TABLE I. Microwave absorption frequencies in ammonia.

J	K	$N^{14}H_3$	$N^{15}H_3$
1	1	23,694.49 mc/s	22,624.96 mc/s
2	2	23,722.63	22,649.85
2	1	23,098.79	22,044.28
3	3	23,870.13	22,789.41
3	2	22,834.17	21,783.98
3	1	22,234.53	21,202.30
4	4	24,139.41	23,046.10
4	3	22,688.29	21,637.91
4	2	21,703.36	20,682.87
4	1	21,134.29	—
5	5	24,532.98	23,421.99
5	4	22,653.00	21,597.86
5	3	21,285.27	20,272.04
5	2	20,371.46	—
6	6	25,056.02	23,922.32
6	5	22,732.43	21,667.93
6	4	20,994.61	—
6	3	19,757.57	—
7	7	25,715.17	24,553.42
7	6	22,924.94	21,846.41
7	5	20,804.83	—
8	8	—	25,323.51
8	7	23,232.24	22,134.89
8	6	20,719.21	—
9	8	23,657.48	22,536.26
9	7	20,735.44	—
10	9	24,205.29	23,054.97
10	8	20,852.51	—
11	10	24,881.90	—
11	9	21,070.70	—
12	11	25,695.23	—

in a million. The standard frequency transmissions of the National Bureau of Standards Station, WWV, were used as the basis of these measurements in order to obtain the necessary precision. These absorption lines should now provide excellent secondary frequency standards in the region mentioned.

Harmonics, from a 240 mc/s crystal controlled oscillator, falling in the above region were used as frequency markers. Interpolation between the markers was done with a calibrated communication receiver. It is estimated that the values in Table I are accurate to ±0.02 mc/s.

[1] B. Bleaney and R. P. Penrose, Nature **157**, 339 (1946).
[2] W. E. Good, Phys. Rev. **70**, 213–218 (1946).
[3] C. H. Townes, Phys. Rev. **70**, 665 (1946).
[4] D. K. Coles and W. E. Good, Phys. Rev. **70**, 979 (1946).
[5] Dailey, Kyhl, Strandberg, Van Vleck, and Wilson, Phys. Rev. **70**, 984 (1946).

Proton-Proton Scattering at 10 Mev

ROBERT R. WILSON*
Harvard University, Cambridge, Massachusetts
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THE 10-Mev protons available from the sixty-inch cyclotron at the Radiation Laboratory of the University of California have been used for studying the scattering of protons by protons. Coincidences between the scattered and recoil protons were measured as a function of the angle of the scattered protons utilizing the same method and equipment which had been developed at Princeton University[1] for use with 8-Mev protons.

A scattering foil of Nylon $(C_{12}H_{22}N_2O)_x$ about 2×10^{-4} cm thick was placed at the center of the scattering chamber such that the normal to the plane of the scattering foil made an angle of 30° with the direction of the incident proton beam which was 2.0 mm in diameter. The scattered protons were counted with a proportional counter, the "defining counter," which had a circular aperture of 1.65 mm diameter 7.8 cm from the center of the scattering foil. Another proportional counter, the "monitor counter," was mounted such that it received the recoil protons at 90° with respect to the defining counter. It had an oval aperture $\frac{3}{16}''$ wide and $\frac{3}{8}''$ high and was only 3.7 cm from the scattering foil. Coincidences between the defining counter and the monitor counter were registered. The solid angle of the monitor counter was large to insure that the recoils of all protons entering the defining counter would also enter the monitor counter.

The proton integrator consisted of a 2.10 mf condenser which was connected between the Faraday collector cup and ground. The charge collected on the condenser during a ten-minute run was measured at the end of each run using a ballistic galvanometer.[2] The system was not accurately calibrated since all the measurements were relative. It was not convenient to vary the magnetic field or frequency of the sixty-inch cyclotron, which customarily accelerates deuterons of α-particles, so molecular hydrogen ions of unit charge were accelerated to 20 Mev, thus giving the equivalent of 10-Mev protons. This energy was determined by accurate range measurements made on the scattered protons. Thus the protons scattered at 30.5° had a mean range of 0.097 mg/cm² in aluminum, corresponding to an energy of 7.35 Mev or 9.9[4] Mev for the incident protons.

At each angle of scattering a counting plateau was established by measuring the number of coincidences per incident proton as a function of the discriminator biases. Good plateaus were found for all angles except 12° where it was necessary to extrapolate to zero bias, thereby introducing a correction of 9 percent. The accidental

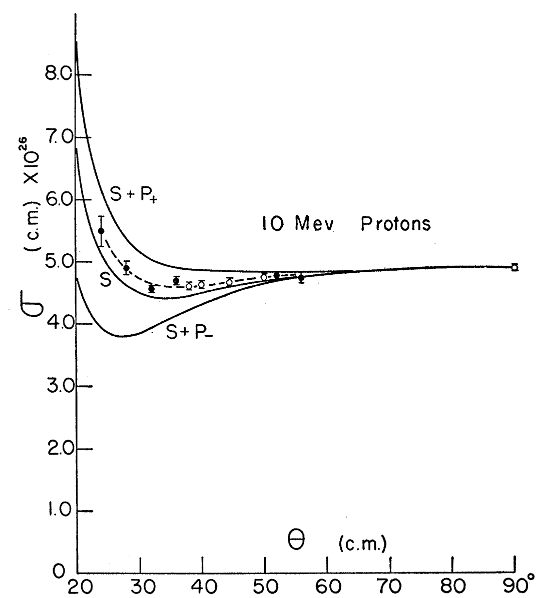

FIG. 1. The cross section per unit solid angle in the center of mass system as a function of the scattering angle in the center of mass system. All the data are relative.

coincidence rate was always kept to less than 0.5 percent of the true coincidence rate. At 12° a geometrical correction of 3 percent was also applied because at that angle not all the scattered and recoil protons could enter both counters.

The results are given in Fig. 1 in which the cross section per unit solid angle is plotted as a function of the scattering angle; both coordinates are in the center of mass system. The open circled points are adjusted to 4.90×10^{-26} at 90° (center of mass). The other points were taken later with a different scattering foil and have been adjusted to the open circled points in the neighborhood of 50° (center of mass). The vertical lines represent the mean square error as determined experimentally from the consistency of the data.

The solid curves were calculated by L. B. Eisenbud and L. L. Foldy on the assumption of a square well of depth 10.5 Mev and width (e^2/mc^2). The central curve is for S wave scattering only; the upper curve is for S plus P wave scattering with the potential for the P wave repulsive; and the bottom curve shows the P wave effects for an attractive interaction. The data are clearly inconsistent with any attractive P wave effects. They indicate, and are consistent with, the dotted curve which could correspond to a small admixture of repulsive P wave effects—perhaps one-third the full effect—or to pure S wave scattering but using a different potential well.

It is a pleasure to thank Professor E. O. Lawrence and Dr. J. G. Hamilton for their hospitality and help in making the facilities of the Crocker Laboratory at Berkeley available to me. I am also indebted to the crew of the sixty-inch cyclotron for their cheerful help and interest in the experiment.

* Now at Cornell University, Ithaca, New York.
[1] R. R. Wilson and E. C. Creutz, Phys. Rev. 71, 339 (1947).
[2] Equivalent but more refined electronic circuits were borrowed from the Los Alamos Laboratory for use in the Berkeley experiments.

Range, Straggling, and Multiple Scattering of Fast Protons

ROBERT R. WILSON
Harvard University, Cambridge, Massachusetts
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THE following is a short report on the details of calculations used in a paper[1] considering the radiological properties of fast protons, i.e., 10 to 200 Mev.

The rate of loss of energy per cm of path of charged particle is given by Bethe's expression[2] which can be written

$$-\frac{dW}{dx}=\frac{2\pi e^4z^2}{mc^2E_0W}NZ\ln\left(\frac{4mc^2W}{I}\right), \tag{1}$$

where W is the ratio of the kinetic energy of the particle to its rest energy E_0, N and Z are, respectively, the number of nuclei per cm^3 and the atomic number of the stopping material, which has an average excitation energy I; ez is the charge of the ion, and mc^2 is the rest energy of the electron. Bloch[3] has shown that I is proportional to Z for atoms heavier than air; the constant of proportionality has been determined[4] to be 11.5 ev. It can be observed that

$$\ln\left(\frac{4mc^2W}{11.5Z}\right)\approx 2.2W^{0.2}\ln(3.6\times 10^3/Z) \tag{2}$$

in the interval $0.01<W<0.2$. Hence the range R is

$$R=\int_0^W\frac{dW}{|dW/dx|}=\frac{10^{24}E_0W^{1.8}}{NZz^2\ln(3.6\times 10^3/Z)} \tag{3}$$

where E_0 is now in Mev.

Because of the unknown effects of the L and M electrons, Eq. (3) is not expected to hold accurately for heavy elements with $Z>50$. The approximation Eq. (2) is not too good for light elements with $Z<5$; in any case a value $Z'=I/11.5$ should be used in Eq. (3) for such light elements. For protons in air, Eq. (3) agrees to within a few percent with the Cornell range curve[2] down to a few Mev, and with Smith's recent range calculations[5] up to 200 Mev. A more convenient formula for the range of protons in air in meters is $R_m=(E/9.3)^{1.8}$ where E is in Mev.[6]

Livingston and Bethe[2] give an expression for straggling which for high particle energy reduces approximately to

$$\delta R^2=(4\pi e^4z^2NZ/E_0^2)\int_0^W dW/|dW/dx|^3, \tag{4}$$

where δR^2 is the mean square fluctuation of the range. Substituting Eqs. (1) and (2) into (4), integrating then eliminating W, using Eq. (3), and neglecting the variation of the one-half power of the log term in Z times $Z^{0.055}$, we get

$$\delta R/R\approx 4.5E_0^{-\frac{1}{2}}(NZz^2R/E_0)^{-0.055} \tag{5}$$

$$\approx 0.24W^{-0.1}E_0^{-\frac{1}{2}}, \tag{6}$$

where $\delta R=(\delta R^2)^{\frac{1}{2}}$. This is in excellent agreement with the values calculated for air by Livingston and Bethe for the highest energies that they consider, which are about 15 Mev.

Williams[7] has given a formula for multiple scattering which can be written

$$(\theta^2)_{\text{Av}}=\frac{\pi}{E_0^2}\left(\frac{Zze^2}{W}\right)^2Nx\ln\left(\frac{Z^{4/3}Nxh^2}{4m^2c^2W}\right) \tag{7}$$

where $(\theta^2)_{\text{Av}}$ is the mean square angle of deflection a particle receives in passing through a thickness x of stopping material. One can calculate the root mean square angle of deflection of a particle of range R which has passed through a thickness x of stopping material by putting Eq. (6) in the differential form, eliminating W using Eq. (3), then integrating, ignoring the variation of the log terms, and taking the square root to get

$$\theta_{rms}=[(\theta^2)_{\text{Av}}]^{\frac{1}{2}}\approx 18.5\left(\frac{Z}{E_0}\right)^{\frac{1}{2}}\left(\frac{NZRz^2}{E_0}\right)^{-0.055}\times\left[\left(1-\frac{x}{R}\right)^{-0.11}-1\right]^{\frac{1}{2}}. \tag{8}$$

The log terms have been evaluated for 50-Mev protons in air but the errors so introduced are not greater than 5