Maximum β-Energies and the Mass of the Neutrino

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OCCASIONALLY it has been mentioned[1] that one might be able to estimate the mass of the neutrino, ν, from the knowledge of maximum β-energies (in the following denoted $E^{\max}$ and involving the electron rest mass mc^2) and corresponding mass differences (ΔM) between two neighboring isobars, one of which decays into the other by β-decay. The mass of the neutrino might then be determined from the following relation:

$$\Delta Mc^2 = E^{\max} + \nu c^2. \tag{1}$$

It has been hoped that this method would yield a more direct estimate of the mass of the neutrino, than that based on a comparison between experimental β-spectra and the form of these spectra as predicted by the theory of Fermi.[2] Such a comparison gives at present a mass of the neutrino smaller than $\frac{1}{5}$ the electron mass. However, a more detailed analysis of the problem seems to indicate that the maximum β-energy can only be determined from the experimental β-spectra with the same accuracy as can the mass of the neutrino by the original Fermi method, i.e., from the shape of these spectra. If the mass of the neutrino is not zero, but is smaller than the electron mass, this will only affect the shape of the β-spectra near the upper limit. Now, the probability for the emission of a β-particle with an energy near the upper limit is small, and for this reason a determination of the upper limit is usually based on an extrapolation of the curve from points not too near the upper limit. It is true that these points are experimentally more accurately determined, but they are generally not affected by the finite mass of the neutrino, and the extrapolation leads, thus, to a value E^{ex}, which would be equal to that $E^{\max}$ which one would find if the mass of the neutrino were zero. In other words, the extrapolated energy value is equal to the true maximum energy augmented by the energy equivalent to the mass of the neutrino. It is thus not possible to estimate νc^2 using the value E^{ex} instead of $E^{\max}$ in Eq. (1).

Very often one uses the so-called Fermi plot for this extrapolation. The probability $P(p)dp$ for the emission of a β-particle with momentum p within the limits dp, is as a first approximation in the Fermi theory given by the following formula in which the mass of the neutrino has not been neglected:

$$P(p)dp = \text{const}\cdot p^2(E^{\max}+\nu c^2 - E) \times[(E^{\max}+\nu c^2-E)^2-(\nu c^2)^2]^{\frac{1}{2}}\cdot dp \tag{2}$$

and where E is the β-energy corresponding to the momentum p. In the so-called Fermi plot mentioned above, one plots $[P(p)/p^2]^{\frac{1}{2}}$ against E. If the mass of the neutrino is put equal to zero, we get, in all cases where the experimental curve follows the Fermi curve, a straight line cutting the abscissa at E equal to $E^{\max}$:

$$[P(p)/p^2]^{\frac{1}{2}} = (E^{\max}-E). \tag{3}$$

If the mass of the neutrino is not zero, but small, we may remember that the determination of $E^{\max}$ is based on an extrapolation, i.e., on points with

$$E^{\max}+\nu c^2 - E \gg \nu c^2 \sim 0. \tag{4}$$

We may then neglect the small term $(\nu c^2)^2$ in (2), and the Fermi plot gives

$$[P(p)/p^2]^{\frac{1}{2}} = (E^{\max}+\nu c^2-E). \tag{5}$$

It is thus seen that the extrapolated maximum energy E^{ex} is given by

$$E^{\text{ex}} = E^{\max}+\nu c^2. \tag{6}$$

As mentioned before, E^{ex} does not permit a determination of νc^2, and only as far as the shape of the curve near $E^{\max}$ can be determined, can we find the actual values of $E^{\max}$ and, consequently, the value of νc^2.

As an illustration, the theoretical Fermi curves for four different assumptions as regards the mass of the neutrino have been drawn in a

[1] E. M. Lyman, Phys. Rev. **55**, 234 and 1123 (1939); R. O. Haxby, W. E. Shoupp, W. E. Stephens and W. H. Wells, Phys. Rev. **58**, 1035 (1940).

[2] E. Fermi, Zeits. f. Physik **88**, 161 (1934).

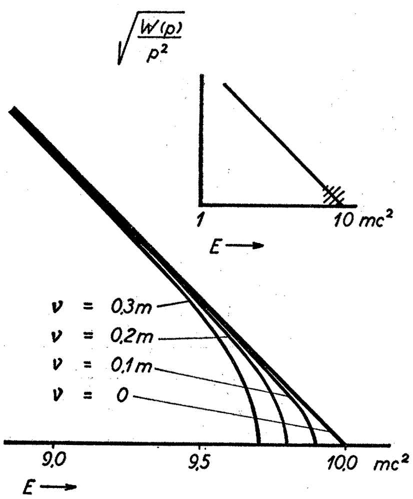

FIG. 1. Theoretical Fermi plots for a β-emitter with $\Delta M = 10 \cdot m$ is shown above to the right. The hatched part of this plot is shown in enlarged scale in the lower part of the figure for four different assumptions about the mass of the neutrino.

Fermi plot in Fig. 1, where the mass difference ΔM has been put equal to $10m$. The deviations from the straight line are small and occur only near the upper limit, practically only in the interval

$$(\Delta M - 2\nu)c^2 < E < \Delta M c^2.$$

The thesis that the maximum β-energy can only be determined with the same accuracy as can the mass of the neutrino, has here been illustrated in the special case of the Fermi theory, which fits experimental data rather closely, but it is not restricted to this theory alone. At any rate, we may say that the probability $P(p)dp$ is a function of the energies and momenta of the neutrino and the electron, which are connected by the law of conservation of energy. The derivation of such an expression is only based on the following assumptions:

(a) The process is one involving three particles, and the mass of the recoil nucleus may be regarded as infinite compared with the two other masses.

(b) The law of conservation of energy and momentum is valid, the conservation of energy being expressed by the following formula:

$$E_\nu + E = E^{\max} + \nu c^2 = \Delta M c^2, \qquad (7)$$

so that $P(p)dp$ may be written

$$P(p)dp = f(p, E, p_\nu, E_\nu), \qquad (8)$$

where the indices ν refer to the neutrino. Inserting (7) in (8) we get

$$P(p)dp = f\{p, E, (E^{\max} + \nu c^2 - E), \times[(E^{\max} + \nu c^2 - E)^2 - (\nu c^2)^2]\}^{\frac{1}{2}} dp, \qquad (9)$$

and it is again seen that E^{ex} may come out in accordance with (6).

Of course, an upper limit for the neutrino mass may be deduced in such cases where $E^{\max}$ may be neglected, e.g., $H^3 \rightarrow He^3$, but the mass difference $H^3 - He^3$ is at present not known with sufficient accuracy. Also, investigations near the upper limit of β-spectra with refined experimental technique might be of interest, and provide a possibility for a better estimate of the mass of the neutrino.

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