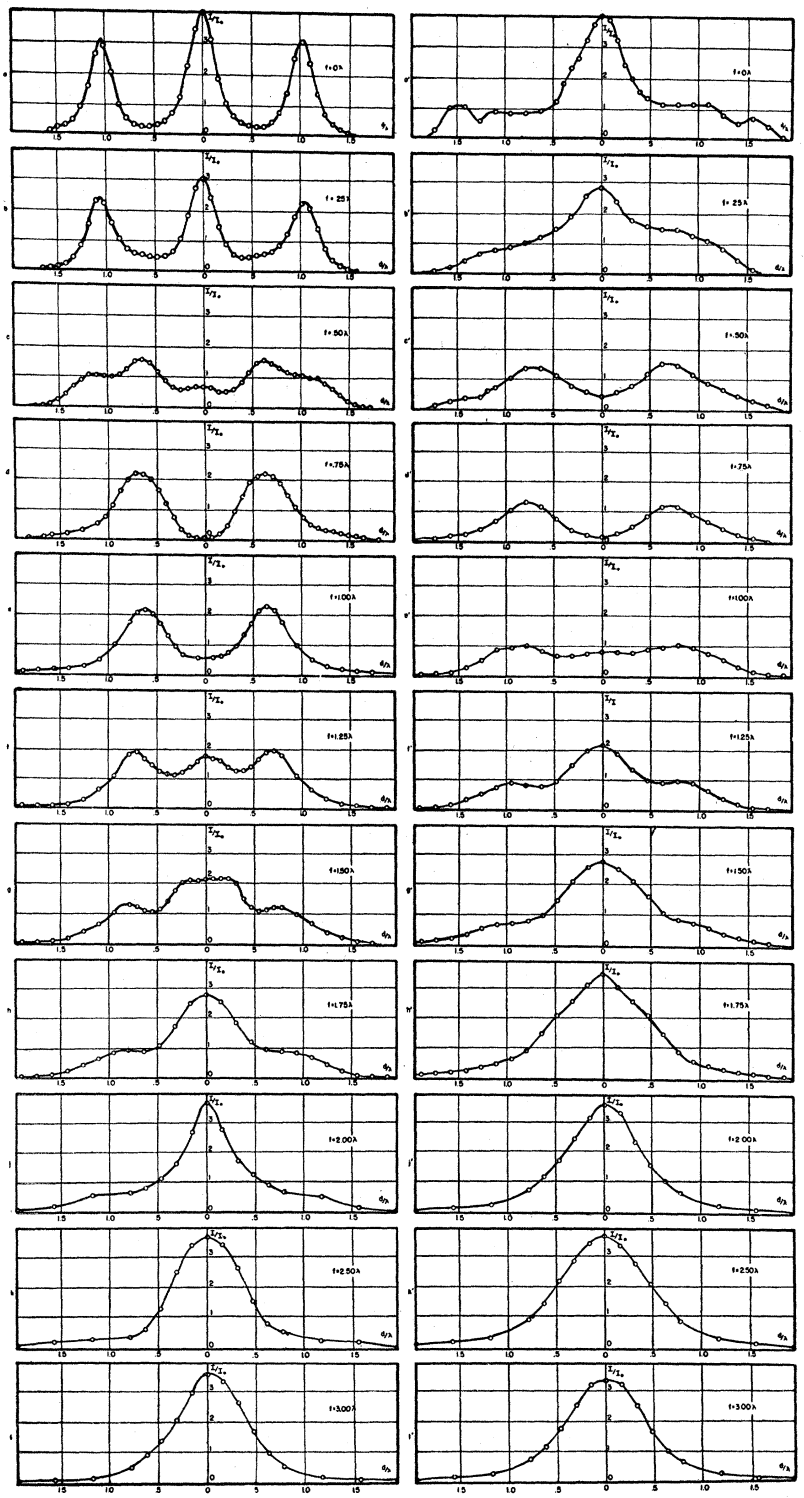

FIG. 6. Intensities in the H and E planes taken along lines parallel to the plane of the aperture when the aperture was three wave-lengths in diameter.

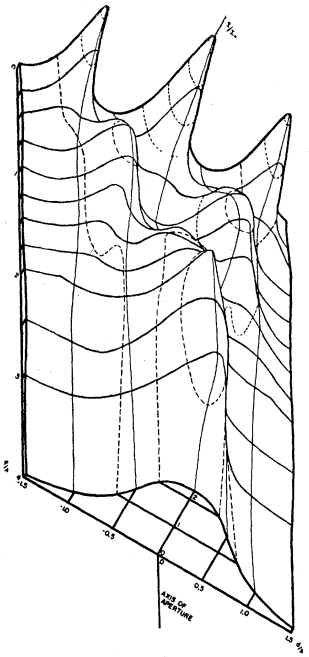

FIG. 7. Three-dimensional plot of intensities in the H plane when the aperture was three wave-lengths in diameter.

tances f are in wave-lengths from the aperture and d in wave-lengths from the axis of the aperture. This plot was shown for the width of the aperture.

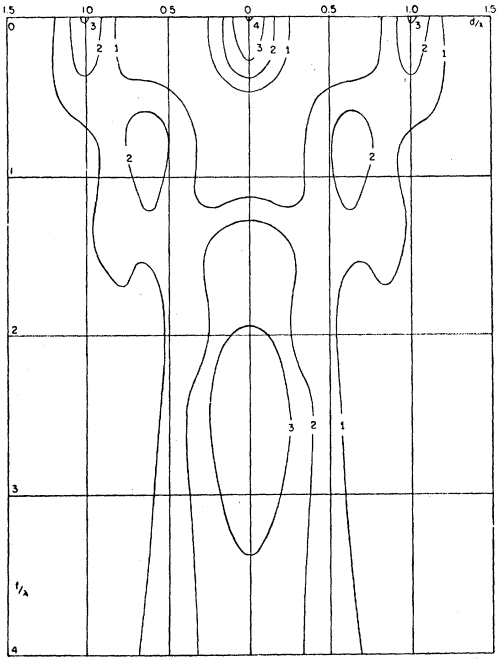

FIG. 8. Contour of intensities in the H plane when the aperture was three wave-lengths in diameter.

For the case of a diaphragm of four wave-lengths diameter, the only data taken were of intensities in the plane of the iris and are shown in Fig. 3d. It would be seen that as the iris was opened from one up to four wave-lengths in diameter, an array of peaks in intensity would move out from the plane of an iris being like an array of ten pins when the diameter of iris reached four wave-lengths.

Figure 8 is a contour map of the intensity in the H plane when the aperture was three wave-lengths in diameter. Figures 7 and 8 and the first column of Fig. 6 gave a complete picture of the diffraction pattern in the H plane when the diameter of iris was three wave-lengths.

A simple case to treat theoretically is that of the intensity along the axis of the aperture. In this case, the intensity is independent of polarization. Moreover the experimental results can be checked more readily with existing theory. Twelve graphs of the intensity at fixed points on the axis at distances f from the aperture were plotted against the diameter of the aperture of the iris. These are shown in Fig. 9, with the distances f indicated in the upper right-hand corner of each graph. The positions of the maxima and minima checked approximately with the predictions on the basis of elementary Fresnel's zone theory. For each distance f the minimum intensities corresponded to apertures of an even number of Fresnel's zones.

These results, as have been previously noted, are surprising when it is noted that in Fresnel's zone theory it is assumed that the intensity over the aperture is constant. From Fig. 3, it is seen that the sharpest diffraction pattern of the circular aperture is in the plane of the aperture. Fresnel's zone theory thus becomes merely a rule of thumb that gives results in close approximation to experiment.

When the zone theory is applied to the ridiculous case of $f=0$, the zone boundaries are circles of radii equal to an integral number of half-wave-lengths. The diameters of the aperture for maximum and minimum intensities at the center of the aperture were correctly predicted. Fresnel did not intend that his theory, which assumed constant intensity in the aperture, would be

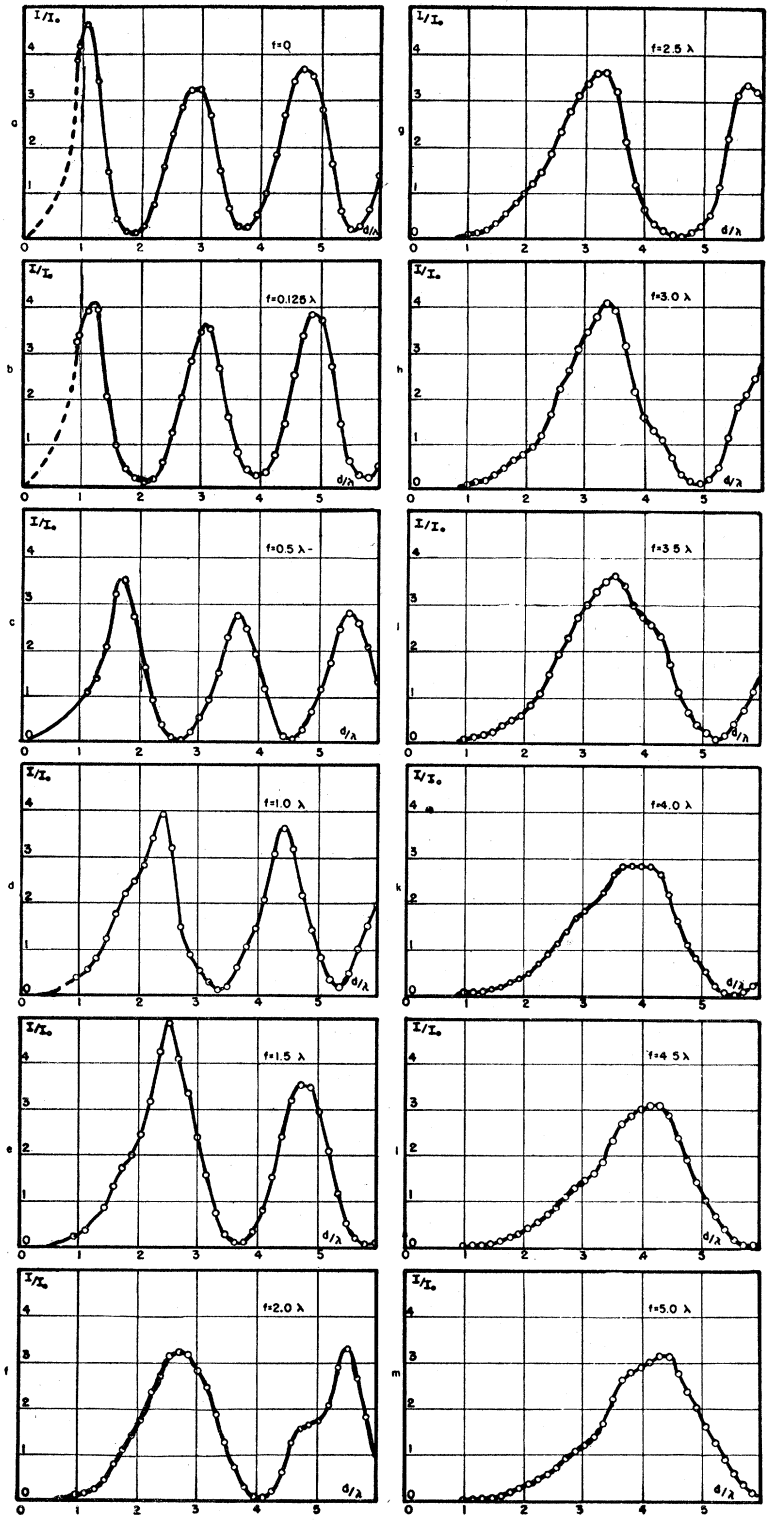

FIG. 9. Intensities at fixed points on the axis of the aperture at distances f from the aperture plotted against diameter of the aperture.

used to calculate the diffraction pattern in that same aperture.

The irregularities in the graphs of Fig. 9 were studied to see if they could be results of experimental error. The same irregularities were repeated with each experimental measurement. It was noted that some of the irregularities appeared systematically along the curves. For instance, in Figs. 9h to 9m, it was observed that a small dip appeared in each curve corresponding to diameters of four wave-lengths. Likewise, although less pronounced, there appeared small humps corresponding to a diameter of three wave-lengths and dips corresponding to diameters of two wave-lengths. That is, dips occurred when there was a minimum at the center of the aperture. Possibly the currents over the surface of the iris were responsible for some of the irregularities. All that can be said definitely is that the irregularities could not all be accounted for as experimental error and that there was order in the irregularities.

THEORY

It is not necessary to mention here all the theoretical papers that have called attention to Kirchhoff's false assumption that the intensity over the aperture is the intensity of the undisturbed beam. Figure 3 is experimental confirmation that the intensity over the plane of the aperture is not constant when a plane wave is incident normally upon the aperture.

The sole virtue of Kirchhoff's theory of diffraction lies in its correct predictions and not in its false assumptions. It has been used to predict correctly the intensity of light at points at distances behind the aperture large compared to a wave-length. Now that measurements have been made near the aperture, the question arises, how badly is Kirchhoff's theory in error in predicting the intensity near the aperture?

Kirchhoff's theory is applied here to the special but important case of calculating the intensity along the axis, a case for which the intensity will be independent of the polarization in the plane of the aperture. It seems desirable to present the results for two reasons:

(1) The results are sufficiently close to experimental observation to serve as a stop gap until a rigid theory is evolved.

(2) The solution of Kirchhoff's theory for the general case is found to be simpler than for the special case in which approximation is made when the distance from the aperture is large compared to the wave-length.

The derivation of Kirchhoff's theorem is found in books on optics[4] and electromagnetic theory.[5] For the special case of a plane wave incident normally upon an aperture, the problem is that of integrating the effects of Huygens' wavelets, from the unperturbed wave in the aperture, at the point at which the amplitude is to be found.

$$u_P=\frac{u_0}{4\pi}\int\int\left\{\frac{2\pi}{\lambda}\frac{e^{-i\phi}}{\rho}(1+\cos\theta)-i\frac{e^{-i\phi}}{\rho^2}\cos\theta\right\}d\sigma, \quad (1)$$

where u_P is the amplitude at the point P resulting from the added effects from all points in the aperture, u_0 is the amplitude of the undisturbed incident wave at the aperture, $d\sigma$ is an element of area in the aperture, ρ is the distance from P to the element $d\sigma$, ϕ is the phase lag of the component from $d\sigma$ behind the component from an element at the base of a perpendicular from the point P to the plane of the aperture, and θ is the angle between the incident beam and a line from $d\sigma$ to the point P.

In books on physical optics, the second term is always omitted since $\rho\gg\lambda$ in all of the cases treated in light. However, in the study of ultra-high frequency radio waves, useful applications occur when ρ is of the same order as the wave-length. Note the direction factor $(1+\cos\theta)$, which was discovered by Kirchhoff. A Huygens' secondary wavelet is not uniform over the whole sphere but has a maximum amplitude in the direction of propagation of the wave that generated it, half maximum at right angles and zero in the direction opposite to the primary wave. A Huygens' wavelet for the second term of the integral has zero amplitude at right angles to the direction of the primary wave and equal amplitude and opposite phase in the directions with and opposite to the direction of the primary wave.

Let the study be restricted to the determina-

[4] M. Born, *Optik* (Verlagsbuchhandlung Julius Springer, Berlin, 1933), pp. 147–151.

[5] J. A. Stratton, *Electromagnetic Theory* (McGraw-Hill Book Company, Inc., New York, 1941), pp. 424–428, 460–464.

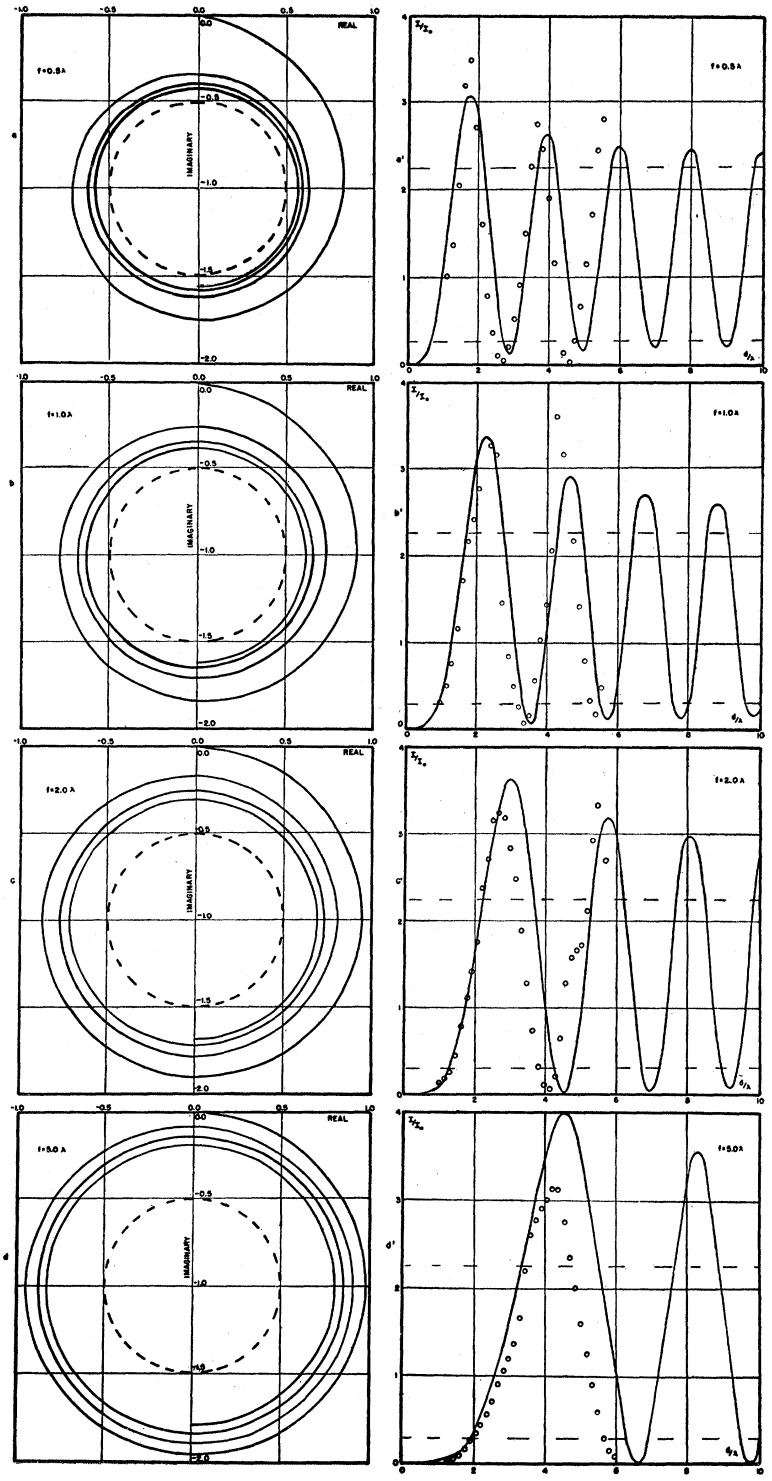

FIG. 10. (a, b, c, d) Spirals, calculated from Kirchhoff's theory, for the determination of amplitudes at fixed points on the axis as the diameter is increased. (a′, b′, c′, d′) Intensities, calculated from Kirchhoff's theory, at fixed points on the axis plotted against diameter and compared with experimental data.

tion of the disturbances along the axis of the aperture, and let f be the distance of P from the aperture. Let the increments of area be concentric rings. For any ring, ρ, ϕ, θ, and the radius r of the ring will be constants for the given ring. R_a is the radius of the aperture. It will be most convenient to express all the variables ρ, θ, r, and $d\sigma$ in terms of ϕ. Applying the transformation

$$\rho=\frac{\phi\lambda+2\pi f}{2\pi},\quad \cos\theta=\frac{2\pi f}{\phi\lambda+2\pi f},\quad d\sigma=\lambda\rho d\phi$$

to Eq. (1) after combining the cosine terms,

$$u_P=\frac{u_0}{2}\int_0^\beta e^{-i\phi}\left\{\left(\frac{1}{\lambda}-\frac{i}{\phi\lambda+2\pi f}\right)\frac{2\pi f}{\phi\lambda+2\pi f}+\frac{1}{\lambda}\right\}\lambda d\phi. \quad (2)$$

β is the phase of the component from the outer ring. Express f in wave-lengths. Let $f=n\lambda$ where n is any positive real number. Thus

$$u_P=\frac{u_0}{2}\int_0^\beta e^{-i\phi}\left\{\left(1-\frac{i}{\phi+2\pi n}\right)\frac{2\pi n}{\phi+2\pi n}+1\right\}d\phi. \quad (3)$$

Note that this solution for the general equation is simpler than the solution when the second term is omitted.

$$u_P=u_0\left[(\sin\beta+i\cos\beta)\left(\frac{\beta+4\pi n}{2\beta+4\pi n}\right)-i\right]. \quad (4)$$

We wish to express the intensity along the axis in terms of the diameter of the aperture. Let d be the diameter. Then

$$d/\lambda=[(\beta/\pi)^2+4n\beta/\pi]^{\frac{1}{2}}. \quad (5)$$

Figures 10a, b, c, and d are plots of real *versus* imaginary components of the amplitude for values of β from 0 to 7π for distances f along the axis from the center of the aperture of 0.5, 1.0, 2.0, and 5.0 wave-lengths. The ratio of the amplitude of the diffracted wave to the amplitude of the undisturbed wave is measured by the distance from the origin to the point on the spiral corresponding to a particular β. The broken circle is the asymptote approached by the spiral.

The square of the ratio of the amplitudes is equal to the ratio of the intensity of the diffracted wave to the intensity of the undisturbed wave I/I_0. From Eq. (5), d/λ the diameter expressed in wave-lengths was computed for values of β from 0.0 to 9.0π. Figures 10a′, b′, c′, and d′ are plots of I/I_0 against diameter of aperture for four distances f along the axis from the center of the aperture of 0.5, 1.0, 2.0, and 5.0 wave-lengths. The envelope of the curve approaches asymptotes indicated by broken lines at I/I_0 equal to 0.25 and 2.25.

The points indicated by circles are of experimental data. As a theoretical development, Kirchhoff's theory does not take into account currents over the surface surrounding the aperture. It assumes constant intensity, constant phase, and constant direction of polarization over the aperture, none of which is experimentally true. Since Kirchhoff's theory contains so many false assumptions, the deviations in Fig. 10 of experimental data from the results of Kirchhoff's theory are to be expected. These results of Kirchhoff's simple theory are still sufficiently precise to be useful.

Any correct theoretical prediction of the diffraction pattern of the aperture would be made on the basis of the known distribution of fields in the plane of the aperture and currents over the surface surrounding the aperture. The distributions along the line of intersection of the H plane and the plane of the aperture shown in Figs. 3a, b, c, d, e appeared simple to analyze. However, because the resistance between the leaves of the iris was variable, the distribution in the E plane was not symmetrical and, therefore, was difficult to analyze. Before the distribution in the E plane can be analyzed, data will have to be taken for a set of apertures in solid metal sheets.

It is a pleasure to acknowledge the support throughout this study of W. C. White and E. D. McArthur of the General Electric Research Laboratory, to thank Professor J. R. Collins of Cornell University for reading the paper and Dr. Emory Cook of Johns Hopkins University for a written status report on the theory of diffraction.