where μ is the chemical potential. As T approaches 0, Y can be developed and the integral evaluated, giving finally:[6]

$$\rho_I=(3\pi^2 n)^{1/3}d^2h/e^2 \text{ e.s.u.}$$
$$=6270\ b^{-1/3} \text{ ohm-cm.}$$

A detailed theory has to be based on a knowledge of both the Hall constant and the resistivity throughout the temperature range, but comparison of our calculations with Estermann's resistivity values indicates good agreement between theory and experiment. See Table I.

* This work has been carried out under contract No. #-36-039-sc-32020 between the Purdue Research Foundation and the Signal Corps.

[1] K. Lark-Horovitz and V. A. Johnson, Phys. Rev. 69, 258, 259 (1946).

[2] K. Lark-Horovitz, A. E. Middleton, E. P. Miller, and I. Walerstein, Phys. Rev. 69, 258 (1946). K. Lark-Horovitz, A. E. Middleton, E. P. Miller, W. W. Scanlon, and I. Walerstein, Phys. Rev. 69, 259 (1946).

[3] E. Conwell and V. F. Weisskopf, Phys. Rev. 69, 258 (1946).

[4] G. L. Pearson and W. Shockley, Bull. Am. Phys. Soc. 21, 9 (1946). I. Estermann, A. Foner, and J. A. Randall, Bull. Am. Phys. Soc. 22, 31 (1947).

[5] $G(\Theta/T)$ is the Gruneisen function; Mott and Jones, *Properties of Metals and Alloys*, p. 261.

[6] It can be seen that ρ_I is of this form at low temperatures by developing the classical formula for ρ_I and setting $kT=(3n/\pi)^{2/3}h^2/8m$.

Possible Use of Thermal Noise for Low Temperature Thermometry*

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FOLLOWING the suggestion of Lawson and Long[1] that the thermal agitation of a quartz crystal be used to establish a law temperature thermodynamic scale, a quantitative investigation of the problem was undertaken. It appeared to be simpler to consider the problem from the circuit point of view[2] than from the detailed analysis of the crystal mechanism. If the complex impedance of the low temperature resonant circuit (crystal or ordinary parallel resonant combination) is given by $R(f)+iX(f)$, then the noise voltage appearing across the element, *on open circuit*, in the range f to $f+df$, is given by the Nyquist formula

$$d\langle e^2\rangle_{\text{Av}}=4kTR(f)df.$$

In practice the resonant circuit is always paralleled by the input capacity, indicated by C_2 in Fig. 1. Associated with this capacity are losses represented by the resistance r_2. The noise voltage appearing across this parallel combination can be obtained from the formula[3]

$$\langle e^2\rangle_{\text{Av}}=\langle e_a^2\rangle_{\text{Av}}\left|\frac{Z_b}{Z_a+Z_b}\right|^2+\langle e_b^2\rangle_{\text{Av}}\left|\frac{Z_a}{Z_a+Z_b}\right|^2,$$

where e is the fluctuating voltage appearing across the parallel combination of an impedance Z_a in series with a generator e_a and an impedance Z_b in series with a generator e_b. Imposing the condition that the noise appearing across the parallel combination of the resonant circuit and the input capacity be characteristic of the low temperature T_a rather than room temperature T_b, one obtains the necessary inequality.

$$C_1\gg\frac{Q_0}{Q_2}C_2\frac{T_b}{T_a}, \tag{1}$$

where C_1, which is in parallel with C_2, is the capacity in the

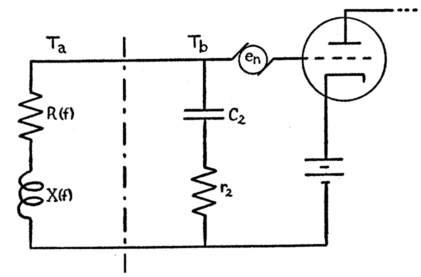

FIG. 1. Simplified circuit for measurement of thermal agitation at low temperature.

low temperature resonant circuit, Q_0 is the Q of this part of the circuit, and Q_2 is the Q of the capacity C_2.

Another condition which must be met is that the low temperature noise be large compared to the tube noise, represented in Fig. 1 by e_n, and which is given approximately by[4,5]

$$d\langle e_n^2\rangle_{\text{Av}}=4kT\left(\frac{3r_p}{\mu}+19.3I_g|Z_g|^2\right)df,$$

where the first term on the right is the term caused by shot noise in the plate circuit, referred to a generator placed on the grid, and the second term is the shot noise due to the grid current I_g in the tube. These two terms result in the inequalities

$$C_1\ll\frac{\pi}{2}\frac{1}{(3r_p/\mu)\Delta\omega}\frac{T_a}{T_b}, \tag{2}$$

and

$$C_1\gg 20I_g\frac{Q_0}{\omega_0}\frac{T_b}{T_a}. \tag{3}$$

In these inequalities ω_0 is the angular frequency at resonance and $\Delta\omega$ is the band width of the amplifier. The minimum value of this band width which will include 99 percent of the energy of the resonant circuit is

$$\Delta\omega=\frac{200}{\pi}\frac{\omega_0}{Q_0}. \tag{4}$$

Equations (1)–(4) lead to estimates of the lowest measurable temperature T_a for any desired accuracy, this accuracy determining the amount by which the left-hand sides of inequalities (1), (2), and (3) must differ from the right. If it is assumed that the background noise voltage cannot be measured or balanced out more accurately than to one percent, the $\gg$ symbols must be replaced by $>$ 4 times, in order to measure T_a to about 2 percent. Assuming $C_2=30\mu\mu$f, $Q_2=10^5$ (a value which seems generous if one either extrapolates from known Q's of air condensers, or calculates roughly on the basis of the power factors of glass and other material which might surround the input lead), and $\Delta\omega>1$ cycle per second (a practical minimum), the lowest T_a is readily calculable.

The lowest temperatures can be attained with electrometer tubes, for which, due to low grid currents, the right-hand side of (3) is less than the right-hand side of (1) so that (1) and (2) can be used to determine the minimum T_a. For a favorable tube such as the Western Electric D-96475, the minimum measurable temperature, under the conditions stated, is about 2°K, and then only if the parallel capacity C_1 of the resonant circuit can be made precisely 0.02μf and the associated network components are designed to give a resonance at 1600 c/s.

It may be possible to reduce the measurable T_a below 2°K. A tube might be designed around Eqs. (1), (2), and (3) which would have better properties for this application than the D-96475; furthermore, if this tube could be inserted in the low temperature bath along with the resonant circuit (R, X in Fig. 1), C_2 might be lowered to 6$\mu\mu$f; perhaps also the background could be measured or balanced out to better than one percent. A scrutiny of these possibilities leads, however, to the conclusion that the ultimate attainable temperature will almost certainly be greater than 0.1°K. It must be noted, however, that these calculations depend on the assumption that the noise arising in the portion of the circuit at temperature T_b can be accurately determined or balanced out. If this is not possible, the minimum temperature measurable by this method may be very much higher.

* Bulletin of the American Physical Society 21, 6 (1946).
1 A. W. Lawson and E. A. Long, Phys. Rev. 70, 220 (1946).
2 This conclusion was arrived at independently by J. B. Brown and D. K. C. MacDonald, Phys. Rev. 70, 976 (1946).
3 F. B. Llewellyn, Proc. I.R.E. 18, 243 (1930).
4 D. O. North, RCA Review 4, 441 (1946).
5 F. E. Terman, *Radio Engineers' Handbook*, first edition, p. 316.

On Bringing the Beam out of a Betatron

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Courant and Bethe[1] have given a brief discussion of their theoretical considerations on bringing the electron beam out of a betatron. I should like to mention that in the autumn of 1944 I made similar considerations and came to the same conclusion that the deflection electrodes for bringing out the beam should be located quite near the point where Br is a maximum. If the simplifying assumption is made that Br follows a parabolic law, for instance $\mathrm{Br}=(\mathrm{Br})_{\max}(1-\alpha\rho^2)$, with ρ indicating the distance from the point r_m where Br is a maximum, while the magnetic guiding field must vary (relative to the induction field) with the time constant T (for example, as $B_{\Delta t}=B_0(1-\Delta t/T)$, then as a first approximation the following differential equation for the electron orbits is obtained.

$$\frac{\partial^2\rho}{\partial t^2}=\frac{V^2}{r_m}\left(\alpha\rho^2+\frac{\Delta t}{T}\right)$$

where V is the tangential velocity of the electrons at the radius r_m.

This corresponds to the so-called Painlevé differential equation $y''=y^2+x$, which cannot be solved by known functions. Figure 1 shows a solution of this differential equation, the initial conditions being so selected that the electrons do not execute any superposed oscillations. It will be seen that the curve for y' (which also corresponds to the separation of the single orbits) rises very steeply when the radius r_m is exceeded (about proportionality to $y^{3/2}$). In order therefore that the divergence of the emerging electrodes should not become too large, the deflecting electrodes must not be located too far outside the circle r_m. On the other hand, in order that too many electrons do not fall on the edge of the deflecting plate, this latter must not be placed too far inwards.

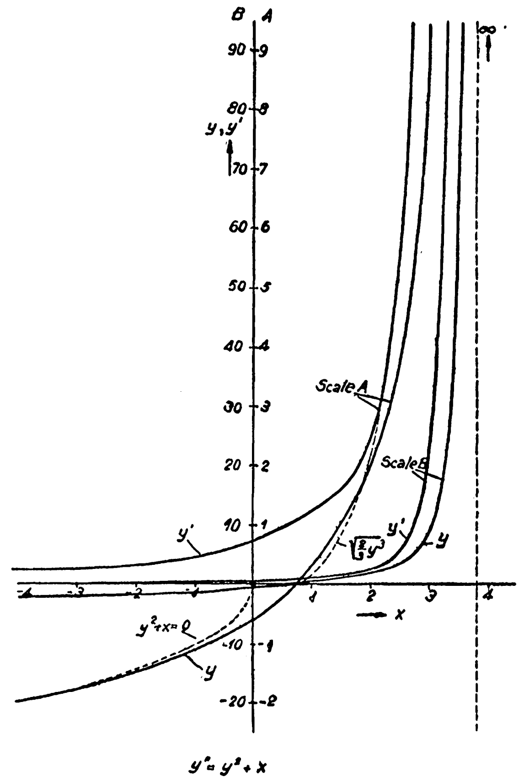

Fig. 1. Graphical form of the solution of the equation $y''=y^2+x$.

These two conditions result in an optimum position for the deflecting plates which, in conjunction with a particular construction of the deflecting field (preliminary deflection by means of a special very thin deflecting electrode), has formed the subject matter of a patent application filed by me in December, 1944. In this patent application also the refocusing of the emerging beam by means of an auxiliary magnetic field has been provided.

1 E. D. Courant and H. Bethe, Phys. Rev. 70, 798 (1946).

On the Dissociation Energy of CO

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Three proposed interpretations of the band spectrum of CO lead to $D(\mathrm{CO})=6.92$ ev,[1] 9.14 ev,[2] and 11.11 ev,[3] respectively. Electron collision experiments in CO give the unique value, $D(\mathrm{CO})=9.6$ ev.[4] Clearly a reconciliation of these conflicting positions is demanded.

$D(\mathrm{CO})=9.6$ ev from electron impact rests upon the appearance potentials of four ionization and dissociation