At 0°K, the electron velocity distribution is uniform within a sphere of radius $(2\mu_0/m)^{\frac{1}{2}}$ in velocity space, where μ_0 = maximum energy of unexcited electrons, and m = electron mass. Assume that the energy of the photon adds to the electron energy in such a way that it increases only the component of electron velocity normal to the surface element. And assume further an excitation probability α proportional to the normal component of the electron velocity. Considering the transformations of the sphere by the excitation energy $h\nu$, the surface potential barrier W_a, and the retarding potential V_e applied at an angle δ to a surface element, one obtains an equation that is readily soluble when $V_e \gtrsim (\mu_0 + h\nu - W_a)\sin^2\delta$. The current from an element is

$$i = (2\pi e m\alpha/h^3)\cos\delta \cdot [\mu_0 + h\nu - W_a - V_e]^2,$$

which is just DuBridge's expression multiplied by $\cos\delta$.

The photoelectric current from a rough surface at 0°K thus shows the same dependence on retarding potential and on energy as that from a smooth surface, providing the surface is not too rough, and the retarding potentials are not too low. Assuming a maximum angle of 45° between surface element normals and the normal to the cathode, the formula is valid for $i \lesssim (2^{\frac{1}{2}}\pi e m\alpha/h^3)(V_e)^2$.

The equation for $V_e < (\mu_0 + h\nu - W_a)\sin^2\delta$ does not reduce to a simple form. From the equations it does follow, though, that the current will be less than DuBridge's expression, indicating that photoelectric saturation current curves for rough surfaces will be rounded even at 0°K.

The writer has examined the data of Overhage[2] on normal energy distribution to see whether the above arguments could explain any of the discrepancy between theory and his curves. They do not.

The writer wishes to express his indebtedness to Dr. W. V. Houston who suggested looking into this problem, and also suggested the mathematical treatment used.

* This work was done at the California Institute of Technology n 1941.

[1] L. A. DuBridge, Phys. Rev. **43**, 727 (1933).
[2] C. F. J. Overhage, Phys. Rev. **52**, 1039 (1937).

Neutron Induced Activities in Lutecium, Ytterbium, and Dysprosium

Mark G. Inghram, Richard J. Hayden, and David C. Hess, Jr.
Argonne National Laboratory, University of Chicago, Chicago, Illinois
January 17, 1947

Nitric acid solutions of Lu_2O_3, Yb_2O_3, and Dy_2O_3 were irradiated with neutrons in the Argonne heavy water pile. Decay curves of the irradiated lutecium sample showed 3.4-hour and 6.6-day activities as reported by Flammersfeld and Mattauch.[1] Decay curves of the ytterbium sample showed a 2.7-hour activity (probably the same activity reported by Marsh and Sugden[2] as 3.5 hours) and also a 102-hour activity not previously reported. No evidence of the 41-hour activity reported by Pool and Quill[3] was found. Decay curves of the dysprosium sample showed the 2.5-hour activity reported by Marsh and Sugden.[2]

Assignment of mass to three of these active isotopes was made as follows. A portion of the sample containing the activity in question was placed on the filament source of a mass spectrograph. Operation of the spectrograph separated the isotopes and deposited them on a photographic plate. The position of the active isotope on this plate was determined by placing a second photographic plate face to face with the original. The disintegration of the active isotopes on the first plate produced developable images on both plates. Comparison of the position of this active deposit with the normal mass spectrum on the original plate determined the mass of the active isotope. This method of mass assignment gives results without the necessity of assuming the nuclear reaction by which the isotope is formed and is therefore frequently valuable in establishing the nuclear reaction. Using these techniques the following masses were assigned: 6.6-day lutecium, 177; 102-hour ytterbium, 175; and 2.5-hour dysprosium, 165.

[1] Flammersfeld and Mattauch, Naturwiss. **31**, 66 (1943).
[2] Marsh and Sugden, Nature **136**, 102 (1935).
[3] M. L. Pool and L. L. Quill, Phys. Rev. **53**, 437 (1938).

Interpretation of Anomalous Larmor Frequencies in Ferromagnetic Resonance Experiment

Charles Kittel*
Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts
January 10, 1947

Recently J. H. E. Griffiths[1] reported an important new ferromagnetic resonance experiment at microwave frequency. The experiment is essentially the ferromagnetic analog of the Purcell-Torrey-Pound nuclear resonance experiment. Griffiths found however the unusual result that the resonance frequencies he observed were greater than the calculated Larmor frequencies for electron spin by factors of about two to six. He attempted unsuccessfully to explain the anomaly by the introduction of the Lorentz cavity force, a step which is definitely not justified.

A theory of the resonance effect is given below which leads to values of the resonance frequency in close agreement with the experimental determination. It is shown that it is important to consider the dynamic coupling caused by the demagnetization field normal to the surface of the specimen. The result is that the appropriate Larmor frequency should be calculated as for a field strength $(BH)^{\frac{1}{2}}$ rather than for H.

A ferromagnetic specimen whose surface is the plane $y=0$ is subjected to a strong d.c. field H_z and a weak microwave field H_x. The magnetization $\mathbf{M}$ and the angular momentum density $\mathbf{J}$ are related by $\mathbf{M}=\gamma\mathbf{J}$, where γ is the gyromagnetic ratio. The equation of motion $\partial\mathbf{J}/\partial t = [\mathbf{M}\times\mathbf{H}]$ may be written

$$\partial\mathbf{M}/\partial t = \gamma[\mathbf{M}\times\mathbf{H}], \qquad (1)$$

where the components of $\mathbf{H}$ are $(H_x, -4\pi M_y, H_z)$. Here $-4\pi M_y$ is written for H_y on the basis of the divergence relation $B_y = H_y + 4\pi M_y = 0$. The Lorentz and Weiss fields are omitted as they are always parallel to $\mathbf{M}$, and hence their vector product with $\mathbf{M}$ is identically zero. It is assumed that saturation obtains, and in fact that the d.c. field is sufficiently strong to outweigh the magnetic

TABLE I. Comparison of observed and calculated resonance frequencies.

	d.c. field $H_z(Oe)$	Angular frequencies for resonance: (rad/sec.) $\times 10^{-10}$ Calculated Larmor frequencies for H_z	for $(B_zH_z)^{\frac{1}{2}}$	Experimental frequencies
Fe	2800	5.0	14.5	15.4
	500	0.9	5.8	5.9
Co	510	0.9	5.3	5.9
Ni	5000	8.8	13.5	15.4
	3800	6.7	10.9	13.2
	1030	1.8	4.9	5.9

anisotropy forces so that in the static case the spin system always points in the direction of the resultant magnetic field. In this limit the whole specimen is supposed to behave as a single domain, so that there are no complications from the domain boundaries.

We have, neglecting products of small quantities,

$$\partial M_x/\partial t=\gamma(M_yH_z+4\pi M_yM_z)=\gamma B_zM_y; \quad (2)$$

$$\partial M_y/\partial t=\gamma(M_zH_x-M_xH_z); \quad (3)$$

$$\partial M_z/\partial t\cong 0; \quad (4)$$

which gives $-\omega^2M_x=\gamma^2B_z(M_zH_x-M_xH_z)$. The susceptibility $\chi_x=M_x/H_x$ is given by

$$\chi_x=\chi_0/[1-(\omega/\omega_0)^2], \quad (5)$$

where the frequency for resonance is found to be $\omega_0=\gamma(B_zH_z)^{\frac{1}{2}}$ and is the Larmor angular frequency in the fictitious field $(B_zH_z)^{\frac{1}{2}}$. The static susceptibility χ_0 is equal to M_z/H_z and varies with the biasing field; here M_z may be taken as the saturation magnetization.

In Table I Griffiths' experimental values of ω_0 are compared with the calculated Larmor frequencies for fields H_z and $(B_zH_z)^{\frac{1}{2}}$. The gyromagnetic ratio is taken as the electron spin value. The induction B_z is calculated using 1700, 1400, and 500 as the saturation magnetization for Fe, Co, and Ni, respectively. The theory given here is seen to account quite well for the experimental results, considering that damping and anisotropy forces have been neglected.

The effect of relaxation is introduced by adding a term $-\lambda(\mathbf{M}-\chi_0\mathbf{H})$ to the right side of Eq. (1). We find

$$\frac{\chi_x}{\chi_0}=\frac{\omega_0^2+\lambda^2\mu_0+j\omega\lambda}{\omega_0^2+\lambda^2\mu_0-\omega^2+j\omega\lambda(1+\mu_0)}, \quad (6)$$

where $\mu_0=B_z/H_z$. From this it is seen that the demagnetizing field acts to increase the apparent damping.

The interpretation of a complex susceptibility in terms of the result of an electrical measurement is somewhat involved, but may be accomplished by the reasoning previously given.[2] The apparent permeability for a resistive measurement (such as a "Q" measurement) is $\mu_R=|\mu_x|(1-\sin\phi)$, where μ_x is the permeability derived from χ_x, and ϕ is the phase angle of μ_x; ϕ is always between 0 and $-\pi$.

* John Simon Guggenheim Memorial Foundation Fellow.

[1] J. H. E. Griffiths, Nature 158, 670 (1946).
[2] C. Kittel, Phys. Rev. 70, 281 (1946).

Biased Betatron in Operation

W. F. WESTENDORP
Research Laboratory, General Electric Company, Schenectady, New York

January 22, 1947

A BIASED betatron[1-3] with closed central core has been successfully operated. The guiding magnetic field consists of a direct-current field component with a sinusoidal component superimposed, while the closed central core carries only a sinusoidal component of flux maintained at the proper magnitude by the current in a bucking coil located in grooves in the polefaces. Figure 1 shows a schematic cross section of the machine and Fig. 2 is a diagram of the principal electrical components of the energizing circuit.[1] The turn ratio of main coils to bucking coil is chosen so that the proper flux ratio and thereby the betatron 2:1 rule is fulfilled. Although radically different from previous betatrons in which the 2:1 rule was established by means of air gaps, this principle of locating the electron orbit offered no difficulties. It has the advantage of a reduction of the size of the capacitor bank since less magnetic energy storage takes place in a machine with a closed central core. The application of a direct-current bias to the main coils to produce an increased guiding field and to the bucking coil to keep the constant component out of the central flux, proved to be not critical and was tried out successfully for all ratios between main coil direct-current and crest alternating-current between 0 and 0.866. The corresponding ratio of current components in the bucking coil was always smaller for maximum x-ray output, but not critical.

No compensating or phase correcting circuits of any kind were used. Even though the a.c. components in main and bucking coils are not exactly 180 degrees out of phase,[2] theory[3] predicts that the shift of the orbit produced is very small during most of the accelerating interval and this was checked experimentally by the use of an orbit tilt circuit by means of which the electrons are made to hit a target located above the orbit.

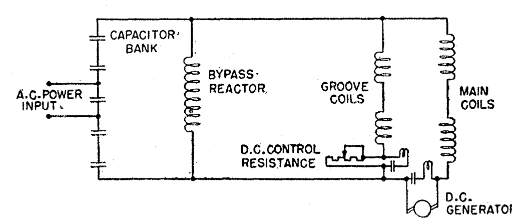

FIG. 1. Biased betatron with closed central core.

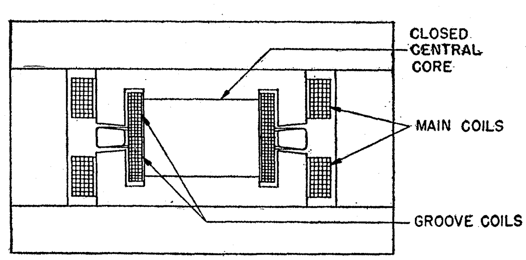

FIG. 2. Circuit diagram of biased betatron.