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Absolute Energies of $K\alpha$-Radiation from Thick Targets of Silver

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$K\alpha$-radiation from a thick silver target bombarded at 40 kev was isolated by Ross filters of 42 Mo and 45 Rh and the total energy measured with a large air-filled ion chamber and found to be 8.6×10^{-12} erg per electron. This standardizes the curve of intensity *vs.* bombardment voltage measured in arbitrary units up to 180 kv by Webster, Hansen, and Duveneck. A quantitative explanation of the curve is developed upon the basis of the absolute cross section of the silver atom for K-ionization by electron impact, known over a considerable range of bombardment voltages from measurements by Webster, Hansen, and Duveneck and by J. C. Clark. From this cross section and the rate of energy loss of the bombardment electron the absolute energy of Ag $K\alpha$ is calculated and found to agree with the observed values within the limits of the errors of observation. The mass scattering coefficient of air at λ0.56A is found by calculation to be 0.244 cm^2g^{-1}.

1. INTRODUCTION

RELATIVE intensities of the characteristic x-ray lines of a variety of elements were measured by Meyer[1] and by Williams[2] but absolute determinations were not attempted. Webster, Hansen, and Duveneck[3] investigated the manner in which the intensity of the $K\alpha$-lines of silver vary with the energy of the electron bombardment which causes their emission, but again all intensities were recorded in arbitrary units. In WHD I and II both thick and thin target observations were presented, and these experimenters showed that a simple relation connects the thin target line intensity, the retardation rate of electrons in the target material, and the manner in which thick target line intensity increases with bombardment energy. Even without absolute intensity data they were able to exploit this relationship so as to show that the loss of energy per unit distance along the path of an electron up to about $\beta=v/c=0.65$ varies approximately as the -1.4 power of the speed.

The present paper reports a measurement of the absolute energy of the $K\alpha$-radiations of silver as emitted within a thick target bombarded at 40 kv. This is followed by a calculation of this line energy based upon an argument like that of WHD II, absolute thin-target energy from the work of Clark[4] and electron retardation laws regarded as approved by theory and experiment. No attempt is made to predict thick-target characteristic radiation from any pure theory of atomic emission because such theories are not yet sufficiently explicit.[5]

2. EXPERIMENT

Radiation from a commercial x-ray tube passed through an appropriate diaphragm, one of a pair of Ross filters, and into a large ionization chamber filled with dry air at atmospheric pressure. The charge of the ions collected within the chamber was measured with a calibrated Compton electrometer whose indications could be expressed in ergs of received radiation energy.

The $K\alpha$-radiation produced in the target was emitted in all directions with spherical symmetry. The portion which proceeded in the direction of observation first underwent absorption in the target, in the windows of the x-ray tube and ion chamber, in the filter, and in the air both without and within the chamber. All losses of this kind had to be evaluated in the process of determining the energy *as emitted*. The filters, one of molybdenum and one of rhodium,

[1] H. T. Meyer, Wissenschaftliche Veröffentlichungen aus dem Siemens-Konzern **7**, 108 (1929).
[2] J. H. Williams, Phys. Rev. **44**, 146 (1933).
[3] D. L. Webster, W. W. Hansen and F. B. Duveneck, Phys. Rev. **43**, 839 (1933), Phys. Rev. **44**, 258 (1933). These papers will be referred to, respectively, as WHD I and WHD II.
[4] J. C. Clark, Phys. Rev. **48**, 30 (1935).
[5] It has been shown, however, for the special case of the emission of K characteristic x-rays by nickel that the agreement between theory and thin-target observations is rather satisfactory. A report by Pockman, Webster, Kirkpatrick, and Harworth is in press.

embraced between their K absorption discontinuities a wave-length band (pass band) containing the $K\alpha$-lines of the silver target but excluding the other lines of the silver K series. This band also included a substantial proportion of continuous spectrum radiation, which proportion was determined. The thickness of the filters was such that they stopped about 85 percent of the radiation at the long wave side of the pass band. Spectrometer observations showed that for the silver spectrum as observed the errors of filter balance produced an error of less than one percent in the pass-band evaluation.

The filters, balanced in the usual way so as to transmit equally at wave-lengths not in the pass band, were placed in the beam between the tube and the ion chamber, one at a time, and the ionizing power of the filtrate radiation was determined by observation of the charge collected in a measured time t. The charge which would have been produced in the ion chamber by the full strength of the pass-band radiation, unattenuated by filtration, was obtained by dividing the difference between the charges collected when the two filters were used one at a time by a constant T which is characteristic of the filter pair. This constant, the transmission coefficient of the pair, is the difference between the fractions of the silver $K\alpha$-radiation passed by the respective filters. T was determined with the help of a Bragg spectrometer and found to be 0.262 ± 0.004.

The bombardment voltage of 40 kv selected for this measurement is high enough to produce an abundant output of $K\alpha$-radiation without being so high as to introduce undesired second order radiation into certain auxiliary spectrometer measurements to be described below.

We define the following quantities:

B = total Ag $K\alpha$-energy (in ergs) produced at the focal spot of a thick silver target per 40 kv bombarding electron. This quantity includes the line radiation resulting from K ionization produced *directly* by electron bombardment and also that produced *indirectly* by the absorption of continuous x-rays.

W = fraction of the $K\alpha$-radiation emitted in the direction of observation which escapes absorption in the target.

Ω = solid angle embracing the observed radiation.

k_w = fraction of the $K\alpha$-energy which survives absorption by the windows of the x-ray tube and the ion chamber.

k_a = fraction of the $K\alpha$-energy which survives absorption by the air lying between the x-ray tube window and the collecting region of the ion chamber.

k_c = ratio of the $K\alpha$-energy which is expended in ion production in the collecting region of the ion chamber to the $K\alpha$-energy arriving at the entrance end of the collecting region.

δ = ratio of continuous to α line energy in the pass band.

T = transmission coefficient of the filters (defined above).

i = target current in amperes.

t = duration, in seconds, of exposure of the ion chamber to x-rays.

Q_{Rh} = charge, in coulombs, collected in the ion chamber when the rhodium filter is in the beam.

Q_{Mo} = charge, in coulombs, collected in the ion chamber when the molybdenum filter is in the beam.

ϵ = x-ray energy in ergs expended in air per ion pair produced[6] $=5.13\pm0.2\times10^{-11}$ erg/ion pair.

A little thought about these quantities establishes the relation

$$B=\frac{4\pi\epsilon(Q_{Rh}-Q_{Mo})}{\Omega W k_a k_w k_c T(1+\delta)it}. \qquad (1)$$

There follows a discussion of the evaluation of the quantities on the right side of this working equation for the determination of B.

Determination of the collected charges Q_{Rh} and Q_{Mo} required knowledge of the voltage sensitivity of the electrometer and of the capacitance to ground of its associated ion-collecting system. Curves of electrometer deflection *vs.* applied potential were obtained under two conditions; first with the potential applied directly to the collecting system, and second, with a standard condenser of capacitance $C_s=42.88\times10^{-12}$ farad connected in series between the potential source and the collecting system. Since the electrometer was to be used ballistically it was calibrated in the same manner, the applied potential being raised gradually to the final value throughout a time interval t. During x-ray measurements C_s was not connected, and the collected charge producing any given electrometer deflection was calculated by the equation $Q=C_s(V-V_0)$, where V_0 and V are the potentials associated with the observed deflections on the first and second calibration curves, respectively.

The time of exposure t, uniformly 20 seconds, was controlled by a pendulum clock and an electrically operated shutter. The deflections, of

[6] W. Binks, Rep. on Prog. in Phys. **3**, 347 (1936).

the order of 20 to 40 centimeters, were quite reproducible. A set of twelve observations with each filter, taken alternately, yielded the values $Q_{Rh}=(23.70\pm0.10)\times10^{-12}$ coulomb and $Q_{Mo}=(8.15\pm0.07)\times10^{-12}$ coulomb, the probable errors indicated being mainly the estimated errors of calibration.

The observed radiation all passed through a reamed hole in a type-metal plate which was mounted 69.7 cm from the center of the small roughened area of the target of the x-ray tube. The hole diameter, according to measurements made with a machinist's hole gauge, was 0.3205 cm. These figures give for the solid angle of observed radiation $\Omega=\pi(5.28\pm0.03)\times10^{-6}$ steradian. The probable error is an estimate which depends principally upon our ignorance of the precise location of the effective center of the focal spot.

The evaluation of the absorption of $K\alpha$-radiation in the silver target was derived from observations by others. It is not possible to deduce the emerging fraction W with precision from WHD II, but their original data, preserved in this laboratory, have afforded the basis for a satisfactory determination. They recorded the intensity, in arbitrary units, of the Ag $K\alpha$-doublet, isolated by a calcite crystal, for the three grazing angles of emergence from the target 5°, 15°, and 25°. For each of these angles they used four different bombardment voltages, respectively, 1.18, 1.5, 2.0, and 3.0 times the K excitation potential at 25.5 kv. None of these data applies exactly to the conditions of the present experiment but fortunately the angles and potentials of Webster, Hansen, and Duveneck bracketed those of this investigation so the required intensities were readily obtained from their data by graphical interpolation.

The treatment of the data for the determination of W follows the method of Kulenkampff.[7] In this method the common logarithms of the intensities for a given bombardment potential are plotted against the cosecant of the grazing angle θ between the emerging radiation and the target face, and the curve (almost straight) is extrapolated to cosecant $\theta=0$. The intensity at this point is taken to be the intensity before target absorption. Used with the data of Webster, Hansen, and Duveneck this process gives $W=0.75\pm0.03$. Wisshak[8] has also recorded intensities of Ag $K\alpha$ emerging at various angles with the target face, and his data provide a check on the W value above. The check is weak in two respects: the original data are unavailable, and the voltage range of his work extended only from 27.5 kv to 31.9 kv. Inferences must be drawn from his Fig. 5 and a drastic extrapolation to 40 kv must be made. Proceeding thus one obtains $W=0.78\pm0.10$. Because of the large indicated uncertainty we shall use the result from Webster, Hansen, and Duveneck, regarding the Wisshak value as confirmatory only.

Air absorption calculations are based upon a mass absorption coefficient of 0.660 $cm^2\ g^{-1}$ at $\lambda=0.560$A, derived from published conclusions of Victoreen[9] and unpublished measurements by David Nicodemus. For air under the conditions of these observations (21°C and 76.3 cm Hg) this gives a linear absorption coefficient $\mu=7.96\times10^{-4}\ cm^{-1}$. The measured air path from the x-ray tube to the collecting region within the ion chamber was 103.3 cm, an absorber passing a fraction $k_a=0.921$.

Window absorptions were directly measured by inserting the absorbing material into the beam entering a Bragg spectrometer adjusted to receive the Ag $K\alpha$-lines. The Cellophane window of the ion chamber absorbed negligibly but the absorption by the wall of the x-ray tube required evaluation. This tube envelope was a reasonably uniform cylinder of Pyrex glass on which one area was rather arbitrarily chosen as the exit port of radiation.

To understand the method by which the absorption at this window was measured without destroying the tube, consider a plane normal to the tube axis and including the chosen window. This plane intersects the tube envelope in a circle, in which an inscribed equilateral triangle may be imagined with one of its vertices coinciding with the center of the window. The x-ray tube was placed before a Bragg spectrometer in such a position that the entering beam of x-rays (from an auxiliary silver-target tube) passed

[7] H. Kulenkampff, Ann. d. Physik 69, 548 (1922).

[8] F. Wisshak, Ann. d. Physik 5, 507 (1930).

[9] J. A. Victoreen, J. App. Phys. 14, 95 (1943).

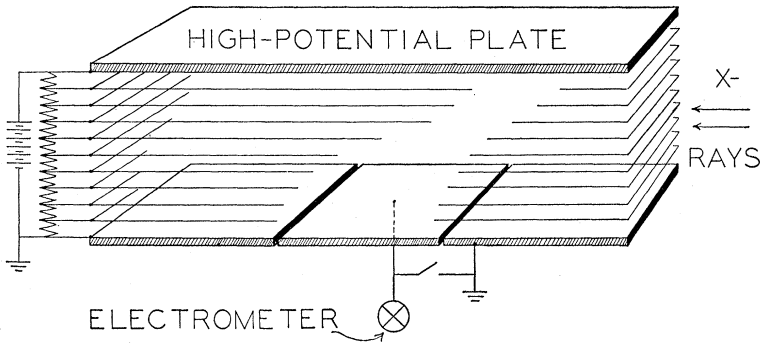

FIG. 1. Internal construction of absolute ion chamber, showing provisions for insuring straightness of field.

along one side of the triangle defined above on its way to the crystal.

The absorption of Ag $K\alpha$ by the tube in this position was measured by making spectrometer observations with the tube in and out of the beam, after which the tube was rotated about its own axis and the measurements repeated with the x-ray beam passing in turn along the other two sides of the triangle. From the three absorption measurements the absorption by the glass tube wall for radial transmission at any of the triangle vertices could readily be calculated. For the area used in the principal measurements of this paper the fraction of Ag $K\alpha$ transmitted was found in this way to be $k_w = 0.785 \pm 0.005$.

The ionization chamber was a steel cylinder 17.5 inches long and 8 inches in internal diameter. The centrally located electrode structure, shown in Fig. 1, consisted of two stiff and parallel aluminum plates 13 inches long, 4 inches wide, and 4 inches apart. One plate was divided into three electrically distinct sections of which the middle one was directly connected to the electrometer, and the others served as guard plates to keep the electrostatic collecting field rectilinear. As further insurance the entire field between the electrodes was surrounded by nine rectangular wire frames of the kind used with success by Taylor and Singer[10] for reducing curvature of their collecting fields. These frames, maintained at properly spaced potentials proportionate to their physical spacings in the electrode gap, together with the guard plates, produced a collecting field straight enough at its entrance and exit boundaries to justify the assumption that the section of the x-ray beam yielding collectible ions was the same in length as the distance ($L = 10.25$ cm) between the centers of the gaps separating the collecting plate from the adjacent guard plates. The over-all potential of the chamber was 171 volts, a potential sufficient to produce sensibly complete saturation in air.

[10] L. S. Taylor and G. Singer, Bur. Stand. J. Research 5, 507 (1930).

3. ANALYSIS OF THE DATA

The ions measured were presumed to be those formed within the rectangular collecting volume bounded on one side by the collecting plate and on the opposite side by a portion of the high potential plate. The chamber was open to the atmosphere through a drying tube, so the ionized medium was dry air at the ambient density. Within the collecting region ions must have been produced by: (*a*) cosmic radiation, (*b*) radioactive materials in and near the chamber, (*c*) photo-electrons ejected from air in the collecting region by the primary beam, (*d*) Auger electrons and fluorescence radiation from the excited atoms of *c*, (*e*) recoil electrons ejected from air in the collecting region by the primary beam, (*f*) fluorescence radiation originating in the guard regions and absorbed in the collecting region, (*g*) radiation scattered by air in all regions of the chamber, (*h*) electrons and x-rays evoked from metal surfaces of the chamber by scattered radiation.

Ions from sources *a* and *b* were negligibly few and furthermore were discarded by subtraction in the differential filter process. The energy W_1 expended in ion production in the collecting region by processes *c*, *d*, and *e* is almost exactly $W_1 = W_0(1 - \exp[-(\tau + \sigma_a)L])$, where W_0 is the Ag $K\alpha$-energy entering the collecting region during the exposure time, τ and σ_a are the linear fluorescence and recoil absorption coefficients of

the air in the chamber for the Ag $K\alpha$-radiation. The inexactitude in the expression for W_1 results from the escape of a portion of the fluorescence radiation emitted within the collecting region.[11] Only the argon K fluorescence, of all the characteristic radiations of air, is hard enough to have a chance of escape, and calculation shows that the amount thus lost is only about one half percent of the total W_1. This loss is in large part compensated by the fluorescence radiation (process f) from the guard regions of the chamber which comes into the collecting region and is there absorbed. Within the accuracy of this experiment W_1 is taken to be the ionizing energy expended in processes c, d, and e.

The coefficient μ covers three processes of removal of energy from the primary beam and may be broken down into the form $\mu=\tau+\sigma_a+\sigma_s$, where σ_s pertains to true scattering, both modified and unmodified. To evaluate the sum $\tau+\sigma_a$ from this equation it is necessary first to determine σ_s.

The linear scattering coefficient σ_s is a combination of the scattering coefficients of the constituent gases. The scattering of x-rays by gases is rather well understood and the present agreement between theory and experiment makes it possible to calculate scattering coefficients with some confidence. For the diatomic gases we have used Woo's[12] equation, which is also conveniently presented by Compton and Allison.[13] This equation, which expresses the intensity of scattering, both modified and unmodified, by a diatomic molecule may be written

$$I_\phi = I(e^4/r^2m^2c^4)(1+\cos^2\phi) \times\{F^2(1+(\sin x)/x)+R(Z-\Sigma f^2)\},$$

where I is the incident intensity, ϕ the scattering angle, r the distance to the point of observation, F the atomic structure factor, Σf^2 the incoherent scattering function, R the Breit-Dirac factor $[1+(h/mc\lambda)\ vs.\ \phi]^{-3}$ and $x=(4\pi s/\lambda)\sin(\phi/2)$ in which s is the distance between the atoms in the molecule. The meaning of the other symbols is obvious. The separation of the oxygen atoms was taken to be 1.22A and that of nitrogen 1.10A, these figures being deduced from Rasetti's[14] spectroscopic determination of the moments of inertia of the molecules. The structure factors of oxygen and nitrogen by Compton and Allison were used as far as possible but they covered only the range of scattering angles from zero to about eighty degrees. The rest of the way as far as 155° was filled out by normalizing Barrett's[15] experimental observations of the scattering by oxygen and nitrogen to fit the calculated angular scattering distribution. From 155° to 180° the curve was drawn in freely, following the known general shape of experimental scattering curves of other scatterers of low atomic number. The calculation is for the wave-length 0.56A whereas Barrett's wave-lengths were 0.48A for oxygen and 0.39A for nitrogen. This discrepancy causes no trouble since the *shape* of the scattering distribution varies quite slowly with wave-length, Barrett's oxygen curves for 0.39A and 0.49A being barely distinguishable.

The linear scattering coefficient may be evaluated by an integration[16] of the angular scattering distribution which must be carried out graphically or numerically in the present case. In this process the ordinates of the distribution curve are multiplied by $\sin\phi$ and since the sine goes to zero at 180° the part of the distribution curve which is most uncertain contributes little to the final result. Graphical integrations yielded the mass scattering coefficients 0.246 for oxygen and 0.242 for nitrogen at λ0.56A. A similar calculation for oxygen was made by Woo and Sun[17] who calculated their own structure factors by the method of Thomas and Fermi. From their curve of atomic absorption coefficient *vs.* wave-length it is possible to deduce for λ0.56A the value $\sigma/\rho=0.246$, in full agreement with the present calculation.

The only other component of air which needs consideration is argon. From the calculations of

[11] Some photo- and recoil electrons will originate in the collecting region but escape into the guard regions, there to expend their energies in the production of uncollectible ions. This loss is almost exactly compensated by the photo and recoil electrons which originate in the guard regions and move into the collecting region to produce ions.

[12] Y. H. Woo, Phys. Rev. **41**, 21 (1932).

[13] A. H. Compton and S. Allison, *X-rays in Theory and Experiment* (D. Van Nostrand Company, Inc., New York, 1935), Eq. (3.58).

[14] F. Rasetti, Phys. Rev. **34**, 367 (1930).

[15] C. S. Barrett, Phys. Rev. **32**, 22 (1928).

[16] After the manner of J. J. Thomson, *Conduction of Electricity through Gases*, second edition, p. 325.

[17] Y. H. Woo and C. P. Sun, Sci. Rep. Nat. Tsing Hua Univ. **3**, 549 (1936).

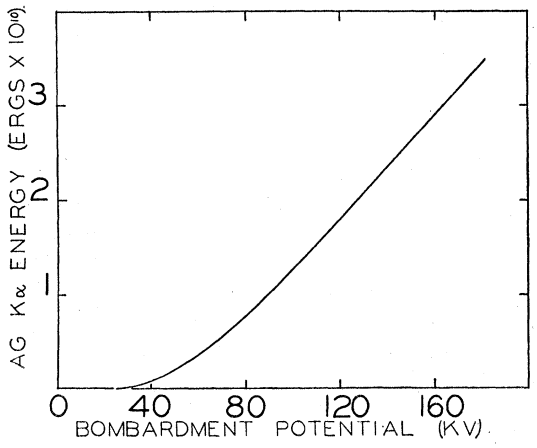

FIG. 2. Total energy of $K\alpha$-radiation produced at the focal spot of a thick silver target per bombarding electron. Observations by the authors and others.

Woo and Sun we deduce $\sigma/\rho=0.374$ for this element at $\lambda 0.56$A. Measurements by Wollan,[18] as interpreted by Woo and Sun, gave 0.395. The value 0.39 is adopted here. From the accepted composition of dry air, the density given above, and the mass scattering coefficients just discussed one obtains directly the mass scattering coefficient 0.244 and the linear scattering coefficient $\sigma_s=2.94\times10^{-4}$ cm^{-1}, which leads to $\tau+\sigma_a=5.02\times10^{-4}$ cm^{-1} and $W_1=0.00514W_0$.

The energy scattered out of the primary beam from the segment of it lying in the collecting region is $W_2=W_0(1-\exp[-\sigma_s L])$. This scattered radiation will traverse the collecting region, having a path length therein of the order of $L/2$, and in so doing will lose by photoelectric and recoil absorption a quantity of energy

$$W_3=W_0(1-\exp[-\sigma_s L])(1-\exp[-\tfrac{1}{2}L(\tau+\sigma_a)]).$$

Scattering takes place also in the guard regions at the ends of the collecting regions. They are approximately the same in size as the collecting region so it may be supposed that each of them sends to the collecting region scattered radiation which expends therein radiation energy W_3. The justification for the roughness of this argument is that $3W_3$ is found by calculation to be a negligibly small quantity. This disposes of ion source g.

Detailed calculations for this chamber show that ions produced by electrons or x-rays from metal parts (source h) cannot exceed a few tenths of one percent of the number formed by primary processes evaluated above so this contribution is neglected. Summarizing these arguments one finds $k_c=W_1/W_0=0.00514$.

When radiation is abundant, air is an excellent gas for use in absolute ionization chambers, since the corrections which cause so much trouble with heavy gases here turn out to be negligible. Furthermore the scattering coefficient may be calculated with a certainty not possible with polyatomic molecules of undetermined structure. It is no disadvantage that the fraction k_c is small.

The ratio δ of continuous to $K\alpha$-line energy in the energy difference between the molybdenum and rhodium filtrates was determined by investigating the pass band region with a spectrometer, using the x-ray tube, tube potential, and ion chamber which were later used in the absolute measurements. Under these conditions δ could be determined by plotting the part of the spectrum lying in the pass band and comparing the line and continuous areas. An error is possible here because of the variation of the calcite spectrometer crystal's reflecting power with wave-length, but this variation is actually slight for good calcite, and the region under consideration has a breadth of only 15 percent of its mean wave-length. No radiations of higher order than the first were included in these spectrometer observations. The observed $\delta=0.920\pm0.02$, where the indicated uncertainty is an estimate taking account of the error just mentioned, the ordinary errors of spectrometer observation, and the uncertainty involved in interpolating a continuous spectrum under a line.

The target current $i=0.0025$ amps., was measured with negligible error with a freshly calibrated milliammeter introduced into the tube circuit so as to measure no corona or other loss currents. Assembling all the numerical quantities we find by applying Eq. (1),

$$B=(8.6\pm0.8)\times10^{-12}\text{ erg}$$

of $K\alpha$-radiation from thick silver per 40-kv electron. This includes all radiation emitted at the focal spot in all directions, the total being taken before absorption. It does not include $K\alpha$-radiation produced at points away from the focal spot by bombardment electrons which

[18] E. O. Wollan, Phys. Rev. **35**, 1019 (1930).

rediffuse from the focal spot by single nuclear scattering and experience radiative collisions elsewhere.

Data on *relative* intensities of $K\alpha$-radiation from thick silver bombarded at potential from 27 to 179 kv were given in WHD II, where curves were presented showing on logarithmic scales the variation of intensity with a stated function of the bombardment energy. Precise relative intensities, however, are not readily extracted from the paper and we have therefore consulted the original data of these authors and have normalized them to the 40-kv value of B given in the preceding paragraph. The result appears in Table I and in Fig. 2.

Argument from Thin-Target Observations

Any theory which would account for the magnitude of B must take account of the K ionization cross section of the silver atom, the relative intensity of the α-lines in the total K spectrum, the intensity of K-radiation from atoms ionized by x-rays rather than by electrons, the silver fluorescence yield, and the rate of loss of bombardment kinetic energy by all processes. Let

$\Phi(\beta)$ = cross section of the silver atom for K ionization by electrons with speed βc.
$-\partial E/\partial x$ = electron energy loss rate in ergs per cm of path in silver.
R = ratio of the number of K ionizations to the number which would have occurred if no electrons had rediffused from the target.
w = silver K fluorescence yield.
P = ratio of directly to indirectly produced $K\alpha$ power.
p = ratio of number of $K\alpha$-quanta to total K quanta emitted.
A = atomic weight of target element.
N = the Avogadro number.
ρ = density of the target substance.

The total number I of K ionizations produced by an electron which enters the target at a speed given by β_0 and slows down therein until it can no longer produce K ionizations is

$$I=-\int_0^{\beta_0}\frac{\partial I}{\partial x}\frac{\partial x}{\partial E}\frac{\partial E}{\partial \beta}d\beta, \qquad (2)$$

in which $\partial I/\partial x=\Phi\rho N/A$, $\partial x/\partial E$ is a function of β to be selected from the theoretical and experimental literature, and the other derivative is obtained from the relativistic kinetic energy expression. All three factors in the integrand are functions of β and the integration must be performed graphically or numerically. The function Φ is known from WHD I and J. C. Clark,[4] and the values obtained by combining the two papers[19] are listed in Table II.

The energy loss rate $-\partial E/\partial x$ has not been measured for electrons in silver with the precision needed here. The nearest thing to an experimental expression which is available comes from Williams'[20] empirical adjustment of Bohr's[21] theory to agree with the observations of White and Millington[22] on the stopping of beta-particles by mica. Williams found the loss rate to be $1.06\,\beta^{-1.4}$kv per milligram/cm^2, which is a rough approximation in that it takes no account of the different stopping properties of the different atoms except as related to the density. The exponent of β was specifically confirmed for silver in WHD II over a range of β applicable here, but the numerical coefficient is supported only by observations on atoms much lighter than silver. For silver Williams' expression may be written

$$\frac{-\partial E/\partial x}{\rho N/A}=3.04\beta^{-1.4}\times10^{-28}. \qquad (3)$$

TABLE I. Energy of Ag $K\alpha$ produced in thick targets. Ergs per bombardment electron.

Bombardment potential (kv)	Ag $K\alpha$ energy B (ergs$\times10^{12}$)
<25.53	0
27.0	0.151
30.1	1.12
38.3	7.14
51.1	23.4
76.6	71.4
102.1	133.
127.6	204.
153.2	272.
178.7	343.

[19] Clark's Φ has been modified by the substitution of more recent values of the silver fluorescence yield and of the ratio p of the number of $K\alpha$-quanta to the total number of K quanta produced.
[20] E. J. Williams, Proc. Roy. Soc. **A130**, 310 (1931).
[21] N. Bohr, Phil. Mag. **25**, 10 (1913).
[22] P. White and G. Millington, Proc. Roy. Soc. **A120**, 701 (1928).

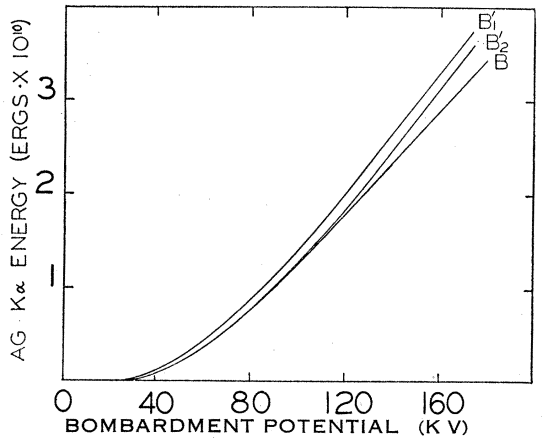

FIG. 3. Total energy of $K\alpha$-radiation produced at the focal spot of a thick silver target per bombarding electron. Curve B, as observed. Curves B_1' and B_2', as calculated from measured K ionization cross sections of the silver atom.

It is possible to use instead of Eq. (3) a formula with a more definite theoretical basis. From the theory of Bethe,[23] based upon Born's approximate collision treatment, one obtains

$$\frac{-\partial E/\partial x}{\rho N/A}=\frac{4\pi e^4 Z}{mc^2\beta^2}\left\{\ln\frac{mc^2\beta^2}{J(1-\beta^2)}-\beta^2\right\}, \qquad (4)$$

where J, the mean excitation energy of the stopping atoms may be taken[24] as $18.4\times10^{-12}Z$ erg. It has been assumed in connection with Eq. (4) that the speed of the incident electron is large compared to the speeds of the atomic electrons. This is certainly not the case with the K electrons of silver but these are of course greatly outnumbered by the slower atomic electrons.

Equation (2) has been used for the graphical evaluation of I, first in combination with Eq. (3) and again with Eq. (4). Each evaluation yields I as a function of β, and it is satisfying to find that the corresponding values of I differ, on the average, by less than ten percent.

The energy of $K\alpha$-radiation resulting from these I ionizations and from the accompanying continuous radiation is

$$B'=h\nu_\alpha pwIR(P+1)/P. \qquad (5)$$

The ratio p of the number of $K\alpha$-quanta to the total number of K quanta is readily evaluated from the known line intensities[25] and is found to be 0.828 ± 0.01. For the K fluorescence yield w Stephenson's[26] value 0.81 ± 0.01 has been adopted. The rediffusion ratio R was estimated by Webster, Hansen, and Duveneck and from Table I of WHD II one obtains by interpolation the values given in Table III below. This table also shows values of $(P+1)/P$ from measurements by Webster.[27] Finally, Table III presents values of B' computed from the tabulated data by the use of Eq. (5). These values of B' are plotted in Fig. 3, together with a curve representing the corresponding directly observed quantity B.

The probable error in the values of B' is estimated to be about ten percent and the probable error for B is estimated at seven percent. In view of these uncertainties, the slight discrepancy between observed and predicted line

TABLE II. Cross sections Φ of the silver atom for K ionization by electrons of speed βc.

β	$\Phi\times10^{23}$ (cm²)
0.306	0
.333	1.60
.367	2.91
.416	3.96
.456	4.37
.493	4.51
.552	4.54
.598	4.45
.640	4.36
.672	4.25

TABLE III. Total energy of $K\alpha$-radiation per incident electron. β_0 is relative speed of an electron at impact with a thick silver target. $B_{1,2}'$ is the number of ergs of resulting $K\alpha$-radiation as predicted from thin target observations. B_1' is calculated by use of Williams' electron-stopping formula, Eq. (3), and B_2' with a formula from Livingston and Bethe, Eq. (4).

β_0	R	$(P+1)/P$	$I_1\times10^3$	$I_2\times10^3$	$B_1'\times10^{10}$	$B_2'\times10^{10}$
0.306			0	0	0	0
0.367	0.863	1.53	0.30	0.24	0.095	0.076
0.416	0.813	1.52	0.90	0.81	0.267	0.241
0.493	0.793	1.51	2.90	2.54	0.834	0.732
0.552	0.781	1.50	5.3	4.89	1.49	1.38
0.598	0.775	1.50	8.1	7.52	2.27	2.10
0.640	0.770	1.49	11.4	10.7	3.13	2.95
0.660	0.768	1.49	13.1	12.6	3.62	3.44

[23] M. S. Livingston and H. A. Bethe, Rev. Mod. Phys. **9**, 263 (1937). This article cites many prior references. The factor 2 in their Eq. (749a) does not apply to the stopping of electrons so it is not retained in Eq. (4) above.

[24] R. Wilson, Phys. Rev. **60**, 749 (1941).

[25] J. H. Williams, Phys. Rev. **44**, 146 (1933).

[26] R. J. Stephenson, Phys. Rev. **51**, 637 (1937). See also E. Arends, Ann. d. Physik **22**, 281 (1935), I. Backhurst, Phil. Mag. **22**, 737 (1936).

[27] D. L. Webster, Proc. Nat. Acad. Sci. **14**, 337 (1928).

intensities is without significance; the agreement is as complete as the quality of the observations will permit. Conclusions about the relative merits of the two electron-stopping formulas would scarcely be warranted by these results since all the curves of Fig. 3 are separated by amounts less than their errors.

The present work needs to be extended in two directions: thick and thin target comparisons for other elements should be made, and the thin target characteristics should themselves be explained by fundamental atomic theory. The latter matter is discussed in a report[28] which is in press at this writing. The former will receive attention in a paper now in preparation on the characteristic radiations of copper and nickel.

[28] D. L. Webster, L. Pockman, P. Kirkpatrick, and K. Harworth, Phys. Rev. **71**, 330 (1947).

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Range Measurements of Alpha-Particles from 94^{239} and 94^{238}†

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The ranges of alpha-particles from 94^{239} and 94^{238} have been measured by comparison with Po alphas, and are found to be 3.68 and 4.08 cm of air.

INTRODUCTION

TO determine the ranges of alpha-particles from 94^{239} and 94^{238} with greater precision than previously reported,[1,2] a direct comparison of these ranges with the ranges of alpha-particles from Po was carried out.

EXPERIMENTAL METHOD

The samples of 94 were deposited on platinum by evaporation. They were separated from the bombarded uranium with a very small amount of carrier. The uranium sample was electroplated on a copper disk. The thinness of all these samples is borne out by the small values of the straggling coefficient observed, which is reported below.

The range of the alpha-particles were measured by comparison with a thin polonium standard. The samples were put in front of, and at a constant distance from, a shallow (0.23 cm) ionization chamber. The chamber and the sample were enclosed in a vessel, the pressure in which could be varied and the temperature measured. We plotted curves giving the counting rate of the samples, at a constant gain of the amplifier and registering circuit, as a function of the air density in the chamber.

The mean range was computed from the air density at which one obtained half the maximum counting rate according to the formula,

$$R = R_{Po}(\delta/\delta_{Po}), \qquad (1)$$

in which R is the range of the alpha-particles of the substance under investigation, $R_{Po}=3.842$ cm (in air at 15°C and 760 mm of Hg)[3] and δ_{Po}, δ are respectively the densities of the air at which half the maximum counting rate is observed for δ_{Po} and for the substance under investigation. A small correction for varying pressure in the ionization chamber is discussed below. The ranges R obtained in this way are clearly mean ranges in air at 15° and 760 mm Hg.

Various runs for each substance were per-

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[1] J. W. Kennedy, G. T. Seaborg, E. Segrè, and A. C. Wahl, Phys. Rev. **70**, 555 (1946).

[2] G. T. Seaborg, A. C. Wahl, and J. W. Kennedy, Phys. Rev. **69**, 367 (1946).