known to have a positive magnetic moment, H_3 has therefore likewise a positive moment.

A second series of observations to obtain a more accurate value for γ_T was performed by keeping the current in the electromagnet constant, and observing the induced signals of the two isotopes for different frequencies. The following table gives the results for the resonance frequencies ν_3 and ν_1 of H_3 and H_1, respectively, in megacycles together with the field B_0 in gauss, at which the observation was carried out and the resulting ratio γ_T/γ_P of the gyromagnetic ratios.

TABLE I. Resonance frequencies ν_3 and ν_1 of triton and proton and the resulting value of γ_T/γ_P. The fourth column represents the result of a repetition for tritium to ascertain that the field stayed constant during the run.

B_0	ν_3	ν_1	ν_3	γ_T/γ_P
9770	44.29	41.51	44.28	1.067
9500	43.08	40.37	43.08	1.067

To summarize we can therefore state that the triton has a spin of $\frac{1}{2}$, and that its magnetic moment is positive, and 1.067 ± 0.001 times larger than that of the proton.

* Work done at Stanford University and at the Los Alamos Scientific Laboratory operated by the University of California under U. S. Government contract.

[1] F. Bloch, Phys. Rev. **70**, 460 (1946). F. Bloch, W. W. Hanson, and M. Packard, Phys. Rev. **70**, 474 (1946).

[2] See Eq. (29) of reference 1.

Transition from Classical to Quantum Statistics in Germanium Semiconductors at Low Temperature

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ANALYSIS[1] of the experimental results[2] obtained with germanium semiconductors in the temperature range from $-180°C$ to about $600°C$ has shown that one can account for electrical conductivity and thermoelectric power of these impurity semiconductors by assuming that lattice vibrations and scattering by singly charged impurity centers[3] are responsible for the observed resistivity ρ, where $\rho = \rho_L + \rho_I$.

$$\rho_L = DRT^{3/2},$$
$$\rho_I = \frac{9\cdot 10^{11}\pi^{3/2}e^2m^{1/2}}{2^{7/2}\epsilon^2(kT)^{3/2}}\cdot \ln\left(1+\frac{36\epsilon^2k^2T^2d^2}{e^4}\right),$$

where $R \sim 1/n$ is the Hall constant, n the number of conduction electrons per cc, m the electronic mass, ϵ the dielectric constant, $d = 0.28n^{-1/3}$ = one-half the average distance between impurity centers D determined from experiments.

In both cases it has been assumed that classical statistics can be applied. This is justified in most cases since the number of electrons, as determined from Hall effect measurements, is small.

If the number of electrons is nearly independent of temperature one may apply the well-known criterion for degeneracy and define a degeneracy temperature

$$T_d = \frac{h^2}{8mk}\left(\frac{3n}{\pi}\right)^{2/3} = 4.2\times 10^{-11}n^{2/3}\ °K.$$

Since n varies from sample to sample, one finds that degeneracy temperatures vary from a fraction of a degree K to about 150°K in the germanium samples studied at Purdue. Therefore, at low temperatures, the behavior of these semiconductors should vary widely, depending upon the number of electrons and the activation energy.

Measurements of such semiconductors down to about 10°K have been reported recently.[4] The observations show that three kinds of samples exist:

(1) Very pure samples with a resistance increasing so sharply with decreasing temperature that the material becomes almost non-conducting (Estermann's "pure" germanium and silicon samples).

(2) Samples for which the resistivity increases with decreasing temperature and in some cases seems to reach a "saturation" value.

(3) Samples with constant resistivity from liquid air temperature to liquid hydrogen temperature. All of the samples of type (3) have degeneracy temperatures of about 100°K or higher; calculations using classical statistics, such as have been used at medium temperatures, are not justified for such samples at low temperatures.

We have, therefore, carried out calculations assuming Fermi statistics instead of classical statistics and can summarize our results as follows:

(*a*) Lattice scattering.[5]

above T_d, $\rho_L = DRT^{3/2}$,
below T_d, $\rho_L = D'RTG(\Theta/T) \rightarrow D'RT^5$ at 25°K.

These expressions, calculated for germanium samples, show a smoothly decreasing resistivity with decreasing temperature and, therefore, contribute little to the observed resistivity at low temperatures.

(*b*) Impurity scattering. By calculating the scattering of electrons by randomly distributed, singly-charged impurity centers, one obtains:

$$\frac{1}{\rho_I} = \sigma_I = \frac{32}{3}\frac{e^2mk^3T^3}{ne^2h^3}\int_0^\infty \frac{x^3\exp(x-\mu^*)dx}{[\exp(x-\mu^*)+1]^2\ln Y},$$

$$Y = 1+\frac{4\epsilon^2k^2T^2d^2x^2}{e^4},\quad x = \frac{mv^2}{2kT},\quad \mu^* = \frac{\mu}{kT}$$

TABLE I.

Sample	Measured by Estermann	Measured at Purdue	Calculated
26 *Z*	0.0051	0.0044	0.0040
11 *R*	0.0040	0.0034	0.0037
26 *E*	0.0037	0.0033	0.0034
27 *L*	0.0034	0.0029	0.0033

(All of the above values represent constant low temperature resistivities measured in ohm-cm.)

Thus the transition from classical to quantum statistics leads to a constant residual resistance due to impurity scattering in degenerate samples in agreement with experiment.

where μ is the chemical potential. As T approaches 0, Y can be developed and the integral evaluated, giving finally:[6]

$$\rho_I = (3\pi^2 n)^{1/3} d^2 h/e^2 \text{ e.s.u.}$$
$$= 6270\, b^{-1/3} \text{ ohm-cm.}$$

A detailed theory has to be based on a knowledge of both the Hall constant and the resistivity throughout the temperature range, but comparison of our calculations with Estermann's resistivity values indicates good agreement between theory and experiment. See Table I.

* This work has been carried out under contract No. #-36-039-sc-32020 between the Purdue Research Foundation and the Signal Corps.

[1] K. Lark-Horovitz and V. A. Johnson, Phys. Rev. **69**, 258, 259 (1946).

[2] K. Lark-Horovitz, A. E. Middleton, E. P. Miller, and I. Walerstein, Phys. Rev. **69**, 258 (1946). K. Lark-Horovitz, A. E. Middleton, E. P. Miller, W. W. Scanlon, and I. Walerstein, Phys. Rev. **69**, 259 (1946).

[3] E. Conwell and V. F. Weisskopf, Phys. Rev. **69**, 258 (1946).

[4] G. L. Pearson and W. Shockley, Bull. Am. Phys. Soc. **21**, 9 (1946). I. Estermann, A. Foner, and J. A. Randall, Bull. Am. Phys. Soc. **22**, 31 (1947).

[5] $G(\Theta/T)$ is the Gruneisen function; Mott and Jones, *Properties of Metals and Alloys*, p. 261.

[6] It can be seen that ρ_I is of this form at low temperatures by developing the classical formula for ρ_I and setting $kT = (3n/\pi)^{\frac{2}{3}} h^2/8m$.

Possible Use of Thermal Noise for Low Temperature Thermometry*

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FOLLOWING the suggestion of Lawson and Long[1] that the thermal agitation of a quartz crystal be used to establish a law temperature thermodynamic scale, a quantitative investigation of the problem was undertaken. It appeared to be simpler to consider the problem from the circuit point of view[2] than from the detailed analysis of the crystal mechanism. If the complex impedance of the low temperature resonant circuit (crystal or ordinary parallel resonant combination) is given by $R(f)+iX(f)$, then the noise voltage appearing across the element, *on open circuit*, in the range f to $f+df$, is given by the Nyquist formula

$$d\langle e^2\rangle_{\text{Av}} = 4kTR(f)df.$$

In practice the resonant circuit is always paralleled by the input capacity, indicated by C_2 in Fig. 1. Associated with this capacity are losses represented by the resistance r_2. The noise voltage appearing across this parallel combination can be obtained from the formula[3]

$$\langle e^2\rangle_{\text{Av}} = \langle e_a^2\rangle_{\text{Av}} \left|\frac{Z_b}{Z_a+Z_b}\right|^2 + \langle e_b^2\rangle_{\text{Av}} \left|\frac{Z_a}{Z_a+Z_b}\right|^2,$$

where e is the fluctuating voltage appearing across the parallel combination of an impedance Z_a in series with a generator e_a and an impedance Z_b in series with a generator e_b. Imposing the condition that the noise appearing across the parallel combination of the resonant circuit and the input capacity be characteristic of the low temperature T_a rather than room temperature T_b, one obtains the necessary inequality.

$$C_1 \gg \frac{Q_0}{Q_2} C_2 \frac{T_b}{T_a}, \tag{1}$$

where C_1, which is in parallel with C_2, is the capacity in the low temperature resonant circuit, Q_0 is the Q of this part of the circuit, and Q_2 is the Q of the capacity C_2.

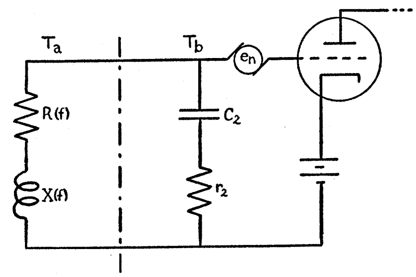

FIG. 1. Simplified circuit for measurement of thermal agitation at low temperature.

Another condition which must be met is that the low temperature noise be large compared to the tube noise, represented in Fig. 1 by e_n, and which is given approximately by[4,5]

$$d\langle e_n^2\rangle_{\text{Av}} = 4kT\left(\frac{3r_p}{\mu} + 19.3 I_g |Z_g|^2\right) df,$$

where the first term on the right is the term caused by shot noise in the plate circuit, referred to a generator placed on the grid, and the second term is the shot noise due to the grid current I_g in the tube. These two terms result in the inequalities

$$C_1 \ll \frac{\pi}{2} \frac{1}{(3r_p/\mu)\Delta\omega} \frac{T_a}{T_b}, \tag{2}$$

and

$$C_1 \gg 20 I_g \frac{Q_0}{\omega_0} \frac{T_b}{T_a}. \tag{3}$$

In these inequalities ω_0 is the angular frequency at resonance and $\Delta\omega$ is the band width of the amplifier. The minimum value of this band width which will include 99 percent of the energy of the resonant circuit is

$$\Delta\omega = \frac{200}{\pi} \frac{\omega_0}{Q_0}. \tag{4}$$

Equations (1)–(4) lead to estimates of the lowest measurable temperature T_a for any desired accuracy, this accuracy determining the amount by which the left-hand sides of inequalities (1), (2), and (3) must differ from the right. If it is assumed that the background noise voltage cannot be measured or balanced out more accurately than to one percent, the $\gg$ symbols must be replaced by $>$ 4 times, in order to measure T_a to about 2 percent. Assuming $C_2 = 30\mu\mu f$, $Q_2 = 10^5$ (a value which seems generous if one either extrapolates from known Q's of air condensers, or calculates roughly on the basis of the power factors of glass and other material which might surround the input lead), and $\Delta\omega > 1$ cycle per second (a practical minimum), the lowest T_a is readily calculable.

The lowest temperatures can be attained with electrometer tubes, for which, due to low grid currents, the right-hand side of (3) is less than the right-hand side of (1) so that (1) and (2) can be used to determine the minimum T_a. For a favorable tube such as the Western Electric D-96475, the minimum measurable temperature, under the