the neutron source. The wall was one-quarter inch of iron and the roof was one inch of glass.

The constant background at low energies probably represents scattering by the walls of the room rather than slow neutrons from the source. A blank run was taken with no carbon on the silver disk and the number of neutrons observed was negligible. Therefore, not more than a few percent of the neutrons observed could have been produced by the molecular beam in the analyzer.

The graph shows that the mean energy of the neutron group is 1.60 Mev. This becomes 1.65 Mev when corrected because recoil protons were measured out to 15°. The mean deuteron energy in the target was 1.96 Mev. This gives an energy evolution of $Q=-0.27\pm0.02$ Mev in the reaction.

CONCLUSIONS

The neutrons from carbon bombarded by deuterons are monochromatic, except for the high energy neutrons from C^{13}, when produced at 2 Mev; and are therefore likely to be monochromatic when produced at lower bombarding voltages.

The Q of the reaction is -0.27 ± 0.02 Mev. This value is believed to be more accurate than previous determinations.[4] This Q value gives a mass of 13.00988 ± 0.00004 for N^{13}.

[4] J. D. Cockcroft and W. B. Lewis, Proc. Roy. Soc. **154**, 261 (1936). $Q\geq -0.28$ Mev (from theshold of reaction).
T. W. Bonner, Phys. Rev. **53**, 496 (1938). $Q=-0.25\pm0.05$ Mev (from recoil protons in cloud chamber).
W. E. Bennett, T. W. Bonner, E. Hudspeth, H. T. Richards, and B. E. Watt, Phys. Rev. **59**, 781 (1941). $Q=-0.19\pm0.05$ Mev (from recoil protons in cloud chamber).

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On the Neutron-Proton Scattering Cross Section

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The sensitivity of the theoretical neutron-proton scattering cross section to possible variations in the quantities defining the potential has been investigated in the energy range from 0 to 6 Mev. It is found that in this energy range the variations in exchange or tensor character and range of the potential which are consistent with what is known about the interaction do not result in so large a modification of the predicted cross section as do variations in shape. A well resembling a Yukawa potential ($A\exp(-\alpha r)/r$) is found to yield a cross section at 6 Mev which is 10 percent lower than that given by a square well fitted to the epithermal neutron cross section and the binding energy of the deuteron. The Yukawa well is in better agreement with experiment than is the square well. One of the constants entering into the determination of the potential is the epithermal neutron scattering cross section in hydrogen. We have determined this by extrapolating the data of Frisch with the aid of the theory, and obtained a value of 20.8×10^{-24} cm^2.

INTRODUCTION

CURRENT nuclear theories attempt to correlate all known facts about nuclear scattering and nuclear binding by means of the assumption that these phenomena are caused by a potential energy of interaction between the fundamental particles. Unfortunately, the present data are too meager and too crude to permit the determination of this potential in any but the most schematic way. As a result, it is desirable to find out how critically the predictions for experimental results depend on each of the many parameters needed to specify the potential, and thus to indicate the range and accuracy of experiments needed to determine any feature of the potential within the limits desired.

In this paper, the neutron-proton scattering cross section is investigated in the energy range

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from 0 to 6 Mev. The object of this investigation is to estimate the extent to which the predicted cross sections will vary when the potential is given the largest possible modifications which seem to be consistent with all data other than that of neutron-proton scattering. When this work was done, there was so little systematic information about the neutron-proton scattering cross section, that our emphasis was placed on estimating the uncertainties in the predicted cross section, on the basis of data then in existence. Since then, fairly complete measurements in the energy range have been made available,[1,2] and our results can now be used not only to provide closer limits on the potential, but also to indicate the direction in which the theory may be modified, if necessary, to obtain better agreement with experiments.

The four main independently specificable features of the potential are: (a) range, (b) fraction of tensor force, (c) exchange character, and (d) shape. The depths of singlet and triplet wells in neutron-proton interaction are most conveniently determined in terms of the other four features with the aid of two reliably known constants, the binding energy of the deuteron, and the epithermal neutron-proton scattering cross section.

Probably the feature of the potential about which the least is known is the shape. In fact, it is found that a square well fits all data other than that of neutron-proton scattering above 3 Mev about as well as any other shape. Since the modifications in cross section resulting from likely modifications in shape are not expected to be large, the analytic simplicity of the square well may be utilized advantageously in investigating the sensitivity of the cross section to features other than the shape. The study of effects of variations in shape will then be deferred until the other three parameters have been taken up.

I. VARIATION IN RANGE

The range of nuclear forces is usually determined by means of a detailed investigation of the variation of cross section with energy. In this way, Breit, Thaxton, and Eisenbud[3] have been able to estimate the range of proton-proton forces with considerable accuracy, obtaining a value of $(2.8\pm0.4)\times10^{-13}$ cm^2, with a square well. In the absence of grounds to the contrary, the tentative assumption is usually made that the range of neutron-proton forces is the same as that of proton-proton forces. This assumption, at present, obtains some qualitative support from the general features of nuclear binding, but its main quantitative support is twofold. First, it is found[3,4] that if the neutron-proton singlet well is taken to be the same as that for proton-proton interaction, the epithermal neutron-proton scattering cross section and other low energy neutron-proton cross sections, can be fitted within experimental error. Second, calculations of Rarita and Schwinger[5] show that a square well of radius 2.8×10^{-13} cm^2 is consistent with the observed quadripole moment of the deuteron. That the range of neutron-proton forces is not much less than that of proton-proton forces is made clear by the calculations of Sen,[6] which indicate that a well of radius 2.4×10^{-13} cm is the narrowest that can yield simultaneously the correct quadrupole moment and magnetic moment.

In the interests of seeking the simplest hypothesis consistent with what is now known about nuclear interactions, it seems desirable to retain the assumption of equality of all nuclear forces, and to try to obtain agreement with experiment within the limits of this hypothesis. If this assumption is made, it is then of interest to investigate the variations in the neutron-proton cross section which may result from the uncertainty of the proton-proton forces, as obtained from proton-proton cross sections. In accordance with proton-proton data, a range of 2.8×10^{-13} cm was taken to be the most probable, with permissable variations as large as 0.4×10^{-13} cm. Calculations showed that this uncertainty in the radius introduced an uncertainty of 5 percent in the cross section at 6 Mev, with

[1] C. L. Bailey, William E. Bennet, T. Bergstrahl, Richard G. Nucholls, H. T. Richards, and J. H. Williams, Phys. Rev. **70**, 118 (1946); Phys. Rev. **70**, 583 (1946).

[2] Bretscher (to be published).

[3] Breit, Thaxton, and Eisenbud, Phys. Rev. **55**, 1018 (1939).

[4] L. E. Hoisington, S. S. Share, and G. Breit, Phys. Rev. **56**, 884 (1939).

[5] W. Rarita and J. Schwinger, Phys. Rev. **59**, 436 (1941) and **59**, 556 (1941).

[6] Sen (unpublished).

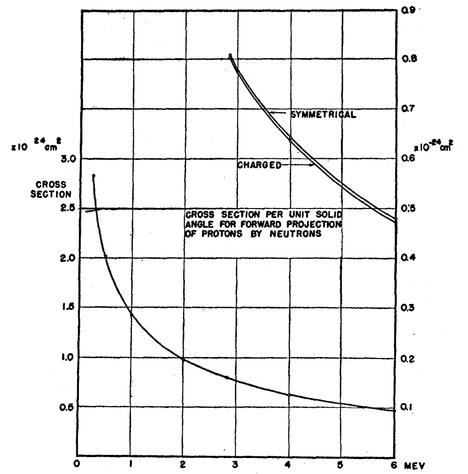

FIG. 1. Upper curves are drawn with expanded ordinate in order to show the extent of uncertainty arising from lack of knowledge of the exchange character of nuclear forces.

correspondingly smaller uncertainties at lower energy. The cross section is larger, the larger the nuclear radius. The reason for the comparatively small sensitivity to nuclear radius, exemplified by the fact that a 5 percent change in cross section accompanies a 30 percent reduction in the "area" of the well, is that the depth is always adjusted to give the correct deuteron binding energy and epithermal neutron-proton cross section.

II. TENSOR FORCES

Tensor forces are introduced into nuclear interactions in order to account for the observed quadrupole moment of the deuteron. Rarita and Schwinger[5] have shown that a variation in the fraction of tensor force of about $\frac{1}{10}$ is consistent with the probable error in the quadripole moment. It is, therefore, necessary to determine whether this uncertainty in the tensor force affects the cross section. It may be expected that the effect of this uncertainty will be small, because at moderate energies, the tensor force behaves very much like an addition to the ordinary radially symmetrical force, except for the production of a small D wave, which is responsible for the quadrupole moment. The expectation is borne out by the calculations of Rarita and Schwinger[5] and Sen,[6] who show that for a radius of 2.8×10^{-13} cm, the tensor force results in a reduction of only 2 percent in the triplet cross section at 0 Mev and at 2.82 Mev. Sen also shows that these cross sections are not changed appreciably when the fraction of tensor force is varied from 0.5 to 1.

The effects of tensor forces at higher energies can be estimated from the results of Rarita and Schwinger,[5] who give the $3S_1+3D_1$ cross section at 15.3 Mev, when the fraction of tensor force is adjusted to obtain agreement with the quadrupole moment of the deuteron. We find that this cross section is only 1 percent greater than that predicted by ordinary forces at this energy. Hence, it seems safe to conclude that, in the range from 0 to 6 Mev, the uncertainty in the fraction of tensor force should produce a negligible uncertainty in the cross section. In fact, even the complete neglect of the effects of tensor forces should result in an error of less than 2 percent in the cross section in this energy range. We have, therefore, decided that, for the purpose of studying the sensitivity of cross section to other parameters, the use of tensor forces is not necessary. Their effect will then be taken into account approximately, however, by means of a uniform reduction of 2 percent in all $3S$ cross sections.

III. EXCHANGE FORCES

Exchange forces are introduced into nuclear interactions for the purpose of explaining the saturation character of nuclear binding. A fairly wide range of combinations of exchange and ordinary forces is, however, consistent with the present experimental data. It is, therefore, necessary to find out whether the cross section depends appreciably on the precise exchange character of the forces in the range from 0 to 6 Mev.

The combinations studied in this paper were, in the terminology of Rarita and Schwinger, the "charged" and the "symmetric" theories, which represent about the widest possible variation in the exchange character consistent with the existence of saturation. The results of the calculations showed that the contribution of the "P" wave to the total cross section was so small that the exchange character did not make an appreciable difference in this cross section. Even the angular dependence varied by less than 1 percent (see Fig. 1). In order to understand the smallness of the angular dependence, it is helpful to analyze the sources of the P scattering. On account of the spin dependent tensor forces which operate only in the triplet state,[5] the phases are different for $3P_0$, $3P_1$, and $3P_2$. The calculations showed that although the $3P_0$ phase was fairly large from 4 to 6 Mev, the angular dependence arising from it was almost cancelled by that arising from the $3P_1$ and $3P_2$ phases, which are smaller, but which occur with greater statistical weight. The reason for this cancellation is that the tensor forces average out to zero in the first approximation. The net result is that practically all the angular dependence arises from the singlet state, which occurs with only one-fourth of the total weight. Consequently, both the angular dependence of the cross section and its uncertainty arising from the uncertainty in the exchange character, are very small below 6 Mev.

IV. SHAPE

Up to this point, the sensitivity of the cross section to possible variations in the parameters specifying the potential has been studied with the aid of the simplifying assumption of a square well. It is now necessary to estimate the effects of possible derivations from the square shape. The principal deviations from the square form which are significant for moderate energy scattering are (a) a tail and (b) a concentration of force near the origin. Hoisington, Share, and Breit[4] have studied such variations in the case of proton-proton scattering, and have found that the best agreement with experiment is obtained by making these two modifications simultaneously. For example, they find that the exponential well, which is characterized mainly by a long range tail without a great concentration of force near the origin, predicts both a wrong energy dependence for "S" scattering and excessive "P" anomaly. On the other hand, the Yukawa potential ($A\exp(-\alpha r)/r$), which embodies both the tail and the necessary concentration of potential, is found to fit the proton-proton data as well as the square potential, while it is also in very good agreement with the epithermal neutron-proton cross section. It is necessary, however, to choose a range corresponding to a meson mass of 300 electron masses. The significance of the disagreement of this value with that of 200 electron masses obtained from cosmic-ray experiments is not clear at present, particularly because of the uncertain status of the details of all current meson theories. In any case, it seems that a Yukawa potential with this range represents about the widest variation in shape from that of a square well which is consistent with the hypothesis of equality of all nuclear forces, and it is, therefore, the most promising one for study.

In order to simplify the calculations, and to separate clearly the effects of the tail from those of the concentration of force near the origin, it is obviously permissible to represent these two main features by an L-shaped potential, consisting of a deep short range well followed by a shallow long range well. The proportions of the L should be chosen to give a general resemblance to the above Yukawa potential in the range of importance, which is in the region from 10^{-13} cm to 5×10^{-13} cm. This method of fitting yields somewhat ambiguous results, but it turns out that almost any proportions of the L which lie within the range of ambiguity yield practically the same cross sections, if the depths are adjusted to yield the deuteron binding energy and epithermal neutron-proton cross sections. The small adjustment of singlet depth needed to give the epithermal neutron-proton cross section is easily within the range of the ambiguity in fitting. Since the L well now resembles the Yukawa potential which also gives the correct proton-proton and epithermal neutron-proton scattering, and because this agreement is not critically dependent on the precise proportions of the well, it may be concluded that the small ambiguity in shape has no important effect on the cross section.

A further requirement which the L-shaped potential must satisfy is that it gives the correct quadrupole moment of the deuteron. According to the work of Sen, the radius of the concentration of the force near the origin which we have chosen (2.2×10^{-13} cm) is definitely below that needed without a tail to give simultaneously the correct quadrupole moment and magnetic moment. The tail is, however, very effective in producing quadrupole moment, which is proportional to the mean value of r^2. Because the magnetic moment is proportional to r, the tail is less effective in producing it Approximate calcu-

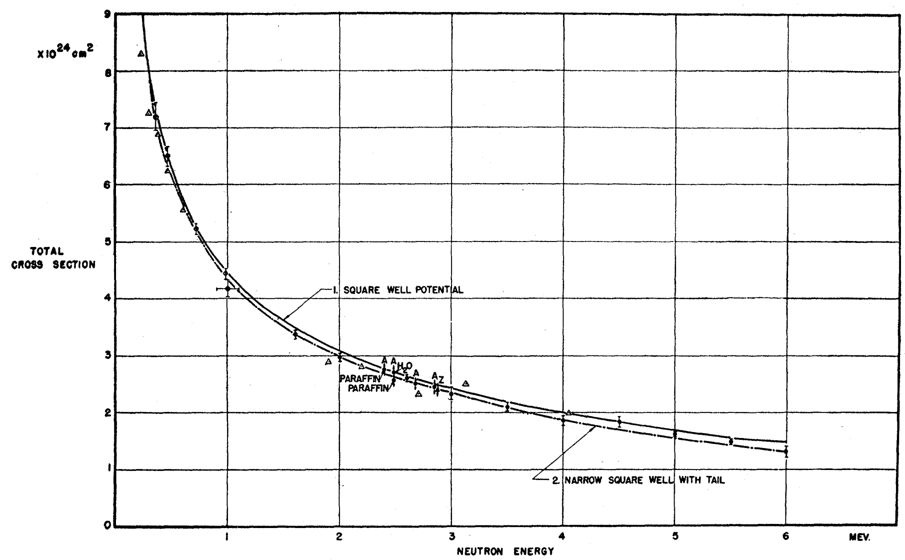

FIG. 2. Comparison of cross section for neutrons-proton scattering for two different well shapes. Specification of wells.—1. Square well-range 2.8×10^{-13} cm. Singlet depth: 11.85 Mev; Triplet depth: 21.2 Mev. 2. Narrow square well with tail: a. Square well-range 2.2×10^{-13} cm. Singlet depth: 18.39 Mev. Triplet depth: 30.33 Mev. b. Square tail-range $2.2\text{–}5.6\times10^{-13}$ cm. Single depth: 0.3 Mev. Triplet depth: 0.5 Mev. ϕ Experimental points. ϕ Williams, *et al.* △ Bretscher, *et al.* *A*—Aoki. *Z*—Zinn, Seeley and Cohen.

lations show that the well chosen by us can yield the correct quadrupole moment and magnetic moment of the deuteron within experimental error, and that the fraction of tensor force needed to obtain the agreement is about the same as that obtained by Rarita and Schwinger[5] for a square well of radius 2.8×10^{-13} cm.

V. RESULTS

The cross sections were calculated in the usual way, which is too well known to require further discussion.[7] Our results (Fig. 2) show that the L-shaped well yields a cross section which is at 6 Mev, 10 percent lower than that predicted by the square well, with correspondingly smaller differences at lower energies. Below 1 Mev, the difference in cross sections is practically undetectable. It is of interest to understand why the cross sections behave in this manner. The agreement at low energies is, of course, a consequence of the fitting of the binding energy of the deuteron and the epithermal neutron-proton scattering cross section. At higher energies, the "tail," which is only 0.5 Mev deep, begins to have very little effect on the wave function, and the main part of the scattering is then due to the deep narrow well at the center.

Because the tail is very effective at low energies, however, the concentration of potential near the origin is not actually deep enough to produce by itself the correct binding or epithermal neutron scattering cross section. If the depth were such as to give the correct deuteron binding and epithermal scattering, then the narrowing of the effective radius would result in a considerable decrease in the triplet cross section, and a slight decrease in the singlet cross section.[8] The decrease of effective depth, however, decreases the slope of the wave function at the edge of the well,[8] and thus introduces a tendency to increase the cross section. In the singlet state, this increase is the main effect, but in the triplet state, it is considerably over-balanced by the decrease resulting from the narrowing of the well. Because the increase of singlet cross section is less than the decrease of triplet cross section, and because of the greater statistical weight of the triplet

[7] Mott and Massey, *Theory of Atomic Collisions*, Chapter II.

[8] H. A. Bethe and R. F. Bacher, Rev. Mod. Phys. **8**, 83 (1936).

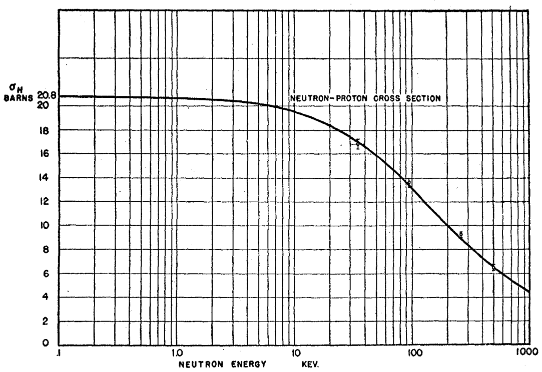

FIG. 3.

state, the net effect is to decrease the cross section at high energies.

VI. COMPARISON WITH EXPERIMENTS

The best early experiments are those of Aoki,[9] and those of Zinn, Seely, and Cohen,[10] which cover a range from 2.1 to 2.9 Mev. Later experiments[1,2] done after this work was completed, cover a range from 0.35 Mev to 6 Mev. In general, the experimental cross sections are in much better agreement with the L well than with the square well, wherever the two predictions differ. Although experimental errors are great enough to permit the assignment of a square well of range 2.4×10^{-13} cm, which would give a 6 Mev, a cross section 5 percent below that given by the well of range 2.8×10^{-13} cm, the mean of the data is in definitely better agreement with the predictions of the L-shaped well. Besides, the range of 2.4×10^{-13} cm is probably the extreme limit of error in the proton-proton range, and not a particularly likely value. Since the square shape has nothing to recommend it, except analytic simplicity, it seems preferable to assume tentatively that the well resembles a Yukawa potential.

One of the most important parameters used in the choice of a potential is the epithermal neutron-proton cross section. A very accurate value of this quantity was obtained by extrapolating to zero energy with the aid of the theory, the neutron-proton cross sections of Frisch,[11] which were made in the energy range from 35 kev to 490 kev. Because the uncertainties in the potential do not appreciably affect the cross section at these low energies, the extrapolation is quite reliable. This is verified in Fig. 3, which shows that the variation of cross section with energy in this range is in very good agreement with the theory. The extrapolated value of the epithermal cross section is 20.8×10^{-24} cm^2, as compared with the previously accepted value of $(20\pm2)\times10^{-24}$ cm^2.[12]

More work on this problem is desirable, particularly with regard to obtaining a more systematic study of the sensitivity of cross section to shape. Now that a Yukawa potential seems to give better agreement with experiment than a square well, an effort should be made to fit a Yukawa potential directly to all known data, including proton-proton scattering, neutron-proton scattering, binding energy of the deuteron, and quadripole and magnetic moments of the deuteron. The prediction of the cross sections at energies above 6 Mev, where the exchange and tensor character of the forces is important, is particularly significant. Some theoretical work has already been done on this problem.[13] It is necessary at higher energies to consider the possibility that the fundamental interactions cannot be described in terms of a potential, which is a function only of the relative coordinates.

We are greatly indebted to Professor Bethe and Mr. Sen for sending us some very helpful manuscripts. It is also a pleasure to acknowledge gratefully the help and encouragement received from Professor Oppenheimer and Professor Serber in the solution of the problem. This work was carried out under the auspices of the Manhattan District.

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[9] H. Aoki, Proc. Phys. Math. Soc. Japan **21**, 232 (1939).

[10] W. H. Zinn, S. Seely, and V. W. Cohen, Phys. Rev. **56**, 260 (1939).

[11] D. Frisch, Phys. Rev. **70**, 589 (1946).

[12] V. W. Cohen, H. H. Goldsmith, and J. Schwinger, Phys. Rev. **55**, 106 (1939); H. B. Hanstein, Phys. Rev. **57**, 1045 (1940).

[13] J. Leites Lopes, Phys. Rev. **70**, 5 (1946). A list of references to the latest theoretical work on this subject is contained in this paper.