

TABLE I.

Element	$(R_i \pm \delta R_i) \cdot 10^{-13}$ cm	Statistical error Standard deviation
4 Be	3.22 ± 0.074	1.5
5 B	4.30 ± 0.24	—
12 Mg	5.40 ± 0.13	0.55
13 Al	5.54 ± 0.11	0.63
16 S	5.02 ± 0.14	2
26 Fe	6.63 ± 0.06	1.2
28 Ni	6.47 ± 0.074	1.5
29 Cu	6.75 ± 0.15	0.3
30 Zn	6.96 ± 0.17	3
34 Se	7.31 ± 0.20	0.55
47 Ag	7.81 ± 1.09	1.3
48 Cd	8.23 ± 0.07	2.3
50 Sn	8.50 ± 0.085	4
51 Sb	8.33 ± 0.096	10
80 Hg	9.50 ± 0.16	0.58
82 Pb	9.00 ± 0.071	0.87
83 Bi	9.08 ± 0.097	3.7

about half of the cases we found that this ratio was smaller, and in the other half was greater than unity, so we believe that the main error is purely statistical. This means that the probability for the real value to deviate by an amount ΔR_i from the observed value may be represented by means of a normal distribution (Gaussian law).

Under this reasonable assumption, we may calculate the probability that a given distribution will deviate from the interpolated straight line as much, or more, than our observed points.

Such a probability is obviously given by

$$P = \int_{(\chi^2)}^{\infty} e^{-\frac{1}{2}\chi^2} \chi^{n-1} d\chi / \int_0^{\infty} e^{-\frac{1}{2}\chi^2} \chi^{n-1} d\chi, \quad (3)$$

where

$$\chi^2 = \sum_1^n (\Delta R_i / \delta R_i)^2 \quad (4)$$

and n is the number of measured points used for comparison with the interpolated straight line of Eq. (1); in our case $n=17$. The deviations of the observed points from the interpolated line are indicated with ΔR_i and the corresponding experimental errors are δR_i (see the second column of Table I).

The function defined by (3) is formally identical to the function P introduced by Pearson for his " χ^2 test," although in our case the definition of χ^2 is slightly different from his. In order to calculate P , the most convenient linear representation (1) is the one that makes χ^2 a minimum. According to the definition (4) of χ^2 , this obviously means that the constants b and r_0 of (1) must be determined by the least-squares method, considering the different precision δR_i of the single points.

In this way we get

$$\begin{aligned} b &= (0.696 \pm 0.082) \times 10^{-13} \text{ cm}, \\ r_0 &= (1.52 \pm 0.02) \times 10^{-13} \text{ cm}, \end{aligned} \quad (5)$$

that is, two values quite different from those obtained with the graphical interpolation (2), in which we intentionally over-rated the points corresponding to heavy elements.

Applying definition (4), with the experimental values of Table I, and through the representations (1) and (5), we obtain for χ^2 the value $\chi^2 \approx 270$ which corresponds, for $n=17$, to a very small value of P defined by (3). From Pearson's tables,³ it is seen that, for $n=17$, P is smaller than 10^{-6} , even for $\chi^2=70$.

Thus we must argue that representation (1) is not a good one if we assume purely accidental errors; in fact, the probability of obtaining a distribution such as the one we found for the deviations of the experimental values from a linear law (1) is exceedingly small. However, it is well known that a small value of P does not necessarily indicate that the experimental deviations from law (1) are systematic, because the "goodness of fit" test has meaning only in the case of purely accidental errors.

We believe that the very small value obtained by us for P gives good evidence for a dependence of the nuclear radius on the number of particles in a nucleus, which is more complex than the simple law (1).

¹ E. Amaldi, D. Bocciarelli, B. N. Cacciapuoti, and G. C. Trabacchi, *Nuovo. Cimento*, in press. Presented at the conference on elementary particles and low temperature at Cambridge, England, July, 1946.

² H. A. Bethe, *Phys. Rev.* **54**, 436 (1938); E. P. Wigner, *Phys. Rev.* **56**, 519 (1939).

³ K. Pearson, *Tables for Statisticians and Biometricians*, Part I.

Search for Element 87

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THE article by F. R. Hirsh, Jr.¹ "The Search for Element 87" came to my notice only recently because of the interruption of postal relations between America and Rumania during the war. I wish to comment on the numerical data employed in his research and on the research itself. He adopts 0.812A and 1.045A for the presumptive wave-lengths of the absorption line L_{III} and the emission line $L\alpha$, respectively. The former departs by 0.011A from the value deduced, by Moseley's law, from the most reliable experimental values obtained for neighboring elements. The latter departs by 0.017A from the value to be expected for $L\alpha_1$, and by 0.011A from the average of the to-be-expected values of $L\alpha_1$ and $L\alpha_2$. Convincing evidence for the element 87 could not be found, even assuming the element to be present in the sample, with such expectations as to the wave-lengths. I suggest that his categorical objections to the inferences of other authors are based on very inexact estimates of the emission-lines which are to be expected.

I find the experiments of Hirsh far from convincing. He used a tungsten filament and two molybdenum anticathodes, and had to clean the anticathodes frequently to remove the pulverized tungsten from them. A photograph taken with an exposure of 36 hours, and having given no indication whatever of the tungsten L lines under the conditions of his observations, should not be expected to reveal the presence of an element present very probably in much smaller quantities than the tungsten remaining on the anticathode even after hourly cleanings. In addition, the $L\gamma$ lines of tungsten would mask the presumptive line $L\alpha_1$ of element 87 if they were both emitted. Similarly, the lines of molybdenum should have made their appearance: whether or not they were sufficiently excited by the narrow residual band of continuous x-rays springing from the primary anticathode under a 25-kv bombardment and falling

on the secondary anticathode, they should have appeared as scattered radiation. What is one to think of the scattering with change of frequency considered by Hirsh and apparently the object of an inexact formula in his text? Myself, I should not have melted the sample even "gently" in order to insert it into the hole in the anticathode, in view of the presumptive properties of element 87.

The search for a stable isotope of element 87 in a mixture of heavy, volatile, and absorbing alkaline substances is difficult, even if the element should be present in proportions of 10^{-4} or 10^{-5} . The methods of spectrum analysis and the conditions of excitation must be the best attainable. Our observations were generally made by primary excitation, with tubes constructed specially for great stability of operation even when volatile substances are present. My spectra do not allow me to come to the same conclusion as that reached by Dr. Hirsh. For the reasons expounded above, I feel that his investigation does not help in the least to settle the question of the existence of a durable isotope of element 87.

¹ F. R. Hirsh, Jr., *Phys. Rev.* **63**, 93 (1943).

The Magnetic Quenching of Superconductivity

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IN a letter with the above title Sienko and Ogg¹ suggest that for the "soft" superconductors Pb, Hg, Sn, In, Tl, CuS, Au₂Bi, Zn, and Cd the threshold magnetic field, H_T , for the destruction of superconductivity, is better represented by the expression $H_T = A(T_c^3 - T^3)$ than by the parabolic relation $H_T = B(T_c^2 - T^2)$. T_c is the temperature at which the metal becomes superconducting in zero field. Careful measurements of the threshold magnetic fields have been made on Sn^{2,4,5}, Hg^{3,4}, In^{3,5}, Tl^{3,5} and Pb^{4,5}. Within the accuracy of the measurements neither the parabolic nor the $\frac{3}{2}$ -power law exactly fits the data. However, the parabolic law is in all cases the better approximation. The variation in the constants A ($\frac{3}{2}$ -power law) and B (parabolic law) between about 1°K, and the transition temperature are: Sn (A 30 percent; B 10 percent), Hg (A 18 percent; B 3 percent), In (A 12 percent; B 8 percent), Tl (A 9 percent; B 5 percent), Pb (A 19 percent; B 8 percent). The values quoted are the averages obtained by the various experimenters. In general the variations among the measurements from different laboratories are less than the deviation from either the parabolic or $\frac{3}{2}$ -power relation.

It is true that the constant A is more nearly the same for different elements than is B , but A as determined from the initial slope of H_T vs. T varies from 90 for Cd⁶ to 46 for Sn, a variation well outside the experimental error and sufficient to make it doubtful that A is a combination of universal constants as Sienko and Ogg suggest.

From an expression for the threshold curve and the well-known thermodynamic relation

$$C_s - C_n = (VT/4\pi) \{ (dH_T/dT)^2 + H_T(d^2H_T/dT^2) \},$$

one can calculate the difference in heat capacity, $C_s - C_n$,

between the superconducting and normal states. V is the volume. For the $\frac{3}{2}$ -power relation one obtains

$$C_s - C_n = (VA^2/4\pi)(3T^2 - \frac{3}{4}T_c^3T^{\frac{1}{2}}),$$

and for the parabolic relation

$$C_s - C_n = (VB^2/2\pi)(3T^3 - T_c^2T).$$

Measurements on Sn⁷ and Tl⁸ indicate that the heat capacity of these two metals in the superconducting state obeys a T^3 law. For metals in the non-superconducting state the experimental heat capacities at low temperatures are in agreement with the theoretical prediction of an electronic heat capacity proportional to the temperature plus a lattice heat capacity which in simple cases varies as T^3 . The parabolic relation predicts a difference in heat capacity of superconducting and normal metal in agreement with these considerations. The absence of a term linear in temperature from $C_s - C_n$ obtained from the $\frac{3}{2}$ -power law is inconsistent with the experimental and theoretical results for the heat capacity of metals in the non-superconducting state.

¹ M. J. Sienko and R. A. Ogg, *Phys. Rev.* **71**, 319 (1947).

² W. J. de Haas and A. D. Engelkes, *Physica* **4**, 325 (1937).

³ A. D. Misener, *Proc. Roy. Soc. A* **174**, 262 (1940).

⁴ J. G. Daunt and K. Mendelssohn, *Proc. Roy. Soc. A* **160**, 127 (1937).

⁵ J. G. Daunt, A. Horseman, and K. Mendelssohn, *Phil. Mag.* **27**, 754 (1939).

⁶ N. Kurti and F. Simon, *Proc. Roy. Soc. A* **151**, 610 (1935).

⁷ W. H. Keesom and P. H. Van Laer, *Physica* **5**, 193 (1938).

⁸ W. H. Keesom and J. A. Kok, *Physica* **1**, 175 (1933).

Bounded Linear Harmonic Oscillator and Phase Transitions of Second Order

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IN a recent letter in the *Physical Review*, Corson and Kaplan¹ have put forth the idea that the concept of a linear bounded harmonic oscillator in the theory of solids may be of help in understanding anomalies in the specific heat-curve of a crystalline solid. Similar ideas were developed by the author in 1945 and were reported in a lecture at Central College, Bangalore City, South India. These ideas were not published at the time, but in view of the considerations of Corson and Kaplan, it has seemed desirable to place them on record.

The problem arose in connection with specific heat anomalies at low temperatures in crystalline solids. It was thought that the concept of the linear bounded harmonic oscillator might be of service in studying these phenomena. The relevant theory has been worked out by Auluck and Kothari.² The allowed energy values are no longer $(n + \frac{1}{2})h\nu$, where ν is the frequency of the oscillator, but increase rapidly with decreasing amplitudes of oscillation. However, the present author applied the above concepts to the second-order phase transition or λ -anomalies of Ehrenfest.³ It is well known that such phase transitions are all accompanied by marked changes in the expansion coefficient of the solid. Therefore, if at the λ -temperature the different oscillators in the crystal are bounded to different extents, the oscillators will be distributed among two well-defined