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Theoretical Range-Energy Values for Protons in Air and Aluminum

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In this paper there are presented a discussion of the calculation of range-energy values for protons in air and aluminum and a tabulation of the results. The calculations have been done with considerable accuracy and at sufficiently small energy intervals to allow good graphical interpolation. These figures, as well as those for the rate of energy loss in both cases, have been compiled over a wide range of energies—up to 10^{10} ev.

IN view of the efforts currently being given to the production of very high energy protons and the general interest in mesons of various energies, it will be convenient to have readily available a fairly extensive tabulation of range-energy relations for such particles. Protons up to 10^{10} ev in air and aluminum have been chosen in this instance; the range of mesons up to 10^9 ev can be deduced easily from Tables I and II. Calculations have been made previously to this to obtain similar relations[1, 2]; it is intended herein to provide data compiled either with more careful numerical methods, or with extension of certain energy ranges for the sake of better accuracy, or both.

The ranges proposed are those a proton would have under the circumstance that it lose energy along its path solely through ionization and excitation of the atoms of the stopping material. These processes certainly constitute the most important mechanism for slowing-down over a rather wide range of energies; for example, meson production will not take place until a proton has at least 100 Mev and probably does not become important until a much higher energy. In addition to total ranges, the rate of energy loss by ionization and excitation will be of interest; these figures are included in the tabulation.

The figures in this paper represent *mean* ranges since they are calculated from an expression which was derived to give the average energy loss per unit thickness of stopping material. For a brief review of the theoretical derivation, see references 1 and 2. The complete expression is[3]

$$\frac{dE}{dx}=-\frac{4\pi(ez)^2e^2N}{mv^2}\times\left\{Z\left[\ln\frac{2mv^2}{I}-\ln(1-\beta^2)-\beta^2\right]-C_K\right\},$$

where

ez = charge of the incident particle,
v = velocity of the incident particle,
N = number of atoms per cm^3 of stopping material,
Z = atomic number of stopping material,
I = average ionization potential of stopping material,
m = electron mass,
$\beta=v/c$, c the velocity of light,
C_K is a correction term which must be applied in case v is comparable to the velocity of the K-electron of the stopping material (but large compared to that of all others). Reference 3 gives a discussion of its calculation and figures.

There are several restrictions placed on the validity of this form for dE/dx, the most important of which are (1) that the incident particle be much more massive than an electron, (2) that the incident energy be much less than M^2c^2/m where M is the mass of the incident particle and m that of an electron (less than 10^6 Mev for protons), (3) that the proton have a velocity considerably greater than that of L electrons in the stopping atoms, and (4) that the proton energy be large enough so that electron capture and loss are of no consequence. A minimum figure suggested for the last of these is 0.1 Mev. Thus, all conditions are seen to be well satisfied for a proton with an energy between 15 Mev and 10^4 Mev in air and Al.

Let us consider the ranges of protons in air. An accurate curve of E *vs.* $R(E)$ up to 15 Mev for this case is presented in Fig. 28 of the

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[1] B. Rossi and K. Greisen, Rev. Mod. Phys. **13**, 249 (1941).

[2] J. Wheeler and R. Ladenburg, Phys. Rev. **60**, 761 (1941).

[3] M. S. Livingston and H. Bethe, Rev. Mod. Phys. **9**, 263 (1937).

Livingston-Bethe article. To find ranges beyond this point, one merely evaluates

$$R(E)=R(15)+\int_{15}^{E}\frac{dE'}{-dE'/dx}.$$

Straightforward numerical integration was found to be the only satisfactory procedure. Even for the low energies considered it proved to be not sufficiently accurate and also too laborious to perform the calculation by reducing the integral to the difference of standard exponential integrals.

TABLE I. Rate of energy loss and ranges for protons in air.

E (Mev)	$-dE/dx$ (Mev/cm)	R (cm)	E (Mev)	$-dE/dx$ (Mev/cm)	R (cm)
15	3.574×10^{-2}	2.385×10^{2}	400	0.3308×10^{-2}	754.4×10^{2}
17	3.230	2.975	500	0.2994	1073
19	2.951	3.623	600	0.2789	1420
21	2.721	4.329	700	0.2649	1788
23	2.529	5.092	800	0.2547	2174
25	2.364	5.910	850	0.2507	2372
30	2.040	8.194	900	0.2472	2572
35	1.801	10.81	950	0.2442	2776
40	1.617	13.74	1000	0.2416	2982
45	1.472	16.99	1250	0.2325	4038
50	1.352	20.53	1500	0.2277	5126
60	1.173	28.49	1750	0.2252	6231
70	1.040	37.56	2000	0.2240	7344
80	0.9393	47.69	2250	0.2236	8462
90	0.8594	58.83	2500	0.2237	9580
100	0.7943	70.95	2750	0.2242	10,700
120	0.6950	97.95	3000	0.2250	11,810
140	0.6227	128.5	4000	0.2290	16,220
160	0.5678	162.1	5000	0.2335	20,540
180	0.5246	198.8	6000	0.2378	24,790
200	0.4899	238.3	7000	0.2419	28,950
250	0.4265	348.2	8000	0.2457	33,060
300	0.3840	472.1	9000	0.2492	37,100
350	0.3526	608.0	10,000	0.2524	41,080

TABLE II. Rate of energy loss and ranges for protons in aluminum.

E (Mev)	$-dE/dx$ Mev/ (mg-cm^{-2})	R (mg-cm^{-2})	E (Mev)	$-dE/dx$ Mev/ (mg-cm^{-2})	R (mg-cm^{-2})
1		3.45	60	0.8458×10^{-2}	39.83×10^{2}
1.5		6.69	70	0.7516	52.40
2	11.5×10^{-2}	10.8	80	0.6794	66.42
2.5	9.85	15.6	90	0.6222	81.82
3	8.62	21.0	100	0.5757	98.54
3.5	7.69	27.3	120	0.5047	135.8
4	6.96	34.5	140	0.4530	177.7
4.5	6.37	42.1	160	0.4136	224.0
5	5.88	50.3	180	0.3826	274.3
5.5	5.47	59.0	200	0.3576	328.4
6	5.12	69.1	250	0.3120	478.7
6.5	4.82	79.2	300	0.2813	648.0
7	4.55	90.0	350	0.2593	833.4
7.5	4.31	101.3	400	0.2428	1033
8	4.10	113.2	500	2.201×10^{-3}	1.467×10^{5}
8.5	3.92	125.6	600	2.054	1.938
9	3.75	138.8	700	1.952	2.438
9.5	3.59	152.4	800	1.879	2.961
10	3.45	166.7	850	1.851	3.229
10.5	3.32	181.4	900	1.826	3.501
11	3.21	196.6	950	1.802	3.777
11.5	3.10	212.5	1000	1.785	4.055
12	2.99	229.0	1250	1.721	5.484
12.5	2.90	246.1	1500	1.688	6.952
13	2.816[a]	263.7	1750	1.671	8.441
13.5	2.734	281.8	2000	1.664	9.941
14	2.658	300.6	2250	1.663	11.44
15	2.518	3.393×10^{2}	2500	1.665	12.95
17	2.281	4.228	2750	1.670	14.45
19	2.089	5.146	3000	1.677	15.94
21	1.930	6.143	4000	1.710	21.85
23	1.796	7.218	5000	1.747	27.63
25	1.682	8.369	6000	1.782	33.30
30	1.456	11.57	7000	1.815	38.86
35	1.289	15.23	8000	1.845	44.32
40	1.160	19.33	9000	1.873	49.70
45	1.058	23.85	10,000	1.898	55.01
50	0.9743	28.78			

[a] For remarks on the validity of dE/dx for $E<13$ Mev, see discussion at the end of the paper.

Values of the constants c, e, m, and N were taken from the tables of Birge,[4] the last being made to correspond to conditions of 760 mm Hg and 15°C. The proton mass, M_Pc^2, was computed from these tables to be 937.6 Mev. The average ionization potential in air, 80.5 ev, and $Z=7.22$ are the values as found in reference 3. dE/dx is rather insensitive to changes in I; for small values of E (≈ 15 Mev), a 3 percent change in I is reflected as about 0.5 percent in dE/dx, the sensitivity decreasing with increasing energy.

A similar procedure was followed in calculating the range of protons in Al. For energies up to 13 Mev, they are obtained from the work of Livingston and Bethe; see Figs. 30 and 34 of that article and the accompanying text. Beyond this point, numerical integration was again employed. The same data for C_K as used before are here also satisfactory. This correction due to K ionization is more important initially for Al than for air since the binding energy and velocity of the innermost electrons are greater for Al by a considerable amount. Experiments by Wilson[5] on the stopping power of Al relative to air provide a value for the average ionization potential of 150 ev.

The values for the rate of energy loss and range as given in the following tables are believed to be correct (consistent with the constants cited in this paper) to a few parts in the fourth figure where this is given. There is a serious question as to the validity of the values of dE/dx in Al for very low energies. The condition

$$E\gg(M_P/m)\times(\text{ionization potential of } L \text{ electrons})$$

is not very well satisfied in the neighborhood of $E\approx 2$ Mev. This indicates that a correction "C_L" should be applied, but it is not available. However, in the range above 13 Mev, where ranges are obtained using the theoretically computed energy loss, C_L will have become completely negligible.

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[4] R. T. Birge, Rev. Mod. Phys. **13**, 233 (1941).

[5] R. R. Wilson, Phys. Rev. **60**, 749 (1941).