coincidence rate was always kept to less than 0.5 percent of the true coincidence rate. At 12° a geometrical correction of 3 percent was also applied because at that angle not all the scattered and recoil protons could enter both counters.

The results are given in Fig. 1 in which the cross section per unit solid angle is plotted as a function of the scattering angle; both coordinates are in the center of mass system. The open circled points are adjusted to 4.90×10^{-26} at 90° (center of mass). The other points were taken later with a different scattering foil and have been adjusted to the open circled points in the neighborhood of 50° (center of mass). The vertical lines represent the mean square error as determined experimentally from the consistency of the data.

The solid curves were calculated by L. B. Eisenbud and L. L. Foldy on the assumption of a square well of depth 10.5 Mev and width (e^2/mc^2). The central curve is for S wave scattering only; the upper curve is for S plus P wave scattering with the potential for the P wave repulsive; and the bottom curve shows the P wave effects for an attractive interaction. The data are clearly inconsistent with any attractive P wave effects. They indicate, and are consistent with, the dotted curve which could correspond to a small admixture of repulsive P wave effects—perhaps one-third the full effect—or to pure S wave scattering but using a different potential well.

It is a pleasure to thank Professor E. O. Lawrence and Dr. J. G. Hamilton for their hospitality and help in making the facilities of the Crocker Laboratory at Berkeley available to me. I am also indebted to the crew of the sixty-inch cyclotron for their cheerful help and interest in the experiment.

* Now at Cornell University, Ithaca, New York.
[1] R. R. Wilson and E. C. Creutz, Phys. Rev. **71**, 339 (1947).
[2] Equivalent but more refined electronic circuits were borrowed from the Los Alamos Laboratory for use in the Berkeley experiments.

Range, Straggling, and Multiple Scattering of Fast Protons

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THE following is a short report on the details of calculations used in a paper[1] considering the radiological properties of fast protons, i.e., 10 to 200 Mev.

The rate of loss of energy per cm of path of charged particle is given by Bethe's expression[2] which can be written

$$-\frac{dW}{dx}=\frac{2\pi e^4z^2}{mc^2E_0W}NZ\ln\left(\frac{4mc^2W}{I}\right), \tag{1}$$

where W is the ratio of the kinetic energy of the particle to its rest energy E_0, N and Z are, respectively, the number of nuclei per cm^3 and the atomic number of the stopping material, which has an average excitation energy I; ez is the charge of the ion, and mc^2 is the rest energy of the electron. Bloch[3] has shown that I is proportional to Z for atoms heavier than air; the constant of proportionality has been determined[4] to be 11.5 ev. It can be observed that

$$\ln\left(\frac{4mc^2W}{11.5Z}\right)\approx2.2W^{0.2}\ln(3.6\times10^3/Z) \tag{2}$$

in the interval $0.01<W<0.2$. Hence the range R is

$$R=\int_0^W\frac{dW}{|dW/dx|}=\frac{10^{24}E_0W^{1.8}}{NZz^2\ln(3.6\times10^3/Z)} \tag{3}$$

where E_0 is now in Mev.

Because of the unknown effects of the L and M electrons, Eq. (3) is not expected to hold accurately for heavy elements with $Z>50$. The approximation Eq. (2) is not too good for light elements with $Z<5$; in any case a value $Z'=I/11.5$ should be used in Eq. (3) for such light elements. For protons in air, Eq. (3) agrees to within a few percent with the Cornell range curve[2] down to a few Mev, and with Smith's recent range calculations[5] up to 200 Mev. A more convenient formula for the range of protons in air in meters is $R_m=(E/9.3)^{1.8}$ where E is in Mev.[6]

Livingston and Bethe[2] give an expression for straggling which for high particle energy reduces approximately to

$$\delta R^2=(4\pi e^4z^2NZ/E_0^2)\int_0^W dW/|dW/dx|^3, \tag{4}$$

where δR^2 is the mean square fluctuation of the range. Substituting Eqs. (1) and (2) into (4), integrating then eliminating W, using Eq. (3), and neglecting the variation of the one-half power of the log term in Z times $Z^{0.055}$, we get

$$\delta R/R\approx4.5E_0^{-\frac{1}{2}}(NZz^2R/E_0)^{-0.055} \tag{5}$$

$$\approx0.24W^{-0.1}E_0^{-\frac{1}{2}}, \tag{6}$$

where $\delta R=(\delta R^2)^{\frac{1}{2}}$. This is in excellent agreement with the values calculated for air by Livingston and Bethe for the highest energies that they consider, which are about 15 Mev.

Williams[7] has given a formula for multiple scattering which can be written

$$(\theta^2)_{\text{Av}}=\frac{\pi}{E_0^2}\left(\frac{Zze^2}{W}\right)^2Nx\ln\left(\frac{Z^{4/3}Nxh^2}{4m^2c^2W}\right) \tag{7}$$

where $(\theta^2)_{\text{Av}}$ is the mean square angle of deflection a particle receives in passing through a thickness x of stopping material. One can calculate the root mean square angle of deflection of a particle of range R which has passed through a thickness x of stopping material by putting Eq. (6) in the differential form, eliminating W using Eq. (3), then integrating, ignoring the variation of the log terms, and taking the square root to get

$$\theta_{rms}=[(\theta^2)_{\text{Av}}]^{\frac{1}{2}}\approx18.5\left(\frac{Z}{E_0}\right)^{\frac{1}{2}}\left(\frac{NZRz^2}{E_0}\right)^{-0.055}\times\left[\left(1-\frac{x}{R}\right)^{-0.11}-1\right]^{\frac{1}{2}}. \tag{8}$$

The log terms have been evaluated for 50-Mev protons in air but the errors so introduced are not greater than 5 percent for protons varying from 10 to 200 Mev. Now make the rough approximation that the root mean square displacement y_{rms} perpendicular to the initial direction follows θ_{ms}, i.e.,

$$y_{rms}=\int_0^R \theta_{rms}dx.$$

On substituting Eq. (7) and graphically evaluating the definite integral

$$\int_0^1\left[\left(1-\frac{x}{R}\right)^{-0.11}-1\right]\frac{dx}{R}=0.305,$$

we get

$$y_{rms}/R\approx 5.7(Z/E_0)^{\frac{1}{2}}(NZz^2R/E_0)^{-0.055} \tag{9}$$

or

$$\approx 0.3(Z/E_0)^{\frac{1}{2}}W^{-0.1}. \tag{10}$$

A photograph of the 40-Mev α-particle beam from the 60″ Crocker cyclotron at the Radiation Laboratory in Berkeley shows a total spread of about 4.5 cm at the end of the 115-cm range. This is in fair agreement with the calculated *spread* ($2y_{rms}$) of 4.8 cm given by Eq. (10).

The above formulae show that the range of 150 Mev protons in tissue is about 16 cm, that the range straggling ($2\delta R$) is 0.3 cm, and the transverse spread ($2y_{rms}$) is 0.6 cm. It is evident that such protons have radiological applications[1] since it is possible to treat a volume as small as one cm^3 anywhere within the body, and give to that volume several times the dose of any of the neighboring tissue. Thus 10^9 protons per cm^2 will produce more than 1000 r.e.d.[8] in the last half cm of range, but the skin dose will be less than 100 r.e.d.

One cannot emphasize too strongly the danger of working near the modern high energy accelerators. Something like one billion protons per cm^2 ($\sim 10^{-10}$ amp./cm^2 for one second) could have lethal effects. When one remembers that the range of 150-Mev protons in air is 150 meters, it is apparent that even a small scattering by air could make any point within a moderately large room lethally dangerous if a current of the order of microamperes is free in the air.

[1] R. R. Wilson, Radiology **47**, 487 (1946).
[2] M. S. Livingston and H. A. Bethe, Rev. Mod. Phys. **9**, 263 (1937).
[3] F. Bloch, Zeits. f. Physik **81**, 363 (1933).
[4] R. R. Wilson, Phys. Rev. **60**, 749 (1941).
[5] J. H. Smith, Phys. Rev. **71**, 32 (1947).
[6] W. M. Brobeck first noticed this relationship.
[7] E. J. Williams, Proc. Roy. Soc. **169**, 531 (1939).
[8] Roentgen equivalent dose, 83 ergs absorbed per gram of tissue.