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The Mass of the Mesotron as Determined by Cosmic-Ray Measurements

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All quantitative determinations of the masses of individual cosmic-ray mesotrons have been based on cloud-chamber photographs involving measurements of magnetic curvature plus ionization density, range, or energy of knock-on electrons. The errors involved in the different methods are discussed as well as the possibility of reducing these errors by the choice of experimental conditions. The errors and the mass values themselves of 47 published measurements are reviewed in the light of the discussion. The great majority of the results are statistically reconcilable with a single mass; only a very small fraction of the mesotrons seem to have significantly different masses.

I. INTRODUCTION

IN the ten years that have elapsed since the discovery of the mesotron as a component of cosmic rays, many attempts have been made to determine its mass. Although the mass of the mesotron was the property which led to its discovery, and in spite of the many efforts to measure it, the mass is still not known accurately, nor is it known whether it actually has a unique value or a distribution of values. As it is likely that mesotrons will be produced in high voltage accelerators, it seems worth while to review the cosmic-ray measurements of the mesotron mass to serve as a basis for comparison with the accelerator results. Such a review was given by the author at the September, 1946 Physical Society meeting, and the present article includes and extends the material presented at that time. The accuracy which has been, and can be, obtained in mesotron mass determinations will first be discussed, then all of the published measurements will be reviewed in the light of the discussion.

II. THE ACCURACY OF MASS DETERMINATIONS

All measurements of the mass of individual cosmic-ray mesotrons (as distinguished from values of the average mass resulting from measurements of such quantities as the number of electron secondaries, etc.) have been made by determining the momentum of the particle from its magnetic curvature in a cloud chamber, together with some second measurable quantity. This second quantity, which gives essentially the velocity of the particle, has been the ionization density, the range, the rate of loss of momentum or the kinetic energy transferred to an atomic electron. The mass follows directly from the momentum and velocity, but in practice the velocity is not determined explicitly because of its somewhat complicated dependence on the actual measured quantities. It is possible to express directly the dependence of the mass on the measured quantities in simple graphical form and

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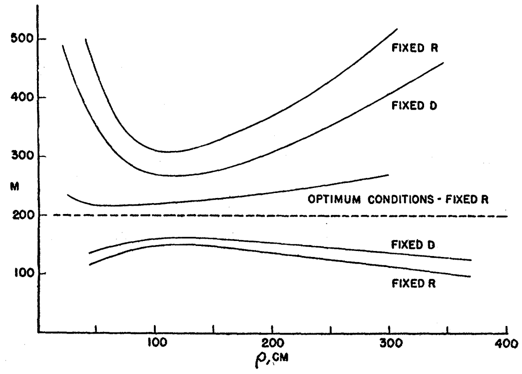

FIG. 1. Variation in mesotron mass (M) as a function of radius of curvature for a true mass of 200 electron masses. Curves labeled "fixed R" and "fixed D" refer to measurements based on range and on ionization density for the chamber described in reference 1. The curve "optimum conditions" is for the same chamber under optimum experimental conditions.

the manner in which mass values depend on the experimental data can thus be clearly shown. A nomograph for this purpose has been given recently as Fig. 1 of an article by Hughes[1] (hereafter referred to as DJH). The nomograph is applicable to cases involving magnetic curvature, change of curvature, density of ionization, and range. Accurate mass measurements based on energy of knock-on electrons are extremely unlikely[2] and the method will not be discussed in this section although several masses based on knock-on electrons will be included in section III.

It was shown in DJH for a particular experimental set-up that the accuracy of mass determination, whether based on curvature and range or curvature and ionization density, is greatest for a particular momentum (curvature) value. For smaller momenta the error caused by spurious curvature due to multiple scattering in the chamber gas lessens the accuracy; for larger momenta the errors in curvature due to gas movements, finite track width, etc., again lessen the accuracy. The calculation of the accuracy as a function of momentum for any chamber can be made by using the formula of Williams[3] for ρ_s (the average radius of curvature caused by multiple scattering), assuming a value for the error in measuring the curvature of a track arising from chamber distortions, and determining the resultant error in mass from the nomograph. The simplified form of Williams' formula applicable to a particular chamber[4] given as Eq. (1) of DJH can be used if the differences in chamber gas and magnetic field are taken into account.

The accuracy of mass determination with the chamber described in DJH was only about 20 percent at best and even this accuracy would be obtained at only a single curvature value ($\rho = 125$ cm) and under the assumption that no error is incurred in the measurement of the ionization density. The operating conditions of the above chamber can be improved with respect to mass determination, and the highest accuracy to be expected is shown in Fig. 1 as the curve marked "optimum" conditions. The other curves in the figure are from Fig. 2 of DJH and refer to the average spread in measured values based on magnetic curvature plus density of ionization ("fixed D") or range ("fixed R"). The true mass of the particle is assumed to be 200 times that of the electron. The "optimum" conditions refer to the same magnetic field of 1165 gauss, and in addition substitution of helium for argon, reduction of chamber distortion to such a point that the magnetic displacement of the center of a track can be fixed to 0.02 cm, and measurement of range in the chamber with negligible error. The minimum error in M under such conditions is 8 percent and is attainable only at a radius of curvature of 65 cm.

It is instructive to consider the effect of supplying a higher magnetic field in order to increase

[1] D. J. Hughes, Phys. Rev. **69**, 371 (1946).

[2] R. Richard-Foy, Cahiers de Physique **2**, 65 (1942); S. Gorodetzky, Ann. de Physique **19**, 5 (1944).

[3] E. J. Williams, Phys. Rev. **58**, 292 (1940).

[4] H. A. Bethe, Phys. Rev. **69**, 689A (1946), has also calculated the effect of scattering on curvature measurements for this same chamber, mainly for very slow particles. *Note added in proof:* Bethe's results have since been published, Phys. Rev. **70**, 821 (1946), and his conclusions are essentially the same as those of the present paper, although he does not consider the errors which become important at high momenta. Bethe calculates that the chance of a mesotron of *assumed* mass 200 giving the *experimental* value of 30±20 reported in DJH, Fig. 5, is 7 percent and hence compatible with the "normal" mesotron mass. The calculations of section III of the present paper, however, give 0.5 percent for the same probability (see left hand point of Fig. 4), a much less compatible value. Bethe's result (see page 825 of his article) is higher than that of the present paper because of (a) his omission of the stopping power of the alcohol, (b) use of the stopping power for 6 Mev alphas whereas the mesotron velocity is actually higher, and (c) the difference (about 20 percent) between Bethe's and Williams' formulas for ρ_s. If Bethe's calculations are corrected for (a) and (b), then the only difference between his results and those given in DJH and in the present article is that caused by (c).

the accuracy further. The particular momentum at which maximum accuracy is obtained is independent of magnetic field strength, but the error in mass is inversely proportional to field strength. Hence, a fourfold increase of H for the above optimum conditions will result in a lowering of the error in mass to two percent, but again only at the same momentum which now means a radius of curvature of only 16 cm. It is obviously very difficult to work at the optimum conditions under such conditions, for particles with such small radii of curvature would tend to be deflected away from the cloud chamber by the magnetic field. On the other hand, the higher magnetic field will give about the same accuracy (8 percent) as the optimum for the lower field, but for particles of four times the momentum. The higher momentum particles will, of course, have the same radius of curvature as the optimum value for the lower field and hence will have the same probability of reaching the chamber. The advantage of the higher field in such a case is that equal accuracy is attained for particles of higher momentum which are much more numerous in the cosmic-ray momentum spectrum. The disadvantage of the higher field is that the higher momentum particles have much greater ranges which are correspondingly harder to measure because they necessitate more absorbers in the chamber.

Thus by increasing the magnetic field it becomes possible to equal the accuracy of the optimum curvature although using correspondingly higher momentum particles of the same curvature in the chamber. However, as particles of much higher momentum are used and the particle velocity becomes relativistic, the accuracy decreases because the error in M relative to a given error in $H\rho$ increases (assuming R measured accurately). In this way the error in M, which is 1.5 times the error in $H\rho$ for low velocities and for a mass of 200 (see nomograph), becomes 2 times at an $H\rho$ of about 5×10^5. The relationships between M, R, and $H\rho$ are shown in Fig. 2 from which the error in M due to a certain error in $H\rho$ can be calculated. Thus the error shown for $M=200$ at an R of 50 g/cm^2 Pb represents a 10 percent error in $H\rho$ which can be seen to correspond to a 20 percent error in M.

In conclusion it is seen that the highest accuracy is obtained for a gas of low atomic weight, for a chamber of small distortion, for a high magnetic field and at a particular curvature value. As the magnetic field is increased the optimum curvature also increases and it becomes impossible for tracks of such curvature to enter the chamber. It is worth pointing out that the foregoing limitation does not apply if the particles are produced *in* the chamber, which seems to take place for cosmic rays at high altitudes,[5,6] and which can be arranged for particles produced by the high voltage accelerators.[7] The possibility of producing mesotrons inside cloud chambers and hence using high magnetic fields and the optimum curvature seems to be a very promising feature of the mass measurement of artificially produced mesotrons.

III. COSMIC-RAY MEASUREMENTS TO DATE

Up to December 1, 1946 there have been published about 50 measurements made in such a way that the determinations can be considered quantitative measurements of individual mesotron masses. The expected errors of each of these measurements have been recalculated following the principles of section II. The mass value itself has also been recalculated in each case from the original data, using the nomograph of DJH, Fig. 1, to eliminate differences in masses caused by differences in the theoretical formulas used by different experimenters. The main change results

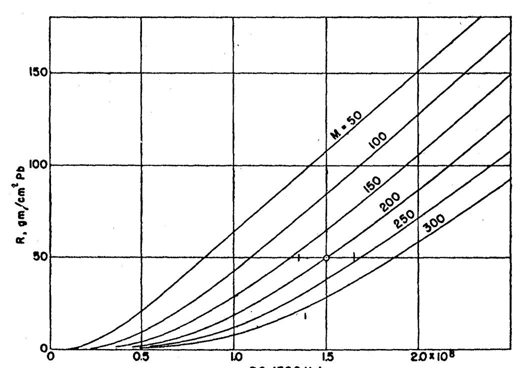

FIG. 2. The relationship between momentum and range for different values of M. The 10 percent error shown in $H\rho$ for a mass of 200 would lead to a 20 percent error in M.

[5] G. Herzog, Phys. Rev. **59**, 117 (1941).
[6] D. J. Hughes, Phys. Rev. **60**, 414 (1941).
[7] G. C. Baldwin and G. S. Klaiber, Phys. Rev. **70**, 259 (1946).

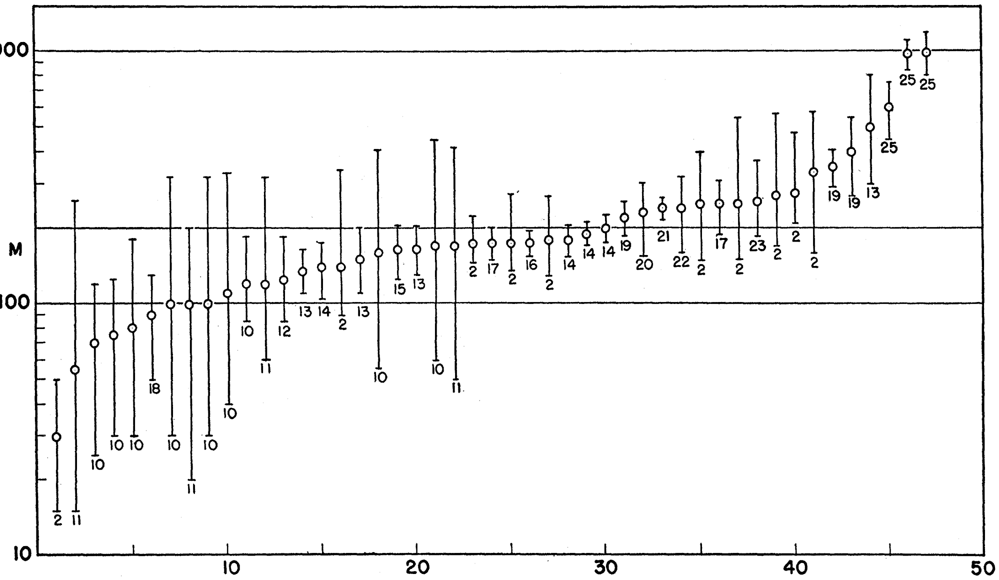

FIG. 3. The experimental determinations of the mass of the mesotron and their expected mean errors. The results are arranged in order of increasing mass and the number identifying each gives the reference for the original publication.

from the use of the theoretical[8] formula for density of ionization, D, as a function of β (approximately $\beta^{-1.8}$ in the usual range) whereas some authors have used experimental values for the exponent of β, such as the value $\beta^{-1.4}$ found by Williams.[9] In many cases the calculated error in M is larger than the error estimated in the original publication because of the omission of some contributory factor by the author. Because pertinent experimental details were sometimes omitted it is undoubtedly true that some of the final estimated errors are themselves in error but it is hoped consistently neither on the pessimistic nor the optimistic side.

The results of the calculations are shown in Fig. 3 where the individual mass determinations are arranged in terms of increasing mass value. The identification number appearing with each mass gives the reference for the original publication.[10–25] Because the formulas[3] for the error in curvature measurement are given in terms of the mean error, and because this error usually predominates, the final errors shown by the vertical lines of Fig. 3 are represented as mean errors also. The mass determinations whose errors are such that only an upper or lower limit for the mass could be obtained have been omitted as have those which give only an average mass value based on some average property of mesotrons (as the average number of knock-on electrons,[26] or the average density in photographic emulsions[27]). Otherwise it is believed that the 47 values of

[8] J. A. Wheeler and R. Ladenburg, Phys. Rev. **60**, 754 (1941) (Appendix).
[9] E. J. Williams, Proc. Roy. Soc. **A126**, 289 (1930); **A172**, 194 (1939).
[10] J. Ruling and R. Steinmauer, Experientia **2**, 108 (1946).
[11] H. Maier-Leibnitz, Zeits. f. Physik **112**, 569 (1939).
[12] R. L. Sen Gupta, Nature **154**, 706 (1944).
[13] E. J. Williams and E. Pickup, Nature **141**, 684 (1938).
[14] C. E. Nielsen and W. M. Powell, Phys. Rev. **63**, 384 (1943).
[15] J. C. Street and E. C. Stevenson, Phys. Rev. **52**, 1003 (1937).
[16] Y. Nishina, M. Takeuchi, and T. Ichimiya, Phys. Rev. **55**, 585 (1939).
[17] J. G. Wilson, Proc. Roy. Soc. **A172**, 521 (1939).
[18] T. H. Johnson and R. P. Shutt, Phys. Rev. **61**, 380 (1942).
[19] S. H. Neddermeyer and C. D. Anderson, Rev. Mod. Phys. **11**, 191 (1939) (Figs. 15, 16).
[20] D. R. Corson and R. B. Brode, Phys. Rev. **53**, 776 (1938).
[21] L. S. Leprince-Ringuet, S. Gorodetzky, E. Nageotte, and R. Richard-Foy, Phys. Rev. **59**, 460 (1941).
[22] P. Ehrenfest, Jr., Comptes rendus Paris **206**, 428 (1938).
[23] E. J. Williams and G. E. Roberts, Nature **145**, 102 (1940).
[24] S. H. Neddermeyer and C. D. Anderson, Phys. Rev. **50**, 263 (1936).
[25] L. Leprince-Ringuet, M. Lheritier, and R. Richard-Foy, Comptes rendus Paris **219**, 618 (1944); **221**, 465 (1945).

Fig. 3 represent all the quantitative mass determinations of the cosmic-ray mesotron that have been published.[28]

The masses of Fig. 3 cover a wide range, from 30 to 1000 electron masses, but because of the large experimental errors associated with many of them, the number of determinations lying within experimental error of a single mass of 200 is 28, or 60 percent of the total. The number of results agreeing with a single mass would have been much smaller had the errors assigned by the authors themselves been used, because of the increase in many of the calculated errors caused by scattering in the chamber gas. The errors also are much larger than the attainable values discussed in section II because in many cases the experimental conditions were not such as to give the highest accuracy. Nine of the mass values differ from 200 by more than twice the mean error and 3 differ by more than three times (the numbers expected from a Gaussian distribution being 5.2 and 0.8, respectively). Thus, although the majority of the results indicate a single mass, or rather a range in mass smaller than the rather large experimental errors, there is evidence for a few masses of greatly different values.

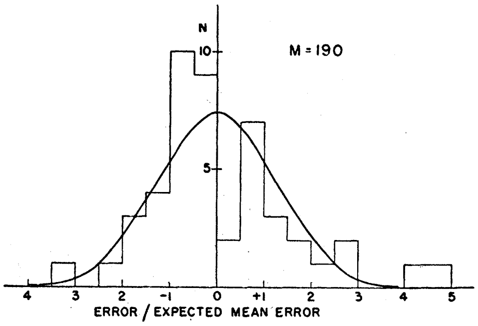

FIG. 4. The errors of the 47 determinations of Fig. 3 relative to the expected mean error of each measurement on the assumption that the true mass is 190. The smooth curve is the Gaussian distribution.

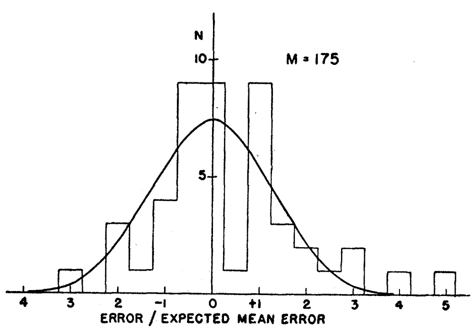

FIG. 5. The same results as in Fig. 4 for a mass of 175.

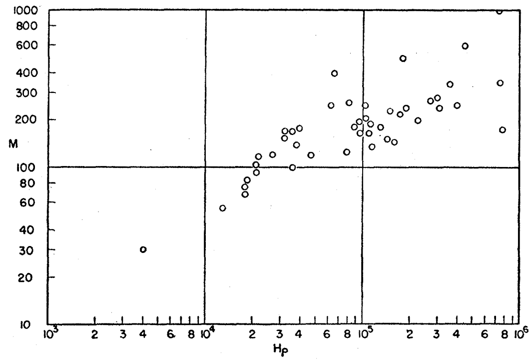

FIG. 6. The experimental mass values plotted against the momentum ($H\rho$) of the mesotron for each measurement.

The errors of Fig. 3 are of course calculated on the basis that the mass involved in each determination is actually the value resulting from the measurement. If it desired to check the possibility that only a single mass is actually involved in all the measurements, then the errors shown in Fig. 3 will be changed somewhat because they are a function of the assumed particle mass. In order to see if the measurements could be reconciled with a single mass, the error in the curvature measurement which would be necessary to give the observed results in each case was calculated under the assumption that the actual mass had a single value. The entire error was assumed to be that of curvature measurement because it is the predominant source of error practically without

[26] H. J. Bhabha, Proc. Roy. Soc. **A164**, 257 (1938).

[27] D. M. Bose and B. Choudhouri, Nature **148**, 259 (1941); **149**, 302 (1942).

[28] The recent measurements of W. B. Fretter and R. B. Brode, which were described at the September, 1946 Physical Society meeting but which have not been published as yet will furnish additional valuable results. *Note added in proof:* This work has since been published, Phys. Rev. **70**, 821 (1946), where 26 mass values are given based on measurement of range and $H\rho$. The errors of individual determinations (expected mean value) are about 18 percent on the average, and the measured masses, if plotted as in Figs. 4, 5 of the present paper, scatter about a single value ($M=202$) in a manner consistent with the expected error. The difference between the mass value of 202 and the 175 of Fig. 4 is not of great significance because of the systematic errors in mass determination (such as the uncertainty in the exact form of the theoretical energy range relation, etc.) which exist in addition to the statistical errors for both values. It should be noted that the mesotrons measured by Fretter all lie in the momentum band 5 to 9×10^5 (see Fig. 5 of present paper and attendant discussion).

exception. The expected mean error for each curvature measurement was also calculated for the particular experimental conditions, again assuming a single particle mass. Although it is true that the expected errors vary with experimental conditions, the results of different measurements can be compared if the errors are taken relative to the expected mean error in each case. The resulting distribution of errors for such a calculation is given in Fig. 4, where the true mass is assumed to be 190, and in Fig. 5 where the mass is taken to be 175. The blocks show the number of measurements which have the indicated errors (in units of the mean expected error, where a positive error signifies a high mass value) under the assumption of a true mass of 190 or 175. The solid curves are the expected Gaussian distributions of errors. The rather close agreement between the observed distribution of "errors" and that expected means that if all the different measurements are considered as repeated trials with varying experimental conditions the distribution of results is about what would be expected. The agreement with mass 175 seems better than with mass 190, but in this connection it must be remembered that the nomograph used to calculate the mass values is based on the theoretical change of ionization with velocity. Experiments[9] seem to show a different change of ionization with velocity and if true would cause the measured masses to increase.

The curves of Figs. 4 and 5 show a few measurements which lie outside the expected distribution but the number is so small that it must be concluded that if there are any mesotrons of mass greatly different from approximately 200 they constitute at most only a few percent of all the mesotrons. This conclusion does not support the suggestion made by Bose and Choudhouri[29] that the average mesotron mass increases with momentum. Actually, if the measured masses are plotted as a function of the particle momentum (Fig. 6), it appears that there is a relationship between mass and momentum. However, the fact that Fig. 6 shows low mass particles only at low momentum can be consistent with either (1), a distribution of masses independent of momentum, because low mass particles would be detectable only at low momentum, or (2) a single mass of about 200, because heavily ionizing tracks ending in the chamber might be neglected unless they show marked curvature which, if owing to scattering, would give erroneously low mass values.

The spread in the observed mass values is much larger than the smallest attainable errors for optimum conditions discussed in section II, hence measurements made under the correct conditions with present apparatus can definitely show whether the spread is real. It is also important, as was pointed out by Blackett at the September, 1946 Physical Society meeting, to measure mesotron masses for as great a range of momentum and altitude as is feasible, because of the possibility that the mesotron mass might vary with these factors.

[29] D. M. Bose and B. Choudhouri, Ind. J. Phys. **18**, 285 (1944).