The sample was irradiated with neutrons from a Ra+Be source, with paraffin as a slowing medium. Fission counting rates of two to three hundred per minute were observed.

The absorption curve for the particles accompanying Pu^{239} fission is seen in Fig. 3. The maximum range of the particle appears to be about the same as that of the alphas from U^{235} fission, within the limits of the experimental error. The curves relevant to the Pu^{239} experiments are also reported in Figs. 2 and 3, together with the curves for U^{235}.

The difference in shape of the curves for U^{235} and Pu^{239} in Fig. 3 may be accounted for by the additional absorber used for Pu^{239}, which has the effect of eliminating the particles which made very small pulses on the U^{235} run. The same arguments used for U^{235} establish also that the ionizing particles emitted by Pu^{239} in coincidence with fission are α-particles.

The number of alpha-particles emitted in coincidence with Pu^{239} fission is computed from the data exactly as before. The result obtained is about 2 alphas per 1000 fissions.

This investigation will be followed up with the purpose of finding for both U^{235} and Pu^{239}:

(1) The energy distribution of the alphas.
(2) The possible presence of protons.
(3) The time relation of the emission of the charged particles to the fission.

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The Probability of K Ionization of Nickel by Electrons as a Function of Their Energy

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Thin targets of Ni were bombarded with electrons at 12 to 183 kv and intensities of the $K\alpha$ doublet were measured in arbitrary units. The results were converted to absolute cross sections for ionization by comparison with Smick and Kirkpatrick's absolute measurements at 70 kv. At any voltage $V=UV_K$, where V_K is the K ionization voltage, the cross section is well represented empirically as

$$\Phi_K=7.3(e/V_K)^2U^{-0.837}\log_{10}U,$$

with e and V_K both in electrostatic units. Burhop's theory is confirmed with accuracy probably well within the limits of error imposed on it by the Born approximation and neglect of relativity, exchange and other minor factors. The effect of relativity is found by comparison of the cross sections for Ni with ones for Ag, previously measured in this laboratory. Relativity increases the cross sections by moderate percentages, which increase with voltage. Deduction of these percentages yields data for a hypothetical non-relativistic element; and Burhop's non-relativistic theory fits this element best. Smith's cross sections for helium are compared with these non-relativistic cross sections and with those for real nickel. At low U's the cross sections for helium are notably less than would be predicted by simple analogy with the other elements, presumably because of unusually great effects in helium, due to movement of the electron which remains in the atom.

I. INTRODUCTION

BECAUSE of the mathematical complexity of wave mechanics and the physical complexity of most real atoms, atomic hydrogen would be the best element for a test of theories of K ionization by electron impact. Experiments with atomic hydrogen are difficult; but silver, which has been studied both experimentally and theo-

retically,[1–5] provides an approximately hydrogenic field of force for the *K* electron. At the same time, however, silver involves serious relativistic effects. The *K* ionization probability of helium has been measured by Smith.[6] With helium, relativistic effects are negligible; but because its nuclear charge is not large compared to the mutual screening of the *K* electrons, the effective nuclear field is far from hydrogenic.

Nickel avoids to an appreciable extent the relativistic complications associated with silver and the nonhydrogenic character of helium. For this reason along with many others of experimental practicability, nickel was chosen for the tests to be reported here.

II. SURVEY OF EXPERIMENTAL TECHNIQUE

On the experimental side of the test, the ionization cross section was calculated from the intensity of the $K\alpha$ x-ray doublet; so the experiments consisted primarily of measurements of this intensity as a function of the energy of electron impact. The general plan was the same as for silver.[3,4] That is, the measurements were all on intensities of x-rays from extremely thin targets, and they were divided into two parts: one, a series of measurements with a crystal spectrometer, giving only relative intensities, i.e., in arbitrary units, but covering a wide range of tube voltage; the other, a set of absolute intensity measurements with Ross filters, at the voltage best suited to their use. These absolute measurements were made and described by Smick and Kirkpatrick.[7] In this paper the relative measurements will be described and standardized by use of the absolute measurements, and then compared with the theory. Finally, the measurements on silver and helium will be compared with these on nickel, for evidence on the effects of relativity and the other *K* electron.

III. DETAILS OF APPARATUS

Most parts of the apparatus used in this research had been used already in the later relative measurements on thin targets of silver.[3–8] Naturally there were improvements, especially as the apparatus was used for two researches on thick targets, of silver[9] and nickel,[10] before starting the work for this paper. This work, however, followed immediately after the thick-target researches, and was finished by 1938.

Many of the improvements need no detailed description. Among these are several in the vacuum pumping system. Another was the introduction of an indirectly heated cathode, with a flat emitting surface of columbium, to distribute the cathode rays smoothly over the focal spot and reduce the danger of burning holes in the target. Still another improvement, more specifically related to the change from silver to nickel, was the elimination of most of the air from the path of the x-rays, to reduce absorption. This was done partly by adding to the x-ray tube an 18″ neck of metal and glass, projecting toward the spectrometer, and partly by inserting a 12″ tube full of hydrogen between the spectrometer crystal and the ionization chamber. Finally, the gas in this chamber was changed from CH_3Br to SO_2, to prevent too much concentration of ionization at one end in using such soft rays, and yet absorb about 90 percent of them.

Even with all these changes, the Ni $K\alpha$-rays were very weak. So the electrometer was thoroughly overhauled and studied by Pockman[11] and its sensitivity was increased fivefold.

The preparation of nickel targets, thin enough for this work, required a special technique, described elsewhere by Pockman and Webster.[12]

IV. EXPERIMENTAL PROCEDURE

As in the work on thin silver, the experiments consisted primarily of relative measurements of

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[10] L. T. Pockman, Paul Kirkpatrick, and D. L. Webster, Phys. Rev. **45**, 131A (1934). The data reported in this abstract need to be combined with the thin-nickel data of the present paper, as for silver in reference 9, to get the most out of them; so the abstract referred to will be replaced soon by a more complete paper.
[11] L. T. Pockman, Rev. Sci. Inst. **7**, 238 (1936).
[12] L. T. Pockman and D. L. Webster, Rev. Sci. Inst. **12**, 389 (1941).

the intensities of the $K\alpha$-doublet at different tube voltages, an ionization chamber filled with sulfur dioxide being used as the detecting device. It was essential, therefore, that of the total line-spectrum radiation produced in the target, the fraction which went to the ionization chamber should be the same at all voltages. A further essential requirement was that auxiliary measurements should be taken, so that the continuous-spectrum rays accompanying the line rays could be eliminated by subtraction. Except for minor differences, the spectrometric procedure was all as described for silver.[3]

For outgassing the tube and measuring tube voltages and currents also, the procedure was almost the same as for silver.[3–8] Since the preparation of the thin nickel targets was very costly in time and effort, however, they were protected against gas bursts by a non-inductive resistor (made of xylene and alcohol) between the anode and the high voltage system. This required the subtraction of its IR drop (not often over 1 or 2 kv) from the measured voltage; and since it was convenient to work at standard points on the voltmeter scale, the tube voltages differed slightly from day to day.

A more important innovation relative to the targets was in the procedure for guarding against possible errors due to changes in the target during its use. One possible change of this sort would be the formation of holes by burning or tearing. With both silver and nickel, this was guarded against by careful visual inspection of each target after the conclusion of work with it. Another possibility for trouble was contraction of the target, especially of the cellulose acetate which backed the nickel. This would pull more nickel into the focal spot and increase the intensity of the rays. This possibility was suggested by the disappearance of shallow wrinkles, observed in the inspections. As a further check, therefore, a statistical test was introduced. This consisted of adding the intensities measured at 7 or 8 well-distributed voltages in each "run" (or sequence of measurements at decreasing voltages) and comparing the sums found for all the different runs with the same target. This test is open to the objection that there might be a compensation of holes by contraction; but such a coincidence would be improbable, and if it did occur, the holes would show on inspection.

An important question is, how to know how small a change in the sum to consider significant. A quantitative measure of the statistical significance of the deviations of these sums from their mean was developed. As an example of the application of this method, if Target 22 had undergone a 3 percent change of thickness just before its last run, the deviations would have been such as would occur by chance only about once in 50 such sets of data. Other possible cases, of course, would give different probabilities, so it seems difficult to formulate an exact general rule; but the existence of systematic errors of more than a few percent seems unlikely.

For the first four targets (21, 22, 25, 26) used without tearing them, this statistical test indicated that if there was any appreciable increase of thickness by contraction it must have occurred during outgassing, before measurements began.

These four targets were used by Pockman and Webster.[13] Then in 1937 they turned the apparatus over to Kirkpatrick and Harworth, who made a new target holder and some other improvements and used it with three more targets, Nos. 29, 30, and 34. Since the essential parts of the apparatus were the same, or equivalent, the results of both sets of measurements agreed within their limits of erratic error. Therefore they are averaged here as products of a single research.

Because the ionization measurements were in arbitrary units, no effort was made to measure the thickness of the targets with more accuracy than was needed for computing the corrections to be described in the next section.

V. CORRECTIONS

For comparing intensities from different targets, each value of the intensity was normalized in terms of a standard for its target. Letting $i(V)$ stand for the measured intensity (in arbitrary units) at voltage V, and $i_0(V)$ for the corrected intensity, to be described, the normalization consists in dividing each $i(V)$ or $i_0(V)$ by the average of the $i_0(V)$'s at 70, 80, 90, and 100 kv.

[13] If necessary, many further details about them and the rest of the apparatus can be found in L. T. Pockman's Ph.D. thesis, "The Probability of K Ionization of Nickel by Cathode Rays as a Function of Cathode Ray Energy," in the Stanford University Library.

TABLE I. Correction factors, c_s, for single and multiple scattering as calculated for four targets.

V	No. 21(670A)	No. 22(960A)	No. 25(150A)	No. 26(100A)
15			0.932	0.956
25			0.974	0.982
35	0.940		0.986	0.991
45	0.960	0.939		0.995
55	0.972	0.956	0.994	0.996
75	0.983	0.974	0.996	0.998
95	0.989	0.982	0.997	0.998
125	0.993	0.989	0.999	0.999
155	0.995	0.992		
180	0.996	0.994		

TABLE II. Correction c_r, for rediffusion of cathode rays from cellulose acetate.

V	Target 25	Target 26
15	0.993	0.992
20		0.995
25	0.996	
30		0.997
35	0.998	
40		0.999
55	0.999	1.000 at $V>40$ kv
>55	1.000	

This division cancels the arbitrary unit, giving a pure number, either the "reduced intensity" $j(V)$, or the "reduced corrected intensity" $j_0(V)$. $j_0(V)$ is the quantity which should be, and is found to be, the same within limits of erratic error for all targets.

The end product of the experiments reported here, therefore, will be a table of values of $j_0(V)$, obtained by averaging at each V the values from different runs. Going on then from these experiments, $j_0(V)$ will be converted back to $i_0(V)$, but this time in absolute units, not arbitrary, by the use of the absolute measurement of i_0 (70 kv) by Smick and Kirkpatrick.[7] This $i_0(V)$ will then be compared with theories and with values for helium and silver, as outlined in Section I.

In principle the corrections required by the finite target thickness and the proximity of supporting structures are like those for silver[3,4,8] but on account of the lower speeds of impact with nickel the corrections are more important, so the theory used with silver has been developed further.

Four corrections for the target thickness have been considered: (1) retardation of the bombarding electrons; (2) screened nuclear scattering (both multiple and single); (3) absorption of the Ni $K\alpha$-radiation by the nickel target itself; (4) fluorescent radiation due to production and absorption of the continuous spectrum within the target. Corrections (3) and (4) turn out to be negligible for all targets and all voltages, and the correction for retardation is very small. Thus the only important correction for target thickness is (2).

Impacts beyond the nickel film require consideration of the following corrections: (5) fluorescence of the nickel under the rays from the thin backing film of cellulose acetate; (6) fluorescence under the rays from the bottom of the supporting aluminum structure, several centimeters behind the target; (7) rediffusion of electrons from cellulose acetate; (8) rediffusion of electrons from the aluminum structure. Of these corrections only (7) is appreciable. What little rediffusion there is, is primarily by large-angle single scattering. Thus it will be noticed that of the seven corrections considered only three are found to be appreciable.

The increase in path length of fast β-rays through thick films was calculated in good approximation by Bothe[14] as due exclusively to multiple scattering. With the thinner nickel targets at the lower voltages, the assumptions on which Bothe's approximation is based are not valid and it becomes necessary to consider both multiple and single scattering. However, with the 150A target at 15 kv, if the impact parameter chosen to divide multiple from single scattering is changed from 0.008A, where Bothe chose it, to 0.15A, the calculated path length is changed only from 1.073 times the target thickness to 1.081 times the thickness. Of course, the distribution between the parts calculated as single and multiple scattering is radically changed. The remarkable feature is that it makes so little difference in the final result where the dividing line is drawn.

To be sure, the division into single and multiple scattering is an approximation. In between, around 10° in this target at 15 kv, there is a range of plural scattering. This range is somewhat indefinite but calculations indicate it is probably short enough to permit the division into multiple and single scattering.

[14] W. Bothe, *Handbuch der Physik*, Vol. 24, Chap. I (1927).

TABLE III. Reduced corrected intensities from the first four usable targets.

V in kv	Reduced corrected intensity Target 21 (670A)	Target 22 (960A)	Target 25 (150A)	Target 26 (100A)	Wtd. av.	Wtd. percent av. dev.
183.3	0.749	0.709			0.725	2.6
153.1	0.812	0.757			0.780	3.4
124.5	0.864	0.831	0.864	0.922	0.850	2.0
95.1	0.964	0.960	0.970	0.914	0.961	0.6
75.3	1.030	1.035	1.020	1.096	1.034	0.6
55.6	1.170	1.172	1.180	1.048	1.167	1.0
46.2	1.255	1.272		1.138	1.258	1.2
35.7	1.335		1.338	1.236	1.327	1.2
24.8			1.385	1.226	1.353	3.8
14.8			1.125	1.002	1.101	3.5

TABLE IV. Intensities from last three usable targets.

Target 29 2150A			Target 30 1320A			Target 34 110A		
V	$i(V)$	$j_0(V)$	V	$i(V)$	$j_0(V)$	V	$i(V)$	$j_0(V)$
179.3	114.2	0.707	175.0	171.8	0.728	100	87.9	0.952
169.5	120.1	0.743				80	93.0	1.005
159.5	121.7	0.751	155.4	181.1	0.767	60	106.1	1.145
149.5	126.2	0.779	125.5	200.9	0.845	40	122.2	1.311
139.5	127.7	0.785	95.4	226.7	0.944	30	130.3	1.386
129.5	137.0	0.841	75.4	254.0	1.047	25	130.2	1.378
119.5	142.3	0.870	57.73	294.1	1.192	20	126.6	1.330
109.6	145.4	0.885				15	108.3	1.116
99.6	151.8	0.920						
89.6	161.2	0.969						
79.6	173.3	1.031						
69.6	185.6	1.084						

For practical application of the scattering correction a factor c_s has been calculated by which each measured x-ray intensity has to be multiplied to reduce it to the intensity which would have been found if all electrons had gone straight through the target. The factor c_s as computed for the first four usable targets is shown in Table I.

The correction for scattering from the backing film of cellulose acetate depends on those electrons scattered by more than 90°. Therefore it is very small. The correction factor c_r derived from it is shown in Table II.

Finally the correction for retardation of electrons in the target in the distance x gives a correction factor c_x. This correction is very small at the voltages for which scattering can be calculated well enough. For targets 25 and 26 it is negligible at all voltages. For 21 and 22 it is negligible above 75 kv; from these down as far as Table I goes, $c_x=0.999$.

VI. RESULTS OF EXPERIMENTS

The reduced corrected intensities for the first four usable targets are listed in Table III. To save printing, uncorrected intensities have been omitted, but may be found if needed by "uncorrecting" the figures in this table with the corrections listed above. The exact meanings of the headings in this table are as follows:

The column for each target shows averages of the reduced corrected intensities for all runs with that target, with the data from each run weighted in proportion to the tube current for that run. The thickness of each target is shown in angstroms.

The column headed "Wtd. av." shows weighted averages of the averages in the preceding columns. In preparing this column the weight to be given to the data from each target was determined by two independent objective methods which gave essentially the same results.

For Targets 29, 30, and 34 the procedure was similar except that, as only one run was made with each of these targets there was no problem of weighting the data from different runs. Moreover the voltages used for different targets were too different for easy reduction to a common list by interpolation. In Table IV, therefore, there are three columns for each target: voltage in kv, measured intensity in arbitrary units, and corrected reduced intensity.

To combine the results of the two sets of measurements, a first step is to get the measurements from the last three usable targets expressed in terms of the voltages used with the first four. To do this, use is made of an empirical reduced intensity, taken from the work on silver,[3]

$$j_2(V)=\text{constant}\ (U^{-m_2}\log U), \qquad (1)$$

where $U=V/V_K$ and V_K is the K ionization voltage. For silver m_2 was 0.783; and for nickel, as will be shown below, $m_2=0.837$. Finally, a weighted average between the data from the two

TABLE V. Comparison and averaging of values of $j_0(V)$ from the two groups of targets.

V in kv	First group $j_0(V)$	Weight	Second group $j_0(V)$	Weight	Final Wtd. av. $j_0(V)$
183.3	0.725	(7)	0.707	(2)	0.721
153.1	0.780	(7)	0.771	(2)	0.778
124.5	0.850	(16)	0.852	(2)	0.850
95.1	0.960	(16)	0.952	(3)	0.959
75.3	1.034	(16)	1.048	(3)	1.036
55.6	1.167	(16)	1.213	(2.3)	1.173
46.2	1.258	(10)	1.289	(1.85)	1.263
35.7	1.327	(12)	1.363	(1.4)	1.331
24.8	1.353	(9)	1.380	(1)	1.356
14.8	1.101	(9)	1.107	(1)	1.102

groups of targets was computed, weighting the data of each group at any one voltage in proportion to the number of runs made in that group at that voltage. The result of this averaging is the "Final wtd. av." recorded in Table V.

This final weighted average $j_0(V)$ is now ready for standardization by means of the absolute cross section for ionization at $V=70$ kv, namely $(3.38\pm0.2)\times10^{-22}$ cm², measured by Smick and Kirkpatrick.[7] For this standardization, $\log\{j_0(V)/\log U\}$ was plotted against $\log U$ as shown in Fig. 1, and the straight line which seems to fit the data near 70 kv best was drawn there. This line gives three valuable pieces of information, namely: (1) the reliability of the empirical function $j_2(V)$ as shown by the accuracy with which the data are fitted by the straight line; (2) the value of m_2 for Eq. (1); and (3) the best value of $\log\{j_0(V)/\log(U)\}$ at 70 kv, and from it the best value of j_0 (70 kv), namely 1.096. The ratio of Smick and Kirkpatrick's absolute ionization cross section to this value of j_0 (70 kv), is 3.08×10^{-22} cm². The product of this area by $j_0(V)$ at any other V is therefore the absolute ionization cross section $\Phi_K(V)$, so it is tabulated in Table VI.

Theories, both classical and wave mechanical, predict values for this cross section as products of a characteristic area $A_K=e^2/V_K^2$ and a dimensionless function of U which changes relatively little from one element to another. To facilitate

TABLE VI. Final results of experiments.

V in kv	Φ_K in (10^{-22} cm²)	U	Φ_K/A_K
183.3	2.22	22.00	0.746
153.1	2.40	18.37	0.805
124.5	2.62	14.94	0.879
95.1	2.96	11.41	0.993
75.3	3.20	9.04	1.073
55.6	3.62	6.67	1.213
46.2	3.90	5.54	1.309
35.7	4.11	4.28	1.380
24.8	4.18	2.98	1.401
14.8	3.40	1.78	1.140

TABLE VII. Parameters of empirical functions.

Element	Z	m_1	C_1	m_2	C_2
Helium	2	10.8	20.2	Inapplicable	
Nickel	28	4.3	15.4	0.837	7.27
Silver	47	5.25	17.8	0.783	7.06

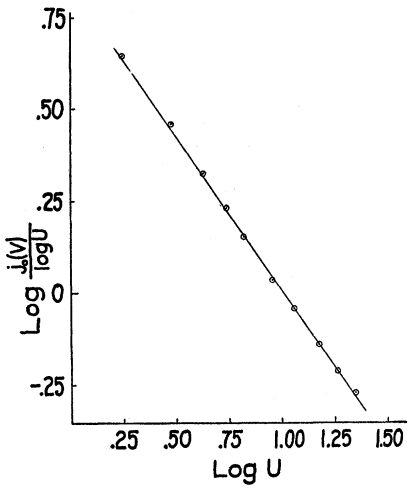

FIG. 1. Graph of $\log_{10}\{j_0(V)/\log_{10}U\}$ against $\log_{10}U$, for finding the value of the parameter m_2 of Eq. (1).

comparisons between different elements, e.g., nickel, silver, and helium, therefore, the dimensionless ratios Φ_K/A_K and U are also tabulated in Table VI. For nickel, $V_K=8.336$ kv $=27.81$ e.s.u., and $A_K=2.98\times10^{-22}$ cm².

Empirical formulas for these cross sections, and probably those of other elements not too far from nickel and silver, may be found by standardizing the $j_2(V)$ of Eq. (1) and another function, $j_1(V)$, also taken from reference 3. Thus we find

$$\Phi_2(V)=C_2A_KU^{-m_2}\log_{10}U \qquad (2)$$

and

$$\Phi_1(V)=C_1A_K(U-1)/(U^2+m_1U). \qquad (3)$$

Of these functions Φ_2 is the more accurate but Φ_1 is clearly more convenient for any calculations involving integration. Their parameters for Ni and for Ag (references 3 and 4) are listed in Table VII. Evidently in the range of Z covered here, the relative change in any one of these parameters is considerably less than that in Z. This fact encourages interpolation. So two other facts must be noted with it: (1) Φ_2 will not fit Smith's[6] data on helium well with any value of m_2, and the value of m_1 for helium seems out of line; and (2) relativity must be much more important at any given U in very heavy elements than in nickel and silver. Therefore extrapolation very far outside the range of Z between nickel and silver is unreliable; but interpolation in this range seems reasonable.

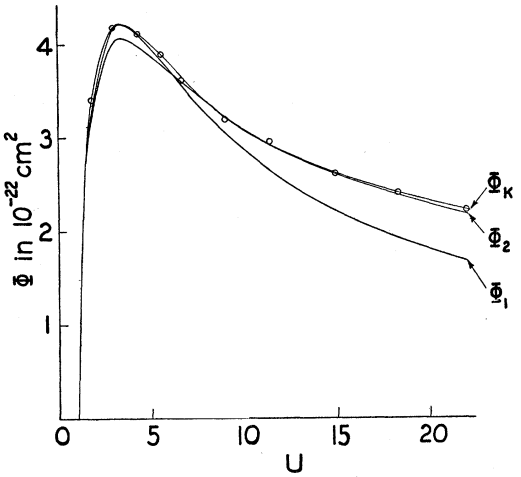

FIG. 2. Graph and points for experimentally determined K-ionization cross sections of nickel atoms (not single K electrons) and graphs of the empirical functions defined in Eqs. (2) and (3).

Graphs of Φ_K, Φ_1, and Φ_2 for nickel are shown in Fig. 2.

VII. COMPARISON WITH THEORIES

That classical mechanics, with some quantizing assumptions, does not work well for K ionization was proved conclusively for silver in reference 3; and if any further proof were needed, it could be found in a similar form with the present measurements on nickel.

Turning to wave mechanics, it seems that most of the theoretical work to date has been done by means of Born's first approximation. This work was started by Ochiai[15] and Bethe,[16] the latter arriving at an equation for the cross section, but only by using approximate methods of integration, which limit its applicability to higher values of U than would be required by Born's approximation alone. For these U's Liska[17] found Bethe's equation very good, but for the range of our data on nickel the approximations additional to Born's must be replaced by more accurate calculations.

This requires very tedious arithmetical integrations. Therefore it was not done at once, but only after intermediate approximations by Massey and Mohr,[18] Wetzel,[19] and Soden.[20] Finally a calculation without any of these intermediate approximations, but with Born's and those inherent in arithmetical integration of a double integral with a highly variable integrand, was made for the K electrons of silver by Massey and Burhop.[5] Comparing their results with Clark's[4] experimental cross sections they found a much better agreement than that of any preceding theory. Therefore Burhop[21] made similar calculations for the K electrons of nickel, silver and mercury and the L electrons of silver and mercury, also with very good results. For the K of nickel his comparison with experiment was with such data as we could give him by letter at that time, i.e., cross sections in arbitrary units. Now, therefore, we must extend the comparison by using absolute units.

In the theoretical calculations a major source of uncertainty is the difference between ordinary atoms, with their full quotas of electrons, and the hydrogenic atoms (either with the true Z and only one electron or with $Z_K=Z-0.30$ to allow for the other one) which are amenable to calculation. This difference becomes especially serious as the voltage gets low. For nickel, at the voltage where the ionization cross section of a "screened hydrogenic" atom ($Z_K=27.7$) would drop to zero, that of an ordinary nickel atom is still greater than $\frac{1}{3}$ of its maximum value. The correction from hydrogenic atoms to ordinary ones is evidently a serious matter.

To make it worse, the difference is due to more than one cause. The ratio of V_K to the screened-hydrogenic ionization potential V_H is 0.800 for nickel, 0.861 for silver, 0.962 for mercury, and 1.006 for uranium. The difference from unity, which is so notable in nickel, is caused primarily by the screening of the nuclear electrostatic field by the outer electrons. But for heavier elements, the lack of so much difference is not primarily caused by any lack of so much screening. Rather, it is caused by relativity, which affects V_K in the direction opposite to the effect of screening, and is more important in heavier elements because the K electrons move faster. Altogether, the transition from hydrogenic atoms to ordinary ones is even more difficult than it looks.

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[21] E. H. S. Burhop, Proc. Camb. Phil. Soc. **36**, 43 (1940).

For hydrogenic atoms, the double integral which Burhop[21] had to evaluate arithmetically can be expressed as a product of a dimensional constant by an integral in which all variables are dimensionless ratios. Those in the integrand are ratios of momenta of other electrons to that of a hydrogenic K electron; and those in the limits of integration are functions of such ratios and the ratio $U_H = V/V_H$. On integration, all these ratios but U_H disappear. Consequently, the cross sections for hydrogenic atoms take the form

$$\Phi_H = A_H f(U_H), \tag{4}$$

with $A_H \equiv e^2/V_H^2$ and f independent of V_H.

For ordinary atoms, such a complete separation into a shape factor f and a scale factor A seems almost too much to hope for; but these questions arise: how near we can come to it, and whether possibly the differences may be within the limits of error imposed on any arithmetic integration by the character of the integrand in the double integral which has to be computed. In answer to these questions, Burhop, in a recent letter, says he would expect the greatest effect of the transition from hydrogenic to ordinary atoms to be in the scale factor. On the shape factor, taking everything now known about it into consideration, he favors using the same function of U to cover nickel, silver, and mercury. Therefore we are following this plan; and to compute this function we are averaging, for each U, the three almost-equal values of $\Phi(U)/\Phi(3)$ tabulated for these three elements in his paper. For the scale factor for each element we are using his absolute $\Phi(3)$ for that element. The resulting theoretical curve for nickel is compared with our experimental points in Fig. 3.

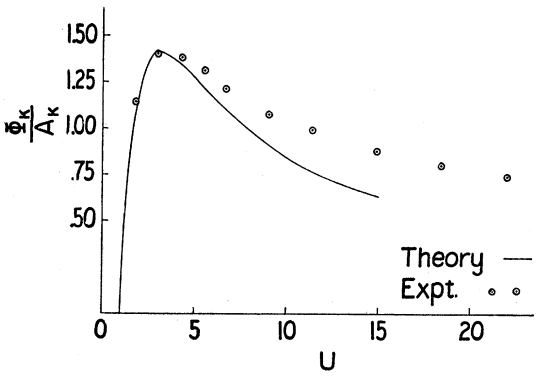

FIG. 3. Comparison of Burhop's theory with experimental cross sections for K ionization of nickel atoms.

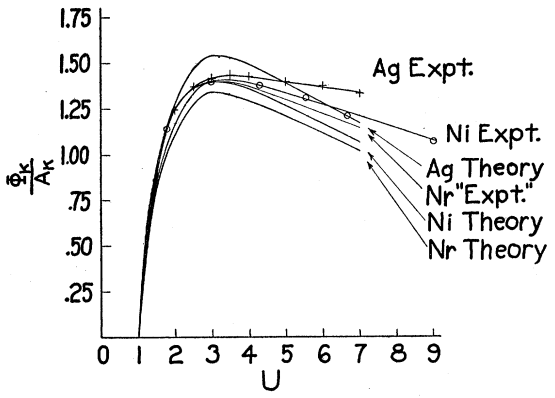

FIG. 4. Comparison of Ag with Ni, showing the effect of relativity; and graphs obtained by extrapolation for a hypothetical non-relativistic element Nr, showing how Burhop's non-relativistic theory fits this element best.

As will be seen there, the agreement is remarkably good, and especially good near the maxima of these functions.

VIII. THE EFFECT OF RELATIVITY

Only at high U's is there any serious disagreement in Fig. 3 between theory and experiment; and it is at these U's that we must expect the greatest effect of relativity, which did not enter into the theory. These facts raise two questions: (1) what is the effect of relativity; and (2) how well would the theory agree with experiment if we could find a strictly non-relativistic element with which to test it?

To answer these questions, in Fig. 4 we are comparing nickel with silver, for which an electron at any given U has almost exactly 3 times as much kinetic energy. At kinetic energies small in comparison with mc^2, most quantities which are changed by relativity in ways independent of the direction of motion are changed from their non-relativistic values by fractions thereof, which are roughly proportional to the kinetic energy. Consequently we may expect the K ionization cross sections Φ_K at any given U to follow this rule; and so will the characteristic areas A_K, and the ratios $F = \Phi_K/A_K$. Moreover the non-relativistic F, like $f(U_H)$ in the preceding section, should be practically the same, at any given U, for both elements. Therefore, at the same U, F_{Ag} should be about 3 times as far from this non-relativistic F

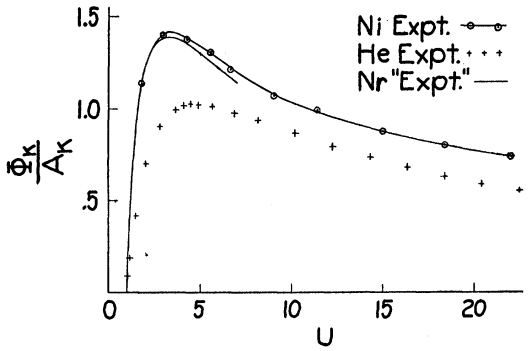

FIG. 5. Comparison of Smith's cross sections for He with those of Ni and the hypothetical non-relativistic element Nr, showing how something, presumably the electron which remains in the He atom, gives He abnormally small ionization cross sections at low U's.

as F_{Ni}. Half the difference $(F_{Ag}-F_{Ni})$, then, should be a fair measure of the effect of relativity in nickel. Roughly, this answers Question (1).

Moreover, it furnishes a basis for answering Question (2). If this measure of the relativity effect is deducted from the observed F for nickel, the result should be the F for the hypothetical non-relativistic element which we should like to use for testing the theory. We shall denote this element by Nr, for non-relativistic, and call the graph for it, derived in this way from the data on silver and nickel, its experimental graph. This graph and the corresponding one from Burhop's theory are shown along with those of the real elements in Fig. 4.

Here it should be noted again that the scale factors are more uncertain than the shape factors, both in experiment and in theory. In experiment the scale factors come from the absolute cross sections. Clark estimated the probable error of his absolute cross section for silver at 70 kv as 9 percent, and we have recomputed his cross section here by using more recent data on the fluorescence yield of silver[22] which we take to be 0.81 instead of 0.72, and on the ratio of the total number of K quanta to the number of $K\alpha$-quanta[23] (1.21 instead of 1.16), thus reducing all absolute cross sections by nearly 8 percent. Smick and Kirkpatrick estimated their probable error for the absolute cross section for nickel at 6 percent. The probable errors in *relative* cross sections are considerably smaller. On the theoretical side also, the scale factor is uncertain, because of the adjustment for the fact that $V_K \neq V_H$; and the extent of this inequality, as noted in Section VII, depends on both relativity and outer electrons. Altogether, the scale factor in Nr seems rather uncertain.

Regarding it therefore as somewhat arbitrary, it is evident that with a change of only a few percent in scale factor the theoretical non-relativistic curve would fit the experimental Nr points extremely well.

IX. THE OTHER K ELECTRON

In elements as heavy as nickel the use of the screening constant 0.30 is probably satisfactory, while the theory is in its present stage of approximation, as an allowance for the effect of the K electron which remains in the atom. This rule holds so long as $0.30 \ll Z$; but in helium the forces exerted on the electrons by the nucleus are so weak that the motion of the remaining electron cannot be neglected, and an allowance for it as a static $0.30e$ is no longer satisfactory. To see what effect it has, therefore, in Fig. 5 we are comparing our data on nickel with those of Smith's[6] data on helium which are within our range of U; and we are replotting also the "experimental" graph for our hypothetical non-relativistic element.

Evidently the helium cross sections, when expressed in the unit A_K, differ notably from those of nickel. Moreover, they show no signs of approaching agreement at U's above our range. With the hypothetical non-relativistic element, on the other hand, helium differs notably at low U's, but their graphs in Fig. 5 look likely to come into agreement at high U's.

[22] E. Arends, Ann. d. Physik **22**, 281 (1935), value 0.795 ± 0.024; R. J. Stephenson, Phys. Rev. **51**, 637 (1937), value 0.81; I. Backhurst, Phil. Mag. **22**, 737 (1936), value 0.838.

[23] J. H. Williams, Phys. Rev. **44**, 146 (1933).