Relativistic Corrections to Magnetic Moments of Nuclear Particles

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ACCORDING to measurements made by W. R. Arnold and A. Roberts[1] the magnetic moment of the deuteron is accounted for by vector addition of proton and neutron moments. They show that the inclusion of the effect of the D state according to Schwinger and Rarita[2] gives agreement with theory within better than the experimental error of ± 0.0012 nuclear Bohr magnetons. In this comparison no correction for relativistic effects to the nuclear moments is made. On the other hand, a recent paper by P. Caldirola[3] indicates as probable a value of -0.006 for the relativistic correction to be added to the sum of the proton and neutron moments in agreement with an older estimate of Margenau.[4] It will be seen that the absolute value of this correction can conceivably be smaller than 0.006 and that a better understanding of interactions between nuclear particles is needed before one can claim that the correction is greater in absolute value than 0.001. The possibility remains open, therefore, that the moments are additive to within about 0.001 of a magneton. The results of Roberts and Arnold[1] when considered in the light of the astounding checks with the measurements of Rabi[5] can thus be reconciled with a simple picture of vector addition of moments of the proton and neutron. The reasoning which leads to this conclusion will now be sketched.

1. The proton moment will be supposed to be partly owing to an intrinsic moment of the type introduced by Pauli[6] and referred to as the "Pauli part." Besides one magneton will be attributed to the Dirac current. This contribution will be called the "Dirac part." The neutron moment will be considered as owing entirely to the Pauli part, as by Caldirola. For the proton, however, other interactions than the four vector type are considered.

2. The relativistic correction factor to the Pauli part is

$$\int [|\psi_3|^2+|\psi_2|^2-|\psi_1|^2]d\tau = 1-2\int|\psi_1|^2 d\tau = 1-\frac{\langle T_z\rangle}{Mc^2}, \quad (1)$$

where the spin is oriented along the z axis and $\langle T_z\rangle$ is the mean value of the part of the kinetic energy due to motion along the z axis. The ψ_μ are the four components of Dirac's wave function with Dirac's original choice of matrices. Since the whole correction in Eq. (1) is owing to ψ_1, it suffices to have a first approximation to ψ_1 in order to ascertain the correction factor to the order v^2/c^2 where v is the velocity of the particle. For spherically symmetric orbital motions (1) becomes

$$1-\langle T\rangle/3Mc^2. \quad (1')$$

The transformation properties of the field which binds the particles do not enter. The magnetic field at the neutron due to the magnetic moment of the proton is neglected in (1) and (1′). This effect is small for the mean distance between proton and neutron and amounts then to a correction of the order $(e^2/r)/Mc^2$ of the momentum. For the mean r this effect is negligible. For very small r it is large. The effect for very small r is neglected here since it involves questions of existence of solutions to the equations. Otherwise one makes the approximation

$$\psi_1 = -\frac{1}{P_0+Mc}\frac{\hbar}{i}\frac{\partial}{\partial z}\psi_1 \cong -\frac{1}{2Mc}\frac{\hbar}{i}\frac{\partial}{\partial z}\psi_1. \quad (2)$$

This approximation is harmless because P_0 differs from its non-relativistic value only in terms of the order v^2/c^2 of itself.

The signs of the correction term are given inconsistently by Caldirola in successive formulas.

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[1] Wayne R. Arnold and Arthur Roberts, Phys. Rev. **70**, 766 (1946) (LE).
[2] W. Rarita and J. Schwinger, Phys. Rev. **59**, 436 (1941).
[3] P. Caldirola, Phys. Rev. **69**, 567 (1946).
[4] H. Margenau, Phys. Rev. **57**, 383 (1940); G. Breit, Nature, **122**, 649 (1928).
[5] J. M. B. Kellogg, I. I. Rabi, N. F. Ramsey, and J. R. Zacharias, Phys. Rev. **56**, 728 (1939).
[6] W. Pauli, *Handbuch der Physik* (Verlagsbuchhandlung, Julius Springer, 1933), Vol. 24/1, p. 221.

His arithmetic shows that he used a plus rather than a minus sign before the correction term in (1′). In the application to the deuteron this makes little difference because the Pauli parts of the moments of proton and neutron are nearly equal and opposite. According to (1′) the correction to the sum due to the Pauli parts is in nuclear Bohr magnetons

$$-\tfrac{1}{3}(1.79-1.93)\langle T/Mc^2\rangle=+0.05\langle T/Mc^2\rangle.$$

Even for the high value $\langle T/Mc^2\rangle\sim 0.012$ used by Caldirola this correction is only 0.0006.

In Eqs. (1), (1′) the kinetic energy that enters is that of the particles separately and not that of their relative motion. The latter is twice the former.

One can verify (1) by (1′) by a consideration involving no quantum theory. A magnetic doublet of unit length directed at angle θ with respect to line of flight suffers a Lorentz contraction changing its projection on the line of flight by $-[1-(1-v^2/c^2)^{\frac{1}{2}}]\cos\theta$ which gives a change $\cong -(v^2/2c^2)\cos^2\theta$ along the doublet's axis. Multiplying numerator and denominator by M one obtains Eq. (1) and averaging over θ one finds Eq. (1′). The quantity T is again seen to be the kinetic energy of an individual particle. Partly for this reason the number 0.012 is too high for $\langle T/Mc^2\rangle$.

3. The relativistic corrections to the Dirac part of the moment depend on transformation properties of the binding energy. The expression for this part of the moment is a sum of contributions containing ψ_1, ψ_2 and their complex conjugates as factors. It is necessary, therefore, to know ψ_1, ψ_2 in the second rather than only in the first approximation. The correction factor

$$1-(2/3)\langle T/Mc^2\rangle \tag{3}$$

has been obtained by Margenau who extended previous calculations[4] to non-inverse square fields of force. This factor corresponds to s states in central fields on the assumption of an interaction of the four vector type. Its difference from unity can be broken down into parts as follows: (a) $-\langle T\rangle/2Mc^2$ due to the effect of ψ_1, ψ_2 on normalization, (b) the same amount as in (a) is contributed by the first correction term to $1/(E-V+Mc^2)$ where V is the potential energy, (c) a term in rdV/dr contributes $\langle T\rangle/3Mc^2$.

For a particle moving in a central scalar[7] field U one can make a similar calculation. The sign of the function U will be chosen so that it will replace V in the non-relativistic approximation. The combination Mc^2+U occurs as a unit everywhere. A breakdown of the difference of the correction factor from unity appears as a consequence as follows: (a) unchanged from previous case, (b) a factor $2Mc^2/(E+U+Mc^2)\cong 1+(T-2W)/2Mc^2$ contributes $\langle T-2W\rangle/2Mc^2$; here W is $E-Mc^2$, (c) in place of rdV/dr there occurs $-rdU/dr$; the sign of this term is reversed in comparison with the four-vector equation. The factor becomes

$$1-\langle W+(T/3)\rangle/Mc^2. \tag{4}$$

Here there is partial cancellation of W and $T/3$. For $W=-2.17$ Mev and $\langle T/Mc^2\rangle=0.012$ one obtains $0.0023-0.0040=-0.0017$ for the correction term in Eq. (4). When combined with the correction to the Pauli part this gives $-0.0017+0.0006=-0.0011$ for the correction to the sum of the moments and is on the limit of experimental error.

4. The value which must be used for W in Eq. (4) is subject to question. A central field was assumed, contrary to reality. It would be better to use the approximately relativistic equations[8] to order v^2/c^2. The choice of available equations is considerable, however. Instead one can use a model in which the proton and neutron move in the same central field and do not interact with each other. For this W is $-2.17/2=-1.08$ Mev and $F(r)$ = radial function of relative motion times r can be correlated with the corresponding quantity F_m for the model by requiring agreement of

$$\frac{d^2F}{dr^2}+\frac{M}{\hbar^2}[W-V(r)]F=0,$$
$$\frac{d^2F_m}{dr_m{}^2}+\frac{2M}{\hbar^2}\left[\frac{W}{2}-\frac{1}{2}V_m(r_m)\right]F_m=0 \tag{5}$$

which give the same characteristic values for W provided

$$V_m(r)=V(r),\quad F_m(r)=F(r). \tag{5'}$$

[7] See reference 6 and W. H. Furry, Phys. Rev. **50**, 784 (1936).

[8] G. Breit, Phys. Rev. **51**, 248 (1937); **53**, 153 (1938).

The model attributes to the proton one-half the kinetic and potential energies of the relative motion of the two particles within the deuteron. The scalar field gives, including the Pauli part,

$$\Delta(\mu_p+\mu_n)=\langle -W_p-0.285T_p\rangle/Mc^2 \qquad (6)$$

where p stands for "proton" and μ is the magnetic moment. For a "square well" the kinetic energy of relative motion is

$$T_p+T_n=2T_p=\alpha a(D+W)/(1+\alpha a) \qquad (7)$$

with

$$\alpha=(-MW/h^2)^{-\frac{1}{2}}, \quad a=\text{radius of well.} \qquad (7')$$

For $a=e^2/mc^2$ and $D=21$ Mev one finds, from Eq. (7), $2T_p=14.4mc^2$ corresponding to $T_p=0.0039Mc^2$, and Eq. (6) gives $\Delta(\mu_p+\mu_n)=0.0000$. The four-vector field gives

$$\Delta(\mu_p+\mu_n)=(-0.67+0.05)T_p/Mc^2 = -0.62\times 0.0039=-0.0024.$$

This is also very small.

The model is not believed in as a reality. It has properties in common, however, with Eq. (18.1) of the first paper[8] and gives[7] the inverted fine structure of nuclear levels in agreement with experiment.

5. For a "square well" interaction and range $a=e^2/mc^2$ the proton and neutron spend 45 percent of the time within $r<a$. The additivity of nuclear magnetic moments indicates, therefore, either the retention of individuality by the proton and neutron within the range of force, or a mistaken idea regarding the range of force or a compensation of changes in the moments.

6. It is not intended to say that the relativistic corrections are negligible, but it is believed that the variations in these corrections due to the sensitivity of the Dirac part of the moments to assumptions regarding the interaction between particles have been shown to be so large as to make estimates of the corrections uncertain to within practically their whole magnitude.

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Note on the Calculation of Angular Distributions in Resonance Reactions

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This note is concerned with pointing out some labor saving devices in the calculation of transformation coefficients arising in the composition of angular momenta which are needed for the calculation of angular distributions of disintegration products arising in nuclear resonance reactions.

THE angular distribution of disintegration products in resonance reactions has been treated theoretically by several authors.[1] The present note is concerned with pointing out a few labor saving devices in the technique of making the necessary calculations of transformation coefficients arising in the representation of the coupling of angular momenta by means of wave functions. It is realized that these coefficients are written out for the general case in Wigner's book on the applications of Group Theory to Quantum Mechanics. The formulas applicable to the general case are rather lengthy, and it is often desirable to have some other way of checking the results or of obtaining them.

Resonance reactions involve transitions from the initial state of the colliding particles *via* one or more states of the compound nucleus to the final state of the disintegration products. The nature of the disintegration products, their yield, and angular distribution depend upon the proper-

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[1] E. Gerjuoy, Phys. Rev. **58**, 503 (1940); C. L. Critchfield and E. Teller, Phys. Rev. **60**, 10 (1941); E. Eisner, Phys. Rev. **65**, 85 (1944).