10. It is shown that the reflectivity may be decreased if the atoms in the crystal are polyisotopic and if the isotopes for a given element are present in comparable abundance, possess nearly equal scattering cross sections and opposite phases. More explicitly the effective scattering strength of a given element is proportional to

$$\sum_i \pm g_i(\sigma_s^i)^{\frac{1}{2}},$$

in which g_i is the fractional abundance of the ith isotope, and σ_s^i is its scattering cross section. The contribution from isotopes with opposite phases tend to cancel one another. The presence of isotopes may also add a diffuse background to the inelastically scattered radiation.

11. It should be emphasized that the present paper does not contain a discussion of the coherent inelastic scattering of neutrons from crystals. This type of scattering actually can be very important in practical cases and can alter some of the qualitative viewpoints which were obtained.

We are indebted to our colleagues, L. Borst, S. M. Dancoff, E. P. Wigner, E. O. Wollan, W. H. Zachariasen, and W. H. Zinn for many interesting discussions of this topic.

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A Proposed Focusing Cosmic-Ray Telescope

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A magnetic lens which will focus charged particles entering its aperture parallel to the axis can be constructed in the form of a toroidal winding. For an air or iron core toroid, the cross section of the winding is parabolic. If partial iron filling is used, trapezoidal or rectangular cross sections may be employed to produce sharp focusing. The focal length is proportional to the particle momentum, and hence a telescope of this type is also a spectrograph. By using coincidence counters and anticoincidence guard counters for observations, a collimating tube effect can be obtained, permitting full use of the properties of the telescope lens without a background due to particles which do not pass through the lens. There are some advantages to an installation in a deep shaft in the earth having two lenses. With such a device charged and uncharged components can be studied separately.

INTRODUCTION

As a consequence of Maxwell's equation

$$\text{curl } H = 4\pi \text{i},$$

it is clear that the magnetic field strength inside a long straight solenoid is uniform over the cross section of the solenoid, regardless of the shape of the cross section. If the solenoid is formed into a toroid, H varies inside the cross section as $1/r$, regardless of the shape of the cross section of the toroidal winding.

If a toroidal coil is to serve as a lens to focus a parallel beam of particles of like energies and e/m through the point F (Fig. 1), it is necessary that the radial momenta imparted to the particles increase directly as r. This can be accomplished in either of two ways.

(1) Air core toroid. Since in an air core toroid the field varies as $1/r$, the width of the cross section must increase as r^2, in order that the time in which a particle is accelerated radially may compensate the decrease in field strength and yield a radial impulse which is proportional to r. This leads to a toroidal lens having the parabolic shape shown in Fig. 1.

(2) Toroidal lens containing iron. If iron is distributed inside the toroid, and the iron is regarded as being transparent to the particles, several choices of lens design are possible. In this case we must deal with the continuous magnetic

flux B in the magnetic circuit, rather than the magnetic field H.[1]

(a) The interior of the toroidal coil may be uniformly filled with iron. The resulting flux distribution is the same as for the air core case (1) above. (The situation is actually the same as if the permeability of all space were changed by a uniform amount.) Thus the only advantage of the use of iron in this way is the large saving in power required to operate the lens, due to the smaller current necessary to obtain the desired flux.

This type of lens would not be very useful when small magnetic fields are required. In addition to the collision effects in the iron, poor reproducibility of results might constitute a further disadvantage. Since the magnetic circuit would be completely closed, the demagnetizing force would be low, and the slight coercive force of the material (*ca.* 0.1 oersted) would be sufficient to prevent the flux from being a unique function of the magnetizing field at low flux densities. However, complete iron filling will be necessary for particles of very high momentum, such as 10^{10}-volt protons.

(b) The design is more flexible, and the above objections are overcome, if the toroid is only partially filled with iron. If, for example, the volume inside the winding is half filled with iron for which $\mu \gg 1$ (ca. 3000), the interior can be regarded as having a permeability of 2 for the purpose of calculating total flux. We are now privileged to choose to make the flux density uniform inside the toroid, which requires that the toroid have a trapezoidal cross section for focusing. The iron and air volumes would alternate, the iron pieces having the shape of trapezoidal wedges, and the air gaps would be trapezoids of uniform width, as shown in Fig. 2. A large number of relatively thin wedges should be used to reduce fringing flux effects.

(c) A toroid of rectangular section can be used if the iron is so disposed that the flux density increases directly as r. This result can be achieved by increasing the slope of the wedges, so that the air gaps become narrower in proportion as r is increased. See Fig. 3.

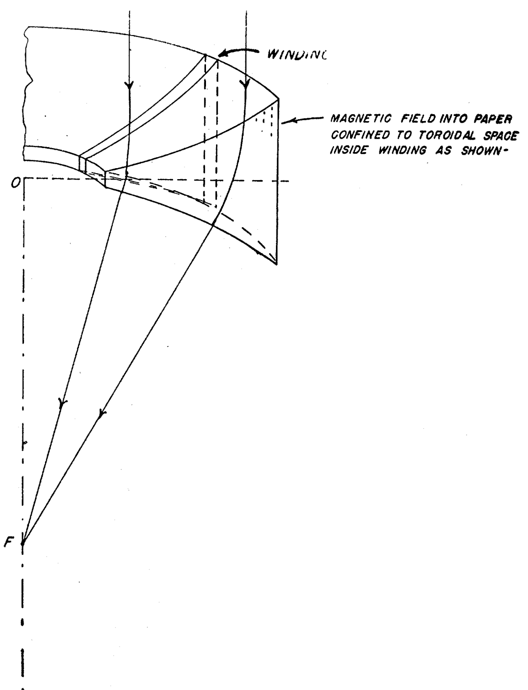

FIG. 1. For an air core lens or a homogeneously iron filled lens, the toroidal magnetic field has a parabolic cross section.

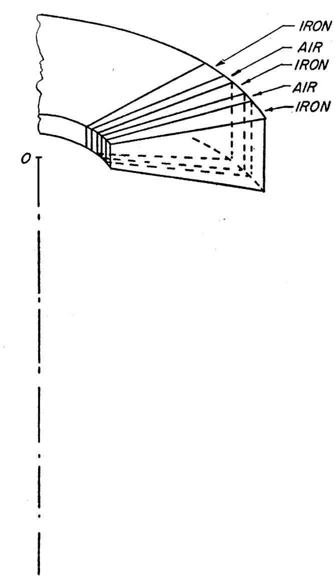

FIG. 2. Toroidal magnetic lens partially iron filled so as to produce a constant flux density. In this case the lens thickness must increase directly as r.

[1] Recently some doubt has been raised regarding the equivalence of the interaction of charged particles with a given magnetic flux inside iron and in space. If these proposals had any real basis the two partially iron filled forms of telescope lenses described below would be unsatisfactory. See G. Wannier, Phys. Rev. **67**, 364 (1945) and D. L. Webster, Phys. Rev. **70**, 446 (1946).

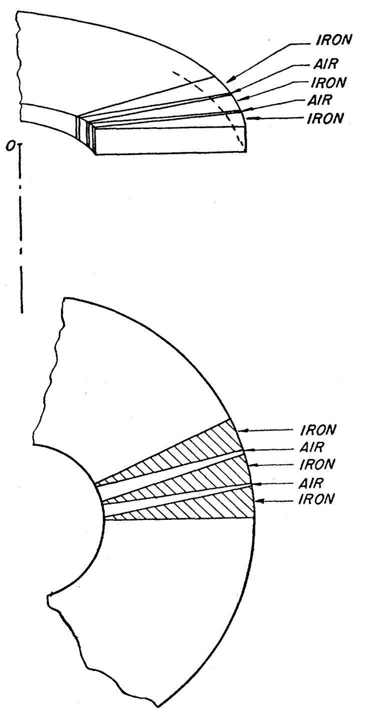

FIG. 3. Partially iron filled lens in which the magnetic flux increases as r.

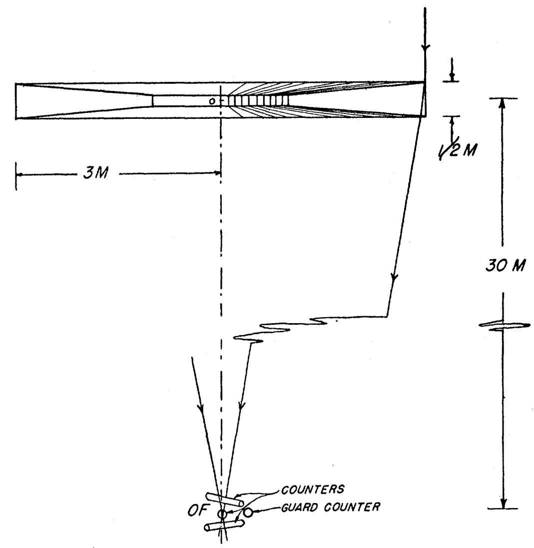

FIG. 4. Sketch illustrating computed example. Counter arrangement to prevent detection of particles which do not pass through the lens is shown. Instead of cylindrical counters, parallel disks with a few fine wires in the median plane between them would be more suitable.

In order to test the practicability of the toroidal lens the magnetic field strength requirements are of primary interest. As an example, we assume an $f:5$ lens for 10^8-volt protons 50 cm thick at the outer periphery and half filled with iron. (See Fig. 4.) Taking β as 0.44, the time to traverse the magnetic field at the outer edge of the lens is given by $50=0.44ct$, or $t=3.8\times10^{-9}$ sec. The radial momentum, p_r, imparted to the particle is $(Bev/c)t$. If we set this equal to 0.1 of the momentum of the particle $H\rho e/c$, we have $3.8\times10^{-9}Bv=0.1H\rho$. For 10^8-volt protons $H\rho$ is approximately 1.4×10^6. Hence the magnetic flux required is

$$B=1.4\times10^5/(3.8\times10^{-9}\times1.3\times10^{10})=2800 \text{ gauss.}$$

Hence the requirements are reasonable, although at energies between 10^9 and 10^{10} ev it may become difficult to make the focal length for heavy particles short enough to preserve the usefulness of the device.

CONSTRUCTION

As visualized at present, the magnetic lens should consist of a large wound doughnut, preferably with the windings open and arranged for forced air cooling. Square section copper bars would be employed for the winding, and the size of the center hole would be determined by the construction. The windings might be single layer, close wound at the inner limit, and spaced at the outer periphery, or they might be multilayer at the inner limit with a smaller number of layers at the outer limit. The use of iron in the lens has previously been discussed. For studying low energy particles an air core lens would be desirable, but for studying very high energy particles iron would be desirable.

For most purposes, lenses probably should be rather large. A $\frac{1}{2}$- to 1-meter radius for the inner hole, and a 3-meter radius to the outer periphery is suggested. This would have a useful focal range of perhaps 5 to 50 meters. The projected area of the active lens would be of the order of 10 square meters. This would give an expected concentration of a homogeneous component coming from above of approximately 5000, if good focusing and a coincidence counter bank of 20 cm² active area are assumed. (It should be noted, however, that the earth's magnetic field will displace and blur the focal spot. A rather clumsy Helmholtz type coil arrangement can be used to approximately neutralize the earth's field in the telescope "tube" and remove this difficulty.) This concentration, the high directivity, and the chromatic abberation which renders the device a spectrometer are the particular advantages of the device.

The power required to produce a uniform magnetic flux of 2800 gauss in a toroid of 3 meters outer radius and center hole of 1 meter radius, the toroid being half filled with iron, and single layer wound with 1″ square copper bars (or an equivalent volume of copper) as suggested above is 970 kw. If the toroid were completely filled with iron the power required would be less than 1 kw. Increasing the volume of copper decreases the power consumption in direct proportion.

It is well to point out that the above discussion has been confined to particles which approach the lens parallel to its axis. In order to study particles which approach at only a small angle to the axis in an experimental set-up it is necessary to eliminate detection of particles which come from other directions. This can be accomplished by two different methods. (1) Banks of coincidence counters which count only particles passing through the aperture of the lens can be employed, together with off-axis anticoincidence counters to eliminate counts due to showers. For the coincidence counters parallel plate pancake counters with an open grid of wires in the central plane are suggested. In this way a collimating tube effect is obtained, and full use of the directivity, dispersion, and focusing effect of the lens may be utilized. Experiments using absorbers between the counters would assist in determining the masses of the particles.

(2) A second collimation method which might be feasible would utilize a deep mine shaft in which to install the telescope. Some such sites may be available at moderate elevations above sea level.

The mine shaft type of installation, of course, could not be trained. However, it apparently has advantages. For example, if the shaft were sufficiently deep, two telescope lenses could be installed, one above the other. (See Fig. 5.) The upper lens could be used to study charged particles and as a filter to deflect charged particles out of the aperture of the lower lens, and hence permit more clearcut use of the lower telescope, with an absorber above the lens, to study uncharged components.

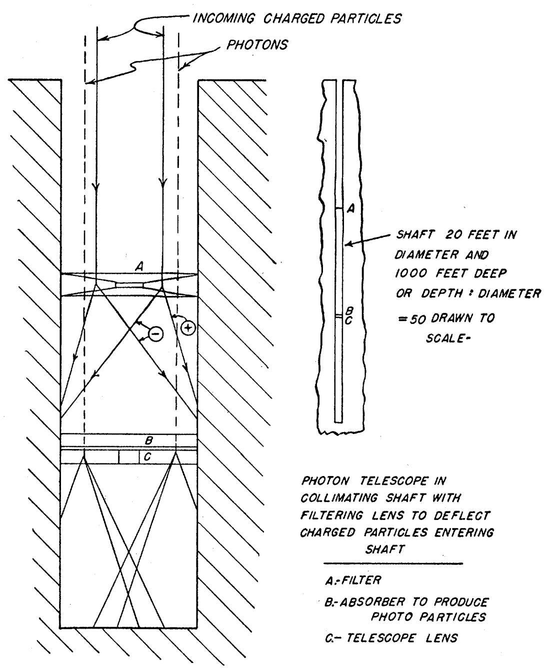

FIG. 5. Sketch of a proposed installation in a deep shaft employing two lenses. Lens A serves as a charged particle spectrograph and also deflects charged particles out of the "tube" so that the absorber B and lens C can be used to study uncharged components.

If there were any reason to believe that an uncharged primary cosmic-ray component existed, this feature might be particularly important, since it would permit the study of discrete sources, if such sources exist, without the complicating effects of magnetic fields in space. In any case, however, the telescope with two lenses should be particularly useful in studying uncharged secondary radiation.